Nuclear Magnetic Moments and Their Origin

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The origin of the nuclear moments is in most cases one proton or one neutron only. The analysis of hyperfine structure leads to the following magnetic moments: proton ~ 2.0 magnetons and neutron ~ -0.6 magneton.

THE purpose of this article is to give a report on the theoretical interpretation of the observed mechanical momenta j (in units $h/2\pi$) and magnetic moments μ (in units $he/4\pi mc = 1$ nuclear magneton) of various nuclei throughout the periodic system. The theory was outlined already in a letter to the *Physical Review*[1] and applied to a group of cases. Meanwhile Tamm and Altschuler[2] extended our scheme to a new group of cases. The basic idea is this: *One* particle only, one proton or one neutron, is responsible for the total spin and the magnetic properties of the whole nucleus, the rest of it forming closed shells in general. The observed vector j is thus composed in general by the orbit l and the spin s of one particle only. Since s is known to be $\frac{1}{2}$ for the proton as well as for the neutron, l can be only $j \pm \frac{1}{2}$; hence there is no arbitrariness in this 2-vector model. This model fails, however, in some cases. The exceptions can be accounted for by the supplementary assumption of Tamm and Altschuler that here two neutrons line up their spins to $s=1$ instead of forming a closed shell $s=0$. The arbitrariness of this additional third vector $s=1$ is greatly reduced by assuming that it reappears in all cases. This theory is in contradiction to that of H. Schueler[3] who starts from the beginning with a 3-vector model taking the core as the third vector. Although Schueler uses the basic idea that one proton or one neutron is mainly responsible for j and μ beside the core, he ascribes, for instance, to the neutron a magnetic moment of -1.65 magnetons instead of Tamm-Altschulers and our value of about -0.5 or -0.6.

The theory is based on a division of all nuclei into four types belonging to different charge numbers Z and mass numbers M.

Type 1: Z even, M even. Nuclei of this type are most abundant. They consist of an *even* number of protons and an *even* number of neutrons. Since they show no mechanical momentum j and no magnetic moment μ at all we suppose them to consist of closed shells.

Type 2: Z odd, M odd. This type differs from the first one by one additional *proton* only. We suppose then that this one proton, by its orbit l and its spins, is responsible for the j and μ of the whole nucleus.

Type 3: Z even, M odd. This type differs from type 1 only by one additional *neutron*. We suppose then that this one neutron is responsible for the j and μ of the whole nucleus.

Type 4: Z odd, M even: This type differs from the closed shell type 1 by one additional neutron plus one proton. In general this type is unstable and occurs only in four instances $H_2{}^1$, $Li_6{}^3$, $B_{10}{}^5$, $N_{14}{}^7$, among them the deuton. Since here two different particles are cooperating it is not possible to obtain a unique interpretation of their j and μ values. Conversely the deuton would be the most inadequate object for an analysis of the properties of its two components. If one wants to know about the proton one has to inquire higher nuclei of type 2, and to know about the neutron one has to inquire type 3.

In some cases Tamm and Altschuler assume that one pair of neutrons does not form a closed shell. One has thus:

Type 2': Z odd, M odd, differing from the closed shell type 1 by one proton plus two neutrons. The latter are supposed to form $j_{2\nu}=1$ out of $s_{2\nu}=1$ and $l_{2\nu}=1$ in all cases.

Type 3': Z even, M odd, differing from type 1 by three neutrons forming $s_{3\nu}=\frac{3}{2}$.

[1] A. Landé, Phys. Rev. **44**, 1028 (1933).
[2] I. Tamm and S. Altschuler, Academy U.R.S.S. **1**, 455 (1934); D. Inglis and A. Landé, Phys. Rev. **45**, 842 (1934).
[3] H. Schueler, Zeits. f. Physik **88**, 323 (1934). A thorough discussion of the experimental g-values is given in Schueler's paper.

The theoretical g-values of type 2 and 3 are both taken from the generalized g-formula

$$g = g_l \frac{j(j+1)+l(l+1)-s(s+1)}{2j(j+1)} + g_s \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}. \quad \text{(a)}$$

In *type 2* we put $s=\frac{1}{2}$, $l=j\pm\frac{1}{2}$, $g_l=1$. To fit the observation best one has to choose $g_s \sim 4$, according to a *magnetic moment of the proton of about* $4\cdot\frac{1}{2}=2$ *magnetons.* In *type 3* we put $s=\frac{1}{2}$, $l=j\pm\frac{1}{2}$, $g_l=0$. To fit the observation best we choose $g_s=-1.2$ according to a *magnetic moment of the neutron of* $-1.2\times\frac{1}{2}=-0.6$ *magneton.*

To calculate g in *type 2′* for $j_\pi j_{2\nu}$-coupling T. and A. first calculate separately g_π for the proton and $g_{2\nu}$ for the neutron pair, and then insert g_π and $g_{2\nu}$ into the formula

$$g = g_\pi \frac{j(j+1)+j_\pi(j_\pi+1)-j_{2\nu}(j_{2\nu}+1)}{2j(j+1)} + g_{2\nu} \frac{j(j+1)+j_{2\nu}(j_{2\nu}+1)-j_\pi(j_\pi+1)}{2j(j+1)}. \quad \text{(b)}$$

In most cases of type 2′ there is an isotope of the same element belonging to type 2 where the two neutrons form still a closed shell. So $j_\pi s_\pi l_\pi$ and hence g_π are fixed already. Furthermore we assume in all cases of type 2′ $j_{2\nu}=s_{2\nu}=l_{2\nu}=1$, hence $g_{2\nu}=-0.6$. In *type 3′* one uses formula (a) with $s=\frac{3}{2}$ and $g_s=-1.2$ for 3 neutrons. This theory claims to represent the facts only in first order approximation. All finer traits brought about by the interaction of the one acting particle with the rest of the nucleus are neglected. It is significant nevertheless that the observed g-values can be well explained in this way, taking into account that the observed g-values claim at best 10 percent accuracy. For instance Goudsmit[4] calculated with his extrapolation formula the value $g=3.6$ for Tl and marked it even with the best grade A of accuracy, while Schueler[5] later decided for $g=2.94$ using the formula of Fermi-Segrè.[6] This may be kept in mind by judging the observed g-values graded B and C and the aim of getting better agreement between observation and theory by help of more complicated models.

To compare the theory with the observed g-values we give first the theoretical g-Tables I and II for various values of j and l. In Table I the number in the upper left of each square represents type 2, where one proton alone is responsible for g and μ. The number in the lower right of each square in italics accounts for type 2′, where according to T. and A. 2 neutrons participate with $s_{2\nu}=1$. In Table II the number in the upper left of each square represents type

TABLE I. *Theoretical g-values of type 2 (and type 2′) using* $\mu_\pi=2.0$ *and* $\mu_\nu=-0.6$.

l \ j	1/2	3/2	5/2	7/2	9/2
0	4 *−2.133*				
1	0 *−0.8*	2 *1.306*			
2		0.4 *0.133*	1.6 *1.35*		
3			0.57 *0.44*	1.43 *1.28*	
4				0.67 *0.59*	1.33 *1.24*
5					0.73 *0.68*

TABLE II. *Theoretical g-values of type 3 (and type 3′) using* $\mu_\pi=2.0$ *and* $\mu_\nu=-0.6$.

l \ j	1/2	3/2
0	−1.2	*−1.2*
1	+0.4 *−2.0*	−0.4 *−0.88*
2	*+1.2*	+0.24 *−0.24*
3		*+0.72*

[4] S. Goudsmit, Phys. Rev. **43**, 636 (1933).
[5] H. Schueler, Zeits. f. Physik **88**, 323 (1934).
[6] E. Fermi and E. Segrè, Zeits. f. Physik **82**, 729 (1933).

TABLE III. *Observed and theoretical g-values with $\mu_\pi = +2$ magnetons, $\mu_\nu = -0.6$ magneton.*

	j	g observed Schueler-Goudsmit		g theor.	coordination		
$Li_7{}^3$	3/2	2.19 (Breit)		2	$l_\pi = 1$		$j = j_\pi$, $l_\pi = j_\pi \pm 1/2$, $s_\pi = 1/2$ in type 2
$Al_{21}{}^{13}$	1/2	3.86	4.2 (B)	4	0		
$Cu_{63,\,65}{}^{29}$	3/2	1.82	1.7 (B)	2	1		
$Ga_{71}{}^{31}$	3/2	1.82	1.7 (A)	2	1		
$As_{75}{}^{33}$	3/2		0.6 (?)	0.4	2		
$Rb_{85}{}^{37}$	5/2	0.6	0.5 (C)	0.57	3		
$Rb_{87}{}^{37}$	3/2	2.04	1.8 (C)	2	1		
$Tn_{115}{}^{49}$	9/2	1.10	1.2 (B)	1.33	4		
$Sb_{123}{}^{51}$	7/2		0.7 (B)	0.67	4		
$Cs_{133}{}^{55}$	7/2	0.83		0.67	4		
$Tl_{203,\,205}{}^{81}$	1/2!	2.94	3.6 (A)	4?	0?		
$Bi_{209}{}^{83}$	9/2	0.80	0.89 (A)	0.73	5		
$Na_{23}{}^{11}$	3/2	1.42		1.31	$l_\pi = 1$,	$j_\pi = 3/2$	$s_{2\nu} = l_{2\nu} = j_{2\nu} = 1$ in type 2′
$Ga_{69}{}^{31}$	3/2	1.42		1.31	$l_\pi = 1$,	$j_\pi = 3/2$	
$Sb_{121}{}^{51}$	5/2	1.08	1.1 (B)	1.35	$l_\pi = 4$,	$j_\pi = 5/2$	
$Au_{197}{}^{79}$	3/2	0.15		0.133	$l_\pi = 2$,	$j_\pi = 3/2$	
$Cd_{113}{}^{48}$	1/2	−1.26	−1.33 (B)	−1.2	$l = 0$	$s = 1/2$	$s_\nu = 1/2$ in type 3, $s_{3\nu} = 3/2$ in type 3′
$Sn_{119}{}^{50}$	1/2	−1.90		−2	1	3/2	
$Ba_{137}{}^{56}$	3/2	+0.63?		+0.72	3	3/2	
$Hg_{199}{}^{80}$	1/2		+1.10 (B)	+1.2	2	3/2	
$Hg_{201}{}^{80}$	3/2		−0.41 (B)	−0.4	1	1/2	
$Pb_{207}{}^{82}$	1/2		+1.20 (A)	+1.2	2	3/2	

3 where one neutron alone is responsible for j and μ. The lower right numbers in italics account for type 3′, where 3 neutrons line up their spins to $s_{3\nu} = \frac{3}{2}$. The theoretical g-values of Tables I and II are drawn as horizontal bars in Figs. 1 and 2, the normal types 2 and 3 a little more to the left than the abnormal types 2′ and 3′ for each single j. The observed g-values are represented by dots, or vertical dashes on account of their possible error of ± 10 percent in most cases. The admission of the abnormal types is increasing the number of possible g-values considerably as compared with our original simple theory of types 2 and 3. Nevertheless there are so great distances between the possible g-values *for small* j that one still has a unique coordination of theory and experiment. On the other hand *for large* j the g-values of type 2′ come closer and closer to those of type 2; and in type 3′ only the cases $j = \frac{1}{2}$ and $j = \frac{3}{2}$ are realized at all.

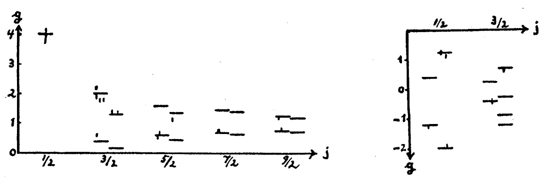

FIG. 1 and FIG. 2. Theoretical g-values of Tables I and II are drawn as horizontal bars.

Aside from every detailed interpretation there remains one main result: The proton has a magnetic moment of $\mu_\pi \sim 2$ and the neutron has a magnetic moment of $\mu_\nu \sim -0.6$, as proved by the hyperfine structure of higher nuclei. There are other determinations of μ_π by Stern and Rabi, namely, 2.5 from molecular rays and 3.2 from atomic rays of hydrogen in inhomogeneous magnetic fields. The Rabi method in particular is a very direct and unobjectionable way of finding μ_π, and one may wonder as to what is responsible for the difference between the two ray methods and the analysis of hyperfine structure.[7] No direct methods are known so far to determine the magnetic moment of the neutron. Scattering experiments seem to indicate that the neutron has no or only a very small magnetic moment.

[7] According to new ideas on the structure of the nucleus a proton may split up into a positron and a neutron, and the neutron may split up into an electron and a proton; then the latter proton may perhaps contain an electron and a positron less than the former proton and may also have a different magnetic moment, a very precarious hypothesis of course, which I do not want to propagate.

Table III gives a more detailed report of the g-values and their origin in various isotopes. It is of interest that the proton (type 2 and 2′) produces j and l-values that are small at the beginning of the periodic table and increase more and more with higher elements.[8] On the other hand the neutrons of type 3 and 3′ retain their small j and l-values throughout the periodic table. One may interpret this in the sense that the single proton is bound to the surface of the nucleus while the single neutrons are bound inside.

[8] Only Tl^{81} is an exception with $j=\frac{1}{2}$. This behavior makes us suspect that our scheme of type 2 and the g-formula (a) does not apply here. The experimental g-value of Tl is rather uncertain, too, depending on whether one applies the interpolation formulas of Goudsmit or of Fermi-Segrè. For these two reasons we have marked Tl in Table III with question marks and have not drawn it in Fig. 1.

Small-Angle Inelastic Scattering of Electrons in Helium, Hydrogen and Mercury

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The inelastic scattering of electrons by helium, hydrogen and mercury is measured for angles very close to the forward direction, ranging from 0° to 15° scattering. Those electrons are studied which have suffered losses of 21.1 volts, 12.6 volts and 6.7 volts, respectively, for the three gases. The initial energies range from 100 to 300 volts. The scattering per unit solid angle is found to reach a maximum at a fairly small angle and then decrease to a lower value at zero angle. The position of the maximum is a function of energy and not of momentum as predicted by the Born theory.

A PRELIMINARY report of some results on the inelastic scattering of electrons of medium energy in the region around the forward direction was made[1] at the New York meeting of the American Physical Society in 1933. Since that time Whiddington[2] and others have published results, first confirming and later failing to confirm the anomalies reported in the behavior of the scattering cross sections at very small angles. It was originally intended to continue the work with some changes in the apparatus, but this has not been possible, so the results already obtained are put on record. The gases used were helium, hydrogen and mercury, those electrons being studied which had lost 21.1 volts, 12.6 volts and 6.7 volts, respectively, in the three gases. The initial energies of the electrons were approximately 100 volts, 150 volts, 200 volts and 300 volts. The range of angles covered was about 0° to 15°.

The apparatus used is shown diagrammatically in Fig. 1. The source of electrons is a cylindrical, indirectly heated cathode. Immediately surrounding it is an electrode whose potential can be varied for focussing purposes. The main accelerating field is between the cathode and the anode. The anode is insulated from the main part of the apparatus, although at the same potential, in order that a galvanometer may be connected between them to measure the current leaving S_1. Scattering takes place in the region R. Scattered electrons go through slits S_2 and S_3 into the electrostatic analyzer,[3] A. Those of the proper energy go on through S_4 to the collector, where they are measured by a two-tube FP-54 amplifier. The dimensions of the slits are: lengths of all, 3 mm; widths, S_1, 0.15 mm, S_2, 0.14 mm, S_3, 0.28 mm, S_4, 0.27 mm. The radii of the plates of the electrostatic analyzer are 5 and 6 cm.

The part of the apparatus containing the cathode and anode is movable about an axis through O. Its movement is read on a divided circle, the readings of which, after the zero has been determined by letting the main beam pass through S_2 and S_3, give the angle of scattering of those electrons entering the analyzer. Gas is admitted to the apparatus at I and pumped out at

[1] S. N. Van Voorhis, Phys. Rev. **43**, 777A (1933).

[2] Whiddington, Emerson and Taylor, Nature **132**, 65 (1933); Poultney and Whiddington, Nature **133**, 685 (1934).

[3] Hughes and Rojansky, Phys. Rev. **34**, 284 (1929).