slightly from the value given. The other lines for which J is high give values that agree reasonably well with the above value. There seems to be little doubt but that the value found is high because an appreciable part of the splitting observed is due to the p and d electrons, while for purposes of calculation it was assumed to be due to the s electron only. Estimations of the contributions of the p and d electrons, by analogy with the results obtained in the different lines in cobalt as well as in other elements, would set as a probable value of the nuclear magnetic moment of cobalt 2 to 3 small magnetons.

In concluding I wish to thank Professor H. E. White for many valuable discussions of this problem.

The Nuclear Magnetic Moment of Lanthanum

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The hyperfine structure of some twenty-five lines has been investigated in order to determine the absolute overall widths of each member of the $5d^26s\ ^4F$ levels. From these a nuclear g-factor of 0.719 and a magnetic moment of 2.5 small magnetons have been computed. The Landé interval rule applied to the hyperfine structure of a three-electron configuration was found to hold. It was found that the interaction energy between the s electron and the nucleus as given by the Bacher-Goudsmit equations is not sufficient to account for the observed intervals but that the interaction energy between the d electrons and the nucleus must also be considered.

As reported earlier,[1] the hyperfine structure of the arc spectrum of lanthanum has been investigated in order to determine the nuclear mechanical moment. All three of the independent methods, maximum number of components in a line involving sufficiently high J values, relative intervals of the hyperfine structure levels and relative intensities of the components, gave good agreement with $I=(7/2)h/2\pi$. In this investigation the h.f.s. separations, chiefly in the metastable ^4F levels, have been determined and from these absolute values the g-factor and hence the nuclear magnetic moment have been evaluated. Also it is possible from these values of the h.f.s. intervals to determine to what extent the Landé interval rule and the Bacher-Goudsmit equations apply to a three-electron configuration.

In their preliminary work, Meggers and Burns[2] found that the $5d^26s\ ^4F$ levels of lanthanum showed wide h.f.s. whereas the normal state $5d6s^2\ ^2D$ was relatively sharp. The widths of some twenty-five lines have now been measured in order to determine the width of each of the ^4F levels. These lines are given by transitions from the triad of quartets $^4D°$, $^4F°$ and $^4G°$ arising from the electronic configuration $5d^26p$ to the metastable $5d^26s\ ^4F$ levels.

The apparatus and experimental procedure are explained in detail in the previous paper.[1] The plates were calibrated for intensity measurements by the use of a step-slit as described earlier. The overall width of each line was carefully measured in several orders in each pattern, then corrected for dispersion and the average value adopted.

From flag patterns, i.e., patterns in which the intensities and intervals degrade uniformly, only the sum or difference of the energy level widths can be determined very accurately unless the absolute width of one of the levels is known. To assume that one level is sharp because many components in a uniformly degrading line are clearly resolved can easily introduce an error as high as 15 percent in the value of the computed level. Hence to determine accurately the widths of the levels of a multiplet, detailed measurements must be made on at least one line which has the off-diagonal components resolved. Most of the lines measured in this investigation degrade uniformly or are only partially resolved. Fortunately, however, two lines in which the off-diagonal components are clearly resolved fix the absolute width of one of the ^4F levels.

[1] O. E. Anderson, Phys. Rev. **45**, 685 (1934).
[2] W. F. Meggers and K. Burns, J. O. S. A. and R. S. I. **14**, 449 (1927).

Experimental Results

Twenty-five lines involving the ^4F levels were carefully measured and the average value of each is given in Table I. The wave-lengths and classification are taken from the analysis of the arc spectrum by Russell and Meggers.[3] In the column headed "remarks" is given the number of clearly resolved components in the line, the "+" sign indicating that there is a smeared tail which could contain at least one and in some cases several more components.

In the notation adopted, the symbol Δ denotes the separation in cm^{-1} between the first and the last h.f.s. levels in a fine-structure level and it precedes the term symbol, e.g., $\Delta^4F_{9/2}=+0.55$ cm^{-1} is the overall width of the $^4F_{9/2}$ level from its lowest to its highest h.f.s. level. A positive Δ indicates that the h.f.s. levels are normal and a negative Δ that they are inverted. If the components of a line degrade towards the violet (e.g., λ5212), Δ of the lower level $-\Delta$ of the upper level is positive and *vice versa* if they degrade towards the red (e.g., λ5106), this difference is negative. Actually the Δ difference does not give the entire width of the line in some cases but since all the measurements were made from the first main component to the point where the tailing off is complete, the measured overall width gives the Δ difference in all cases accurately to within 0.02 cm^{-1} and in most cases more accurately.

Table I. *Overall widths of spectrum lines.*

		Remarks
λ5212	$\Delta^4F_{9/2}-\Delta^4D^\circ_{7/2}=+0.58$ cm^{-1}	7+components
5047	$\Delta^4F_{7/2}-\Delta^4D^\circ_{7/2}=+0.45$	5+
5791	$\Delta^4F_{9/2}-\Delta^4F^\circ_{9/2}=+0.47$	5+
5588	$\Delta^4F_{7/2}-\Delta^4F^\circ_{9/2}=+0.31$	2+
6007	$\Delta^4F_{9/2}-\Delta^4F^\circ_{7/2}=+0.37$	4+
5789	$\Delta^4F_{7/2}-\Delta^4F^\circ_{7/2}=+0.25$	3+
5177	$\Delta^4F_{7/2}-\Delta^4D^\circ_{5/2}=+0.38$	5+
5051	$\Delta^4F_{5/2}-\Delta^4D^\circ_{5/2}=+0.16$	2+
5789	$\Delta^4F_{7/2}-\Delta^4F^\circ_{7/2}=+0.25$	3+
5631	$\Delta^4F_{5/2}-\Delta^4F^\circ_{7/2}=0.0$	1(sharp)
5935	$\Delta^4F_{7/2}-\Delta^4F^\circ_{5/2}=+0.24$	3+
5769	$\Delta^4F_{5/2}-\Delta^4F^\circ_{5/2}=+0.03$	1+
6617	$\Delta^4F_{7/2}-\Delta^4G^\circ_{7/2}=+0.23$	3+
6411	$\Delta^4F_{5/2}-\Delta^4G^\circ_{7/2}=0.0$	1(sharp)
5145	$\Delta^4F_{5/2}-\Delta^4D^\circ_{3/2}=+0.14$	2+
5056	$\Delta^4F_{3/2}-\Delta^4D^\circ_{3/2}=-0.23$	3+
5769	$\Delta^4F_{5/2}-\Delta^4F^\circ_{5/2}=+0.03$	1+
5658	$\Delta^4F_{3/2}-\Delta^4F^\circ_{5/2}=-0.35$	2+
5856	$\Delta^4F_{5/2}-\Delta^4F^\circ_{3/2}=+0.17$	4+
5741	$\Delta^4F_{3/2}-\Delta^4F^\circ_{3/2}=-0.22$	2+
6693	$\Delta^4F_{5/2}-\Delta^4G^\circ_{5/2}=-0.17$	2+
6543	$\Delta^4F_{3/2}-\Delta^4G^\circ_{5/2}=-0.55$	9(exact)
6600	$\Delta^4F_{5/2}-\Delta^4P^\circ_{5/2}=-0.23$	2+
6454	$\Delta^4F_{3/2}-\Delta^4P^\circ_{5/2}=-0.62$	4+
5106	$\Delta^4F_{3/2}-\Delta^4D^\circ_{1/2}=-0.27$	5(exact)
6250	$\Delta^4F_{9/2}-\Delta^4G^\circ_{11/2}=+0.49$	7+
6394	$\Delta^4F_{7/2}-\Delta^4G^\circ_{9/2}=+0.30$	5+

Table II. *Relative widths of adjacent ^4F levels.*

		Average
$\Delta^4F_{9/2}-\Delta^4F_{7/2}$	0.13, 0.16, 0.12	0.14 cm^{-1}
$\Delta^4F_{7/2}-\Delta^4F_{5/2}$	0.22, 0.25, 0.21, 0.23	0.23
$\Delta^4F_{5/2}-\Delta^4F_{3/2}$	0.37, 0.38, 0.38, 0.39, 0.39	0.38

The values given in Table II are obtained by taking the difference of the widths of the lines in brackets in Table I. These give the relative widths of adjacent levels in the ^4F multiplet. As soon as the absolute width of any one of these levels is determined, this table gives quite accurately the absolute values of the remaining three. From $\lambda\lambda$5106 and 6543, $\Delta^4F_{3/2}$ can be evaluated. Immediately the widths of the remaining ^4F levels and the $^4D^\circ$, $^4F^\circ$ and $^4G^\circ$ levels are fixed. The Δ values in Table III therefore depend on the average value of $\Delta^4F_{3/2}$ as determined from $\lambda\lambda$5106 and 6543.

Table III. *Absolute intervals between line components.*

			5106 (15 mm sep.)					
Abs.	0.085	0.064	0.068	0.041	cm^{-1}			
Rel.	5.00	(3.77)	4.00	2.41				
			6250 (8 mm sep.)					
Abs.	0.111	0.097	0.083	0.068	0.058	0.043	0.028	
Rel.	8.00	6.94	5.95	4.90	4.14	3.08	2.00	
			6543 (15 mm sep.)					
Abs.	0.106	0.081	0.089	0.083				
Rel.	6.00	(4.58)	5.05	(3.58)				
	(6.54)	5.00	(5.51)	3.91				
			6543 (7.5 mm sep.)					
Abs.	0.106	0.078	0.085	0.066	0.067	0.054	0.049	0.032
			6454 (12 mm sep.)					
Abs.	0.205	(first to third component)						

By using etalons with a resolving power of 500,000, λ5106, as shown in Fig. 1, is resolved into five components. The intervals and the

[3] H. N. Russell and W. F. Meggers, Bur. Stand. J. Research 9 (RP497) (1932).

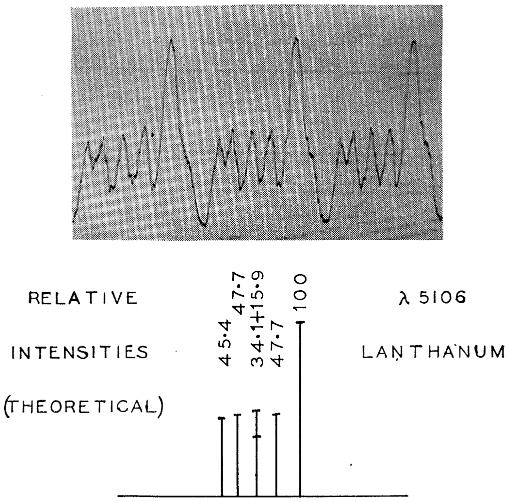

FIG. 1. Microphotometer curve and theoretical intensities of components of λ5106.

intensities of the components indicate that $\Delta^4D_{1/2}{}^\circ = 0.063$ cm^{-1} and $\Delta^4F_{3/2} = -0.208$ cm^{-1}. The calculated theoretical and observed intensities agree to within 4 percent. The fourth interval of λ5106, see Table III, is smaller than would be expected. Undoubtedly the peak of the fifth component is shifted towards the center of the pattern by the overlapping of the fourth component.

The overall width of $^4F_{3/2}$ can be again evaluated from λ6543, $^4F_{3/2} - {}^4G_{5/2}{}^\circ$, shown in Fig. 2. This line shows nine resolved components. The measured intervals and visual estimates of the intensities of the components are in good agreement with $\Delta^4F_{3/2} = -0.194$ cm^{-1} and $\Delta^4G_{5/2}{}^\circ = +0.355$ cm^{-1}. The eight intervals of this line are given in Table III; however, their accuracy cannot be great since the values are taken from only the two orders nearest the center of the pattern where the dispersion is changing rapidly. By using a 15 mm separator and taking an average over many orders a more accurate determination of the first four intervals was obtained. These intervals, given in Table III, along with the overall width of the line, were used to determine the values given above for $\Delta^4F_{3/2}$ and $\Delta^4G_{5/2}{}^\circ$.

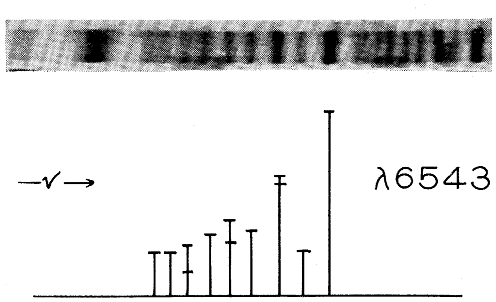

FIG. 2. Etalon pattern and theoretical pattern of λ6543.

Hence both λ5106 and λ6543 indicate that the overall width of $^4F_{3/2}$ is -0.20 ± 0.008 cm^{-1}. Measurements on the partially resolved λ6454 indicate close agreement with this same value. By using an average value $\Delta^4F_{3/2} = -0.20$ cm^{-1}, the absolute values of the remaining members of the multiplet can now be evaluated very accurately (see Table IV). These are also given in Fig. 3, with the energy levels marked "observed."

TABLE IV. *Overall widths of levels.*

$\Delta^4F_{9/2} = +0.55$	$\Delta^4D^\circ{}_{7/2} = -0.04$	$\Delta^4F^\circ{}_{9/2} = +0.09$	$\Delta^4G^\circ{}_{11/2} = +0.06$
$\Delta^4F_{7/2} = +0.41$	$\Delta^4D^\circ{}_{5/2} = +0.03$	$\Delta^4F^\circ{}_{7/2} = +0.17$	$\Delta^4G^\circ{}_{9/2} = +0.11$
$\Delta^4F_{5/2} = +0.18$	$\Delta^4D^\circ{}_{3/2} = +0.04$	$\Delta^4F^\circ{}_{5/2} = +0.16$	$\Delta^4G^\circ{}_{7/2} = +0.18$
$\Delta^4F_{3/2} = -0.20$	$\Delta^4D^\circ{}_{1/2} = +0.07$	$\Delta^4F^\circ{}_{3/2} = +0.01$	$\Delta^4G^\circ{}_{5/2} = +0.35$

THEORETICAL DISCUSSION

From the known intervals of the h.f.s. energy levels, the nuclear g-factor and magnetic moment can now be calculated. Assuming that all the interaction energy is due to the penetrating $6s$ electron, the Bacher-Goudsmit equations[4] give the relative widths of the ^4F levels as shown in the first column of Fig. 3. Qualitatively these separations are also given by the vector diagrams as suggested by White.[5] By assuming $\Delta^4F_{9/2} = +0.55$ cm as correct, the splittings of the other members will be as shown. This results in $a' = 0.141$ cm^{-1} in the interaction energy equation $\Gamma_F' = a'I^*s^* \cos(I^*s^*)$. By using this a', the g-factor can be calculated as suggested by Goudsmit.

By taking the orbital interaction into consideration much better agreement with observation is obtained. We may assume to a first approximation that this additional interaction energy is given by $\Gamma_F'' = a''L^*I^* \cos(L^*I^*)$. Now by assuming $\Delta^4F_{3/2} = -0.20$ cm^{-1} and $\Delta^4F_{9/2}$

[4] S. Goudsmit and R. F. Bacher, Phys. Rev. **34**, 1499 (1929).

[5] H. E. White, Phys. Rev. **34**, 1404 (1929).

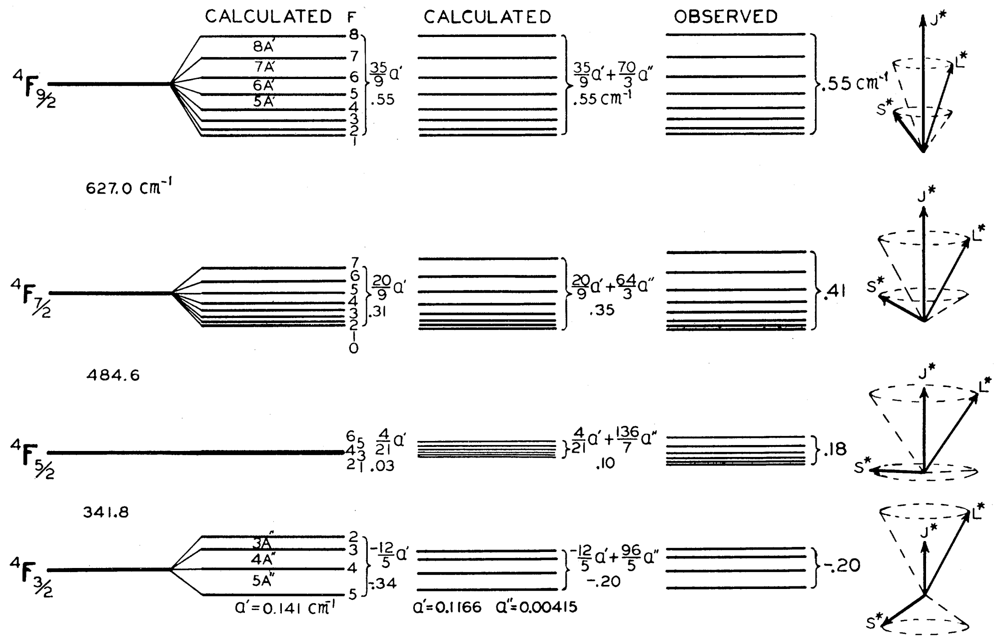

FIG. 3. Hyperfine structure in the ^4F levels.

$=+0.55$ cm^{-1} as correct, a' and a'' can be calculated. The new values computed for $\Delta^4F_{5/2}$ and $\Delta^4F_{7/2}$ are shown in the second column of Fig. 3.

Although the agreement is considerably better in this latter case, it is not yet as good as one would desire it to be. Undoubtedly, a quantum-mechanical treatment applied to the $5d^26s$ configuration will result in still closer agreement between the observed and calculated intervals. Nevertheless, from the latter value of a', a fairly good estimate of the nuclear g-factor and the magnetic moment can be made. Substituting $a'=0.117$ cm^{-1} in Eq. (5) of Goudsmit's paper,[6] results in $g=0.719$ and hence a magnetic moment for lanthanum of 2.5 small magnetons.

The relative values of the intervals in several lines, Table III, show that the Landé interval rule as applied to h.f.s. arising from a three-electron configuration holds to within the limits of error in this experiment. The results also show that the coupling of the penetrating $6s$ electron with the nucleus is not sufficient to account for the h.f.s. in the ^4F multiplet and that the other interactions are large enough to be taken into account.

In concluding I wish to thank Professor H. E. White for suggesting the problem and for the many helpful suggestions that he has given in connection with this work and also Professor F. A. Jenkins for many suggestions pertaining to the intensity work.

[6] S. Goudsmit, Phys. Rev. 43, 638 (1933).

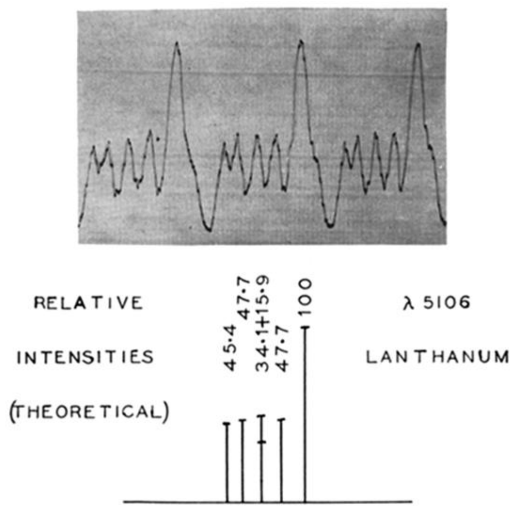

Fig. 1. Microphotometer curve and theoretical intensities of components of λ5106.

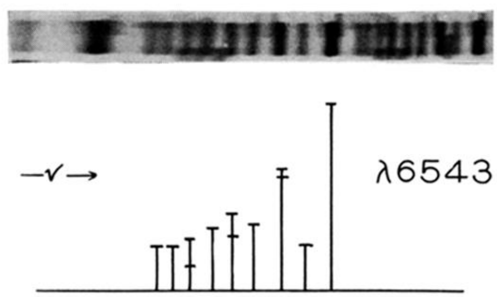

FIG. 2. Etalon pattern and theoretical pattern of λ6543.