

## CONCLUSIONS

It is interesting to note that light excitation was observed in every case well below the ionization potentials reported by Beeck and Mouzon<sup>7, 8</sup> for the corresponding cases.

It is also interesting to note that the light beam was observed at the lowest accelerating potentials for those cases in which Beeck and Mouzon observed lowest ionization potentials. That is, for any gas the light beam was observed at the lowest ion accelerating potential for that ion which is closest to the gas in the periodic table.

The minimum accelerating potential at which Na<sup>+</sup> ions were able to produce a visible light

beam in Ne was approximately 130 volts while Güntherschulze and Keller<sup>9</sup> have found that Ne atoms do not produce excitation in Ne below 300 volts. This agrees with the prediction of Weizel<sup>10</sup> based on the theory of ionization by impact developed by Weizel and Beeck. He states that ionization and excitation of light in the inert gases may be produced at lower energies by alkali ions than ions of the inert gases; applied to this specific case, ionization and excitation may be produced in Ne at lower energies if Na<sup>+</sup> is the moving particle rather than Ne<sup>+</sup>.

In conclusion, the writer wishes to express his appreciation to Professor A. J. Dempster for his valuable criticisms of this work.

<sup>7</sup>O. Beeck and J. C. Mouzon, *Ann. d. Physik* **11**, 737 (1931).

<sup>8</sup>O. Beeck and J. C. Mouzon, *Ann. d. Physik* **11**, 858 (1931).

<sup>9</sup>A. Güntherschulze and F. Keller, *Zeits. f. Physik* **72**, 143 (1931).

<sup>10</sup>W. Weizel, *Zeits. f. Physik* **76**, 259 (1932).

## Relative Multiplet Transition Probabilities from Spectroscopic Stability\*

C. W. UFFORD AND F. M. MILLER, *Allegheny College*

(Received June 11, 1934)

With the method based on the principle of spectroscopic stability, the relative transition probabilities of different multiplets in Russell-Saunders coupling have been calculated for the transitions  $d^3p-d^4$ ,  $dp^2-pd^2$ , and  $d^2p^2-d^3p$ . A formula is given for getting the transition probabilities, in terms of the core transition probabilities, for the pair of electron configurations which are obtained by adding the same non-equivalent  $s$  electron to each core configuration.

## THEORY

THE method, based on the principle of spectroscopic stability, of calculating the relative transition probabilities of different multiplets in Russell-Saunders coupling has been given by Condon and one of us.<sup>1</sup> This method is used here to calculate the transition probabilities for the transitions  $d^3p-d^4$ ,  $dp^2-pd^2$ , and  $d^2p^2-d^3p$ . The results of these calculations are given for  $d^3p-d^4$  in Table I,  $dp^2-pd^2$  in Table II, and  $d^2p^2-d^3p$  in Table III, where the notation is the same as that used in the previous paper.<sup>1</sup>

We have also found a simple formula for getting the transition probabilities which arise when a non-equivalent  $s$  electron is added to a pair of core configurations. These transition probabilities are given in terms of those between the two core configurations. The added  $s$  electron must not be equivalent to any core electrons either in the initial or the final state. The relative transition probability in Russell-Saunders coupling is given by

$$(asN^{2S+1}L, a'sN'^{2S+1}L') \\ = (2S+1)[1/(2S+2)(aN^{2S+2}L, a'N'^{2S+2}L') \\ + (1/2S)(aN^{2S}L, a'N'^{2S}L')], \quad (1)$$

where  $S$  is the total spin angular momentum,  $L$

\* Presented at the Washington Meeting of the American Physical Society, April 27, 1934.

<sup>1</sup>E. U. Condon and C. W. Ufford, *Phys. Rev.* **44**, 740 (1933).

TABLE I. Relative transition probabilities for the transition  $d^3p-d^4$ .

		$d^4$										
		$^5D$	$^3H$	$^3G$	$2\ ^3F$	$^3D$	$2\ ^3P$	$^1I$	$2\ ^1G$	$^1F$	$2\ ^1D$	$2\ ^1S$
$d^3p$	$^5G$	$y$	$x$	$x$	$x$	$x,y$	$x,y$	$x,y$	$x$	$x$	$x,y$	$x,y$
	$^5F$	2800	$x,y$	$x$	$x$	$x$	$x,y$	$x,y$	$x$	$x$	$x$	$x,y$
	$2\ ^5D$	6500	$x,y$	$x,y$	$x$	$x$	$x$	$x,y$	$x,y$	$x$	$x$	$x,y$
	$^5P$	2700	$x,y$	$x,y$	$x,y$	$x$	$x$	$x,y$	$x,y$	$x,y$	$x$	$x$
	$^5S$	$y$	$x,y$	$x,y$	$x,y$	$x,y$	$x$	$x,y$	$x,y$	$x,y$	$x,y$	$x$
	$^3I$	$x,y$	780	$y$	$y$	$y$	$y$	$x$	$x,y$	$x,y$	$x,y$	$x,y$
	$2\ ^3H$	$x,y$	5412	1188	$y$	$y$	$y$	$x$	$x$	$x$	$x,y$	$x,y$
	$4\ ^3G$	$x,y$	9648	5427	4365	$y$	$y$	$x,y$	$x$	$x$	$x,y$	$x,y$
	$5\ ^3F$	$x$	$y$	6345	6615	2160	$y$	$x,y$	$x$	$x$	$x$	$x,y$
	$6\ ^3D$	$x$	$y$	$y$	9180	2700	4020	$x,y$	$x,y$	$x$	$x$	$x,y$
	$4\ ^3P$	$x$	$y$	$y$	$y$	2340	3060	$x,y$	$x,y$	$x,y$	$x$	$x$
	$2\ ^3S$	$x,y$	$y$	$y$	$y$	$y$	1560	$x,y$	$x,y$	$x,y$	$x,y$	$x$
	$^1I$	$x,y$	$x$	$x,y$	$x,y$	$x,y$	$x,y$	1820	$y$	$y$	$y$	$y$
	$2\ ^1H$	$x,y$	$x$	$x$	$x,y$	$x,y$	$x,y$	4420	860	$y$	$y$	$y$
	$3\ ^1G$	$x,y$	$x$	$x$	$x$	$x,y$	$x,y$	$y$	3465	855	$y$	$y$
	$4\ ^1F$	$x$	$x,y$	$x$	$x$	$x$	$x,y$	$y$	4315	805	1320	$y$
	$4\ ^1D$	$x$	$x,y$	$x,y$	$x$	$x$	$x$	$y$	$y$	1700	2100	$y$
	$3\ ^1P$	$x$	$x,y$	$x,y$	$x,y$	$x$	$x$	$y$	$y$	$y$	1380	960
	$^1S$	$x,y$	$x,y$	$x,y$	$x,y$	$x,y$	$x$	$y$	$y$	$y$	$y$	0

and  $L'$  the initial and final values of the total orbital angular momentum,  $a$  and  $a'$  the initial and final core configurations, and  $N$  and  $N'$  are the number of multiplets of the same kind occurring in the given configuration. The  $N$ 's are in general different for the multiplets of different multiplicity. In some cases either the first term or the second term on the right-hand side of Eq. (1) does not exist and so is omitted in determining the new transition probability. Thus to illustrate Eq. (1)

$$\begin{aligned}
 & (d^3ps10\ ^2D, d^4s\ ^2F) \\
 &= 2\left[\frac{1}{3}(d^3p6\ ^3D, d^4s\ ^3F) + 1/1(d^3p4\ ^1D, d^4s\ ^1F)\right] \\
 &= 2\left(\frac{1}{3}9180 + 1700\right) = 9520, \quad (2)
 \end{aligned}$$

where the values of  $(d^3p6\ ^3D, d^4s\ ^3F)$  and  $(d^3p4\ ^1D, d^4s\ ^1F)$  are taken from Table I.

Eq. (1) was obtained as follows. When a non-equivalent  $s$  electron is added to a configuration, the  $L$  values of the terms are unchanged. The values of  $S$  are increased and decreased by  $\frac{1}{2}$  or the multiplicities by unity. Thus the new terms of  $as$  are the sums of the terms of  $a$  having the same  $L$  value but of multiplicity one greater and one less. Now for given values of  $M_L$  and  $M'_L$  the part,  $(LM_L|\mathbf{r}|L'M'_L)$ , of each matrix element<sup>1</sup> of

TABLE II. Relative transition probabilities for the transition  $dp^2-d^2p$ .

		$d^2p$											
		$^4G$	$^4F$	$2\ ^4D$	$^4P$	$^4S$	$^2H$	$2\ ^2G$	$3\ ^2F$	$3\ ^2D$	$3\ ^2P$	$^2S$	
$dp^2$	$^4F$	162	350	76	$y$	$y$	$x,y$	$x$	$x$	$x$	$x,y$	$x,y$	
	$^4D$	$y$	112	200	108	$y$	$x,y$	$x,y$	$x$	$x$	$x$	$x,y$	
	$^4P$	$y$	$y$	324	0	108	$x,y$	$x,y$	$x,y$	$x$	$x$	$x$	
	$^2G$	$x$	$x$	$x,y$	$x,y$	$x,y$	198	162	18	$y$	$y$	$y$	
	$2\ ^2F$	$x$	$x$	$x$	$x,y$	$x,y$	$y$	648	168	87	$y$	$y$	
	$3\ ^2D$	$x,y$	$x$	$x$	$x$	$x,y$	$y$	$y$	507	300	123	$y$	
	$2\ ^2P$	$x,y$	$x,y$	$x$	$x$	$x$	$y$	$y$	$y$	153	162	27	
	$^2S$	$x,y$	$x,y$	$x,y$	$x$	$x$	$y$	$y$	$y$	$y$	$y$	57	0

er which depends on  $M_L$  in the  $LSM_LM_S$  scheme, and the square of which appears as a coefficient in the equation for the part of the matrix element of  $er$ ,  $|(N^{2S+1L}|er|N'^{2S+1L'})|^2$ , independent of  $M_L$ , will be the same for a given pair of states of  $as, a's$  as for the pair of core states of  $a, a'$  with the same values of  $L$  and  $L'$ . In the zero order scheme for given values of  $M_S, M_L$  and  $M'_L$  one group of the states of  $as, a's$  will arise by adding  $s^+$  to the core states with the same values of  $M_L$  and  $M'_L$  where  $M_S$  is less by  $\frac{1}{2}$ , and the other group will arise by adding  $s^-$  to the core states with the

TABLE III. Relative transition probabilities for the transition  $d^2p^2-d^3p$ .

		$d^3p$																			
		$^3G$	$^3F$	$2^3D$	$^3P$	$^3S$	$^3I$	$2^3H$	$4^3G$	$5^3F$	$6^3D$	$4^3P$	$2^3S$	$^1I$	$2^1H$	$3^1G$	$4^1F$	$4^1D$	$3^1P$	$^1S$	
$d^2p^2$	$^5G$	6750	4050	$y$	$y$	$y$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	
	$^5F$	4050	3150	5400	$y$	$y$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	
	$2^5D$	$y$	5400	13500	2850	$y$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	
	$^5P$	$y$	$y$	2850	3600	1200	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	
	$^5S$	$y$	$y$	$y$	1200	0	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	
	$2^3H$	$x$	$xy$	$xy$	$xy$	$xy$	10530	10098	4122	$y$	$y$	$y$	$y$	$xy$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	
	$3^3G$	$x$	$x$	$xy$	$xy$	$xy$	$y$	15642	18063	6795	$y$	$y$	$y$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	
	$6^3F$	$x$	$x$	$x$	$xy$	$xy$	$y$	$y$	32895	20055	13830	$y$	$y$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	
	$5^3D$	$xy$	$x$	$x$	$x$	$xy$	$y$	$y$	$y$	21030	13800	6570	$y$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	
	$6^3P$	$xy$	$xy$	$x$	$x$	$x$	$y$	$y$	$y$	$y$	17820	8370	3780	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	
	$^3S$	$xy$	$xy$	$xy$	$x$	$x$	$y$	$y$	$y$	$y$	$y$	1800	0	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	
	$^1I$	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$xy$	$xy$	2730	390	$y$	$y$	$y$	$y$	
$^1H$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$xy$	3120	1716	774	$y$	$y$	$y$		
$4^1G$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$y$	11094	6291	3135	$y$	$y$		
$3^1F$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$x$	$x$	$xy$	$y$	$y$	5895	4375	2330	$y$		
$6^1D$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$x$	$xy$	$y$	$y$	$y$	8870	5350	3030		
$2^1P$	$xy$	$xy$	$x$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$x$	$y$	$y$	$y$	$y$	2820	720		
$3^1S$	$xy$	$xy$	$xy$	$x$	$x$	$xy$	$xy$	$xy$	$xy$	$xy$	$x$	$x$	$x$	$y$	$y$	$y$	$y$	$y$	1920		

same values of  $M_L$  and  $M'_L$  where  $M_S$  is greater by  $\frac{1}{2}$ . Therefore the matrix elements of  $er$  between states arising from core states of different values of  $M_S$  will be zero since the values of  $m_s$  for the non-equivalent  $s$  electron are different. This  $s$  electron does not jump and therefore must have identical quantum numbers in the initial and final states which can differ by at most one individual set of quantum numbers.<sup>2</sup> Hence for given values of  $M_S$ ,  $M_L$ , and  $M'_L$ , the sum of the squares of the matrix elements of  $er$  in the zero order scheme for the states of  $as$ ,  $a's$  is equal to the sum of the squares of the matrix elements of  $er$  in the zero order scheme for the cores with the same values of  $M_L$  and  $M'_L$  where  $M_S$  is less by  $\frac{1}{2}$  plus the sum of the squares of these matrix elements where  $M_S$  is greater by  $\frac{1}{2}$ . Thus, since the coefficients in the linear equations for the squares of the matrix elements,  $|(N^{2S+1}L|er|N'^{2S+1}L')|^2$ , for the states of  $as$ ,  $a's$ , are the same as in the two

core equations; and since the constant term is the sum of the constant terms for the two core equations, the solution obtained for the states of  $as$ ,  $a's$  will be the sum of the two solutions obtained for the core equations with the same values of  $M_L$  and  $M'_L$  and the components of total spin angular momentum equal to  $M_S + \frac{1}{2}$  and  $M_S - \frac{1}{2}$ . To get the relative transition probabilities, the square of the matrix element,  $|(N^{2S+1}L|er|N'^{2S+1}L')|^2$ , has to be multiplied by factors depending on  $L$  and  $S$  to sum over the initial and final states.<sup>1</sup> As  $L$  is the same for the states of  $as$ ,  $a's$  and the core states of  $a$ ,  $a'$ , the only factors which are different are the multiplicities. The multiplicities,  $2S$  and  $2S+2$ , of the core states must be removed from the transition probabilities of the core states before they are added to give the transition probability between the states of  $as$ ,  $a's$ . Finally the sum must be multiplied by  $2S+1$ , the multiplicity of the states of the configurations  $as$  and  $a's$  containing the  $s$  electron.

<sup>2</sup> E. U. Condon, Phys. Rev. 36, 1121 (1930).