The Pressure of Plasma Electrons and the Force on the Cathode of an Arc

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It is pointed out that the electrons in a plasma, and particularly in that at the cathode spot of an arc, exert a pressure nkT that has hitherto been overlooked. Numerically, the major portion of the force on the cathode spot can be accounted for in this way.

THE force on the cathode of an arc, which has been ascribed both to high temperature evaporation of the cathode material[1] and to an accommodation coefficient, α, less than unity for ions neutralized at the cathode surface,[2] must be largely due to the pressure of the plasma electrons which are reflected from the cathode sheath. This factor must also be very important in the deflecting vane experiments intended to measure accommodation coefficients,[3] although heating effect experiments for the same purpose[4] are not influenced unless electrons are actually collected by the thermal element.

A momentum change accompanies the rebounding of plasma electrons from the positive ion sheath covering a negatively charged electrode. From the simple kinetic point of view this must appear as a pressure $p=nkT$. Since the force causing the velocity reversal is the repulsion between the negative charge on the electrode and the electron, this pressure is exerted on the electrode. The essential difference between the action of the electrons and ions is that the latter acquire momentum in the electric field of the electrode and then return it to the electrode on impact, whereas the former acquire oppositely directed momentum in the same field but deliver it elsewhere—ultimately to the walls of the discharge tube.

Consider the one-dimensional case of a region extending from x_0 to x_1 in which the field E accelerates ions from x_0 toward x_1. Let the ion density be n_p and suppose that the electron density n varies with potential V and electron temperature T according to the Boltzmann law. Subscripts 0 and 1 refer to values of the variable at x_0 and x_1, respectively. Suppose, further, that no ion ever collides with any atom which may be in the space. The gain of positive ion momentum per second per unit cross section in this region is

$$M=e\int_{x_0}^{x_1} En_p dx. \quad (1)$$

Poisson's equation converts this to

$$M=(1/4\pi)\int_{x_0}^{x_1} EE'dx+e\int_{x_0}^{x_1} Endx. \quad (2)$$

By using the Boltzmann equation and integrating, we obtain

$$M=(E_1^2-E_0^2)/8\pi+(n_0-n_1)kT. \quad (3)$$

If we locate x_0 at the potential maximum in the plasma so that $E_0=0$, then M is the total momentum acquired from the field by all the ions reaching x_1. Neglecting the small initial momentum of the ions when they are formed, this is the total momentum carried by the ions to a collector C at x_1.

If, in a neutralizing impact on C, the ions have an accommodation coefficient α, additional momentum beyond M will be conveyed to C to an amount $M(1-\alpha)^{\frac{1}{2}}$.

A third contribution to the pressure is that of the electrons striking C. If all are reflected without loss of energy, this pressure will be n_1kT; if all are absorbed, it will be $\frac{1}{2}n_1kT$; and for a reflection coefficient ρ and accommodation coefficient α_e, the pressure will be

$$\tfrac{1}{2}[1+\rho(1-\alpha_e)^{\frac{1}{2}}]n_1kT. \quad (4)$$

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[1] Tanberg, Phys. Rev. **35**, 1080 (1930); Kobel, Phys. Rev. **36**, 1636 (1930).
[2] K. T. Compton, Phys. Rev. **37**, 1077 (1931).
[3] Lamar, Phys. Rev. **43**, 169 (1933); Compton and Lamar, Phys. Rev. **44**, 338 (1933). The electron pressures on the two sides of the vane should balance when the vane is centered in the tube. Its displacement will make the electron pressures on its two sides unequal and will produce a net force depending on vane position such as was actually observed.
[4] Van Voorhis and Compton, Phys. Rev. **35**, 1438 (1930); **37**, 1596 (1931).

The fourth component of force is the electrostatic pressure $-E_1^2/8\pi$ due to the field at the collector surface.

The total pressure P is the sum of these four parts:

$$P = M[1+(1-\alpha)^{\frac{1}{2}}]+\tfrac{1}{2}[1+\rho(1-\alpha_e)^{\frac{1}{2}}]n_1kT - E_1^2/8\pi, \quad (5)$$

M being given by Eq. (3) with $E_0=0$.

In the simplest case where $\alpha=1$ and n_1 is negligible,

$$P = n_0kT, \quad (6)$$

so that the net pressure is purely electron kinetic. When $\alpha<1$,

$$P = M(1-\alpha)^{\frac{1}{2}}+n_0kT. \quad (7)$$

Now M can be calculated from the positive ion current density I_p to C, for

$$M = (I_p/e)m\bar{v} \cong (2m/e)^{\frac{1}{2}} I_p(V_0-V_1)^{\frac{1}{2}}. \quad (8)$$

The use of V_0 is not quite exact since the ions originate over a small range of potentials. Eqs. (7) and (8) afford a means for determining α.

When the ions make collisions with gas atoms, they strike C with momentum less than M. But the deficiency is made up by the increased gas pressure resulting from the ion-atom impacts. The pressure component $M(1-\alpha)^{\frac{1}{2}}$ will be affected, although not greatly if few collisions occur in the sheath and the potential difference between potential maximum and sheath edge is small compared to that between sheath edge and C.

A rough estimate of the kinetic electron pressure at a mercury cathode spot can be made. Since we have no exact knowledge of the gas density in the cathode glow, but the indications are that the mean free path of the ions is at least comparable with the distances they travel, formulas based on long free paths should give approximately correct results. For this case the positive ion current density is related to the electron concentration in the plasma by the equation[5]

$$I_p = s_0h_0en_0(2kT_e/m_p)^{\frac{1}{2}} \quad (9)$$

with $s_0=0.4046$, $h_0=0.8513$. Denoting the fraction of the arc current carried by electrons at the cathode by f, and the total current density there by j, we have

$$I_p=(1-f)j. \quad (10)$$

Accordingly, under the conditions applicable to Eq. (6), the kinetic pressure of the electrons gives a force per unit current of

$$n_0kT/j = (1/s_0h_0e)(km_p/2)^{\frac{1}{2}}(1-f)T^{\frac{1}{2}} \quad (11A)$$

$$= 2.74(1-f)T^{\frac{1}{2}} \text{ dynes/amp.} \quad (11B)$$

for mercury. Taking $T=5\times10^4$ in the plasma immediately outside the cathode spot and $f=0.9$, the force is found to be 61 dynes/amp. This is to be compared with the average value 40 dynes/amp. calculated from Kobel's measurements[1] on the Hg arc and 17 dynes/amp. from Tanberg's data[1] on the Cu arc.

Finally, it should be noted that the partial pressure of the electrons is present in any arc stream. It may even predominate over the gas pressure itself when the degree of dissociation exceeds a few percent because of the far higher temperature of the electrons.

[5] Tonks and Langmuir, Phys. Rev. **34**, 876 (1929), Eq. (55A) and Table IIa.