Energy of Removal of Neutrons and Alpha-Particles from Nuclei and Alpha-Instability Below the Radio-Elements

Gamow,[1] arguing from the existence of a minimum at about atomic number 50 in his plot of nuclear energy (negative mass defect in formation from alpha-particles, protons and electrons) against number of constituent alpha-particles, discusses the possibility that elements above this point in the periodic system are unstable with respect to alpha-emission. This possibility results as a more definite conclusion from quite different considerations,[2] based upon the approximate theory of Heisenberg, involving more varied and more numerous data. Since the preparation of the paper embodying these ideas, interesting articles by Gapon[3] and Landé[4] which bear upon the point in question have been brought to my attention. The following comments are intended to assist in bringing the various lines of approach into relation, and to emphasize that all indicate the alpha-instability in about the same region of relatively low atomic number.

Gapon and Landé treat the nucleus as composed of alpha-particles, neutrons, and zero or one proton. Gapon deals with quantities which may best be described as mean "apparent" mass defects of the particles within the nucleus. He concludes that for the neutron this quantity is constant in any series of isotopes of a single element, and that it has nearly the same value in the various elements for which data are available. The first of these conclusions can be roughly substantiated, as far as the form of the relation is concerned, by the methods of the previous paper.[2] Thus, the condition for constancy of the binding energy of the neutron to a nucleus (which is approximately equivalent to Gapon's quantity) can be shown to be $Z^2/M^{4/3}=\text{const.}$, where Z is the atomic number and M the mass number. This condition is roughly fulfilled within any series of isotopes. The variation in number of neutrons within such a series is, however, small compared to the total number, and a considerable variation in binding energy is still possible if the entire number is taken into account. The second of Gapon's conclusions is pretty definitely contradicted by the general considerations and numerical results of the former paper. Heisenberg's assumption of a dependence of binding energy on the neutron-proton ratio seems to be well supported as far as the general trend throughout the system of elements is concerned. The numerical results which suggested this conclusion to Gapon are of insufficient accuracy to support it as a close approximation. This affects to some extent the validity of some of Landé's extrapolations.

In his curve of (negative) mass defect of the alpha-particle against number of particles, Gapon finds a minimum in the general region of Gamow's. Insofar as such "apparent" quantities have significance, it is again that the emission of single particles from elements to the right of the minimum is energetically favored, in agreement with the deductions first mentioned. Gapon considers the point at which his mean defects reach zero as the region of "instability" of elements. This type of instability, however, would concern only the explosion of his "alpha-aggregate" into its separate particles. There is no conclusive evidence favoring the idea that such an explosion is energetically possible in any of the known elements.

Landé points out that if the "constituents" of the nucleus be taken as previously stated that the mass defect curve shows no maximum within the range of low atomic number. (The same is true, incidentally, if the constituents be taken as neutrons and protons, or protons and electrons.) Such a maximum may be approached at the largest values of Z, though the error of the measurements is too large to permit this conclusion. This is again not relevant to the question as to alpha-instability. The quantity necessary here is, of course, $(\Delta E/\Delta n_\alpha)_{n_1,\, n_2\, \cdots}$, where E is the energy of formation of a nucleus synthesizable from n_α alpha-particles and n_1, n_2, $\cdots$, particles of any other arbitrarily selected kinds. Gamow discusses schematically curves of a type which exhibit this quantity. Landé in his second article gives a plot of actual data, in part extrapolated (compare above), of the requisite type, and here recognizes instability in the region of atomic number 50, in agreement with the conclusion referred to above.

Because of the importance of this question of stability it is desirable to show exactly what pertinent direct data exist, uninfluenced by extrapolations, and with the attendant error clearly indicated. This is difficult in a small scale plot. The table of reactions (Table I), however, presents them in simple form. This comprises all the data pertaining to reactions involving the removal of alpha-particles alone, in the region of atomic number between 40 and 60. If the value of $(\Delta E/\Delta n_\alpha)$ is negative the reaction indicated is energetically possible. For those reactions involving more than one alpha-particle only mean values may be inferred. In the last column is given the probable error in $(\Delta E/\Delta n_\alpha)$. In the reactions Xe→Sn and Te→Sn the mass defect is favorable to the emission of alpha-particles and of the same order as the error. In all other cases the probable error is at least twice as great as the indicated effect. No more positive statement than that $(\Delta E/\Delta n_\alpha)$ is zero within the error is therefore possible from the table or from the treatments in references 1, 3 and 4. In this respect it is thought that the calculations cited above[2] represent a

TABLE I. *Energy changes in emission of alpha-particles.*

Parent nucleus	Products	Energy change per alpha-particle (g per g at. $O^{16}=16$)	Probable error
${}_{54}\text{Xe}^{124}$	${}_{42}\text{Mo}^{100}+6\alpha$	0.0042	0.009
${}_{54}\text{Xe}^{124}$	${}_{50}\text{Sn}^{116}+2\alpha$	$-$ .0073	.009
${}_{54}\text{Xe}^{126}$	${}_{50}\text{Sn}^{118}+2\alpha$	$-$ .0073	.009
${}_{54}\text{Xe}^{128}$	${}_{50}\text{Sn}^{120}+2\alpha$	$-$ .0083	.009
${}_{54}\text{Xe}^{130}$	${}_{50}\text{Sn}^{122}+2\alpha$	$-$ .0078	.009
${}_{54}\text{Xe}^{130}$	${}_{52}\text{Te}^{126}+\alpha$	.0081	.029
${}_{54}\text{Xe}^{132}$	${}_{50}\text{Sn}^{124}+2\alpha$	$-$ .0083	.009
${}_{54}\text{Xe}^{132}$	${}_{52}\text{Te}^{128}+\alpha$	.0102	.029
${}_{50}\text{Sn}^{114}$	${}_{42}\text{Mo}^{98}+4\alpha$	.0094	.013
${}_{50}\text{Sn}^{116}$	${}_{42}\text{Mo}^{100}+4\alpha$	.0099	.013
${}_{52}\text{Te}^{126}$	${}_{50}\text{Sn}^{122}+\alpha$	$-$ .0238	.029
${}_{52}\text{Te}^{128}$	${}_{50}\text{Sn}^{124}+\alpha$	$-$ .0269	.029
${}_{56}\text{Ba}^{138}$	${}_{54}\text{Xe}^{134}+\alpha$	.0152	.030

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[1] Gamow, Der Bau der Atomkerns u. die Radioaktivität, Leipzig, 1932, pp. 15–24.

[2] Eastman, Phys. Rev. **46**, 1 (1934).

[3] Gapon, Zeits. f. Physik **79**, 676 (1932).

[4] Landé, Phys. Rev. **43**, 620, 624 (1933).

considerable improvement over the data on masses, and constitute the best present evidence that nuclei in this region are actually unstable. Exceptions, in the nature of fluctuations from the general trend, are of course to be anticipated.

The adoption of this view raises the question of the probability of the alpha-emission. Except in cases of extremely small energy changes, this must be very low to account for the failure of direct detection in these elements. This may, of course, be due sometimes to the necessity for multiple emission. Landé, however, suggests that the mechanism of ejection may require that the residual nucleus be left with a considerable excitation. Emission would then be probable only when the total energy of the change exceeds the necessary residual excitation. As pointed out independently in another connection in the former[2] paper there is some indication that the residual nucleus in many of the known alpha-changes is excited, even in cases where no gamma-rays are found. This perhaps offers support to Landé's suggestion.

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Energy States of Doubly Excited Helium

In a letter in the July 1st issue of the *Physical Review*, Fender and Vinti gave the result of their calculation of the lower states of doubly excited helium. They compared their result with the assignments of the two lines in the extreme ultraviolet by Kruger and of the corona lines by Rosenthal.[1] Dr. Goudsmit and the writer, in an attempt to examine the possible relation of the spectrum of doubly excited helium and the corona spectra, have also undertaken similar calculations of the approximate positions of the levels of doubly excited helium. A modified form of the variational method[2] was employed; hydrogenic wave functions were used and two "screening constants," one for each electron, introduced as variational parameters with respect to which the energy integral is to be minimized. As the finding of the exact minimum in some cases requires a considerable amount of numerical computations, we are content at present with approximate values of the minima. The preliminary result of such calculations is given in Table I. The energies in column (1) are in units of R, measured from the state of the naked nucleus, those in column (2) are in wave numbers above the first ionization limit of helium. For comparison, the values obtained by Fender and Vinti are given in column (3).

The relative positions of these levels show some unexpected features. The level $2s2p\,^3P$ is lower than $2s^2\,^1S$, whereas a superficial comparison with the normal helium spectrum might falsely suggest the opposite is the case. That this can be so is easily understood without any detailed calculations. The electrostatic repulsion between two electrons both in the $2s$ state will be greater than for the $2s2p$ configuration and as the energy of a single $2s$ or $2p$ electron in He$^+$ is practically the same, the $2s^2\,^1S$ will be higher than $2s2p$. The exchange energy complicates these considerations somewhat and the calculations show that the $2s2p\,^1P$ state is considerably higher than the ^3P of this configuration, and that the $2s^2\,^1S$ lies between these two multiplets. Another result is that the levels of the $2p^2$ configuration have low positions.

Table I.

State	Energy (1)	Energy (2)	Energy (3)
$2s4d^1\,^3D$	$-1.062\ R$	322,000 cm^{-1}	
$2s4s\ ^1S$	-1.090	319,200	
$2s4s\ ^3S$	-1.102	318,000	
$2s3d\ ^1D$	-1.110	317,000	
^3D	-1.112	316,800	
$2s3p\ ^1P$	-1.126	315,000	
^3P	-1.146	313,000	
$2s3s\ ^1S$	-1.175	310,000	
$2s3s\ ^3S$	-1.222	304,600	
$2s2p\ ^1P$	-1.308	295,000	296,118
$2p^2\,^1D$	-1.320	294,000	
$2s^2\,^1S$	-1.445	280,000	275,000
$2s2p\ ^3P$	-1.504	274,000	274,526
$2p^2\,^3P$	-1.662	256,000	

The spectrum of doubly excited helium calculated from the states in Table I is given in Table II.

Table II.

Possible transition	Wave number
$2s^2\,^1S-2s3s\,^1P$	35,000 cm^{-1}
$2s2p\,^1P-2s4d\,^1D$	27,000
$2s2p\,^3P-2s3s\,^3S$	30,000
$2s2p\,^1P-2s4s\,^1S$	24,000
$2p^2\,^1D-2s3p\,^1P$	21,000
$2s2p\,^1P-2s3d\,^1D$	22,000
$2p^2\,^3P-2s2p\,^3P$	18,000
$2s^2\,^1S-2s2p\,^1P$	15,000
$2s2p\,^1P-2s3s\,^1S$	15,000

These energy levels and wave numbers of possible transition are, on account of the nature of the calculation, only rough approximations. A partial support, however, is given by the two lines observed by Kruger and Paschen in the far ultraviolet spectrum of He (λ320.4 and λ357.5). Using the energy levels calculated above, we find,[3]

$1s2s\ ^3S-2s2p\ ^3P$	314,000 cm^{-1}	observed lines
$1s2p\,^1P-\ 2s^2\,^1S$	311,000 "	312,117 cm^{-1}
$1s2p\,^3P-\ 2p^2\,^3P$	289,000 "	279,715 "

In a separate paper[4] the possible relation of the spectrum of doubly excited He with the solar coroner spectrum is discussed.

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[1] Rosenthal, Zeits. f. Astrophysik **1**, 115 (1930).

[2] C. Eckart, Phys. Rev. **36**, 878 (1930).

[3] This agreement, however, should not be stressed upon too much, for the energy involved in these transitions is in the main part the excitation of an electron from $1s$ to $2s$ or $2p$ state. Furthermore, the numerical error in the calculation of states can easily be 0.005 R corresponding to about 500 cm^{-1}, not to say the inaccuracy inherent in such *methods* of calculation.

[4] Goudsmit and Ta-You Wu, to appear presently in the Astrophys. J.