Neutron Collisions and the Beta-Ray Theory of Fermi

The theory recently proposed by Fermi[1] for β-ray disintegration offers a precise theoretical method for treating the interaction of the neutron with charged particles. In calculations of simple interactions of this kind, three points become immediately apparent: The calculated effects are many orders of magnitude too small to have any bearing on the experimental behavior of free neutrons or on the exchange forces between nuclear neutrons and protons. This is a direct consequence of Fermi's introduction of the small dimensionless number

$$G=g(h/2\pi mc)^{-3}(mc^2)^{-1}\cong 9\times 10^{-13}$$

to fit the empirical slowness of β-ray decay. Secondly, the field of force of the neutron for charged particles is appreciable out to a radius of some 10^{-10} cm, and this is traceable to the fact that the only characteristic length appearing in the formalism is $h/2\pi mc$. Finally, one occasionally finds divergent integrals of the same type as are met with in the quantum theory of the radiation field and of the positron. We have treated two simple cases of neutron interaction with charged particles—the inelastic scattering of a neutron by a heavy charged particle and the elastic scattering of a neutron by a proton—in the first of which the divergence difficulty does not occur, while in the second it can be obviated in a plausible manner.

According to the theory, any charged particle may interact with the neutron by polarizing the charge cloud produced by oscillation ($N\rightleftarrows P+e^-+n$) between the two states postulated for heavy particles by Fermi. In the case that the interacting particle is a proton, there is the additional possibility that this proton itself make a transition of the Fermi type, thus giving rise to an exchange force of the kind postulated by Heisenberg.

The simplest case considered is that of the inelastic scattering of a neutron by a heavy particle: $X+N\rightarrow X+P+e^-+n$. This is to be thought of in the formalism as a transition of the neutron into a proton with the emission of the two light particles, electron and neutrino, followed by a deflection of the created proton by the stationary particle through their Coulomb interaction. The effective cross section of the heavy particle for this type of collision is found to be

$$\sigma_{\text{inelastic}}=G^2Z^2(e^4/m^2c^4)f(v)\cong 10^{-49}Z^2\text{ cm}^2.$$

Here G is the small constant mentioned above, eZ is the charge of the heavy particle, $e^4/m^2c^4\cong 10^{-25}$ cm^2 is approximately the area necessary to give observable scattering, and $f(v)$ is a function of the velocity of the incident neutron which vanishes unless the energy of the neutron is larger than the threshold value $(2+2\Delta)mc^2\cong 2\times 10^6$ volts if the heavy particle is a proton, $m\Delta$ being the mass defect of the neutron with regard to the proton. With an available energy less than this threshold energy the process cannot occur because energy and momentum cannot simultaneously be conserved. If the heavy particle has infinite mass, the threshold is at $(1+\Delta)mc^2\cong 10^6$ volts. For energies above the threshold $f(v)$ is roughly of the order of magnitude unity. The formula should be applicable to neutron energies up to 10^7 volts. The inverse process $N+P\rightarrow 2N+e^++n$ is describable by the same formula with the threshold at $(2-2\Delta)mc^2\cong 0$.

The elastic scattering may be visualized as an exchange phenomenon: An electron and a neutrino are emitted by the neutron and absorbed by the proton, thus transferring momentum from the one heavy particle to the other. The effective cross section of the proton for this kind of collision may be written

$$\sigma_{\text{elastic}}=G^4\alpha^{-2}\left(\frac{M}{m}\right)^2\frac{e^4}{m^2c^4}\,\varphi(v)\cong 10^{-61}\text{ cm}^2,$$

where α is the fine structure constant and M/m is the ratio of the proton mass to the electron mass. $\varphi(v)$ contains the square of an integral which diverges like $\int^\infty dp/p$ for large values of the momentum p of one of the light particles. Since the contributions to the integral decrease with increasing p, it seems perhaps justified, at least for momentum transfers between the heavy particles not exceeding mc/α, to replace the upper limit by mc/α. This contributes a factor $(\log(1/\alpha))^2\cong 24$; for the rest, $\varphi(v)$ is then a function of the velocity of the incident neutron which is roughly of the order unity.

One may further attempt to calculate the mutual potential energy (exchange energy) $\Phi(R)$ of the neutron and proton as a function of the distance $R\times h/2\pi mc$ between them. One would then expect the σ_{elastic} given above to be obtainable from this Φ by the methods of quantum-mechanical collision theory. The expression to which one is led is:

$$\Phi(R)=\frac{G^2mc^2}{64\pi^6}\int\int d\mathbf{k}d\mathbf{k}'\,\frac{\epsilon k'+(\mathbf{k}\cdot\mathbf{k}')}{\epsilon k'(\epsilon+k')}\,e^{\mathbf{k}\cdot\mathbf{R}+\mathbf{k}'\cdot\mathbf{R}},$$

where $d\mathbf{k}$ and $d\mathbf{k}'$ are the volume elements in the k and k'-spaces, $\epsilon=(1+k^2)^{\frac{1}{2}}$, and R is the relative distance in units $h/2\pi mc$. The double integral diverges, being of the form of an infinite sum of increasing terms of alternating sign. One may arbitrarily give a meaning to the expression by inserting factors e^{-ak} and $e^{-ak'}$, performing the integration, and taking the limit of the resulting expression as $a\rightarrow 0$. This gives a finite answer for all non-zero values of R; the force is attractive and the potential decreases like R^{-5} for distances small compared to $h/2\pi mc$ and exponentially for distances large compared to $h/2\pi mc$. This procedure is, however, quite arbitrary and does not lead to the formula given above for elastic collisions, but to one containing a more seriously divergent integral, so that little significance can be attached to Φ thus calculated. One may alternatively calculate the elastic scattering with the expression for Φ written above without first carrying out the integrations indicated, breaking off the resulting logarithmically divergent integral again at momenta mc/α. This gives just the cross section found directly, and corresponds roughly to inserting into the integral for $\Phi(R)$ the factor $e^{-a|\mathbf{k}'-\mathbf{k}|}$. With this definition of $\Phi(R)$, to which perhaps more significance can be attached, the explicit dependence on R is not readily calculable.

[1] Fermi, Zeits. f. Physik **88**, 161 (1934).

The elastic cross section and the exchange force have been calculated on the hypothesis that only the processes $N \rightleftarrows P + e^- + n$ with the electron in a positive energy state can occur. If one admits in addition the symmetrical and perhaps plausible possibilities $P \rightleftarrows N + e^+ + n$, the exchange energy is increased by a factor of 2 and the elastic cross section by a factor of 4.

The writer wishes to acknowledge suggestion and valuable discussion of these considerations on the part of Professor J. R. Oppenheimer.

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July 11, 1934.

Ionization by Positive Alkali Ions as Measured by a Balanced Space-Charge Method

The minimum speeds at which ionization occurs when gas atoms are struck by accelerated positive alkali ions has been investigated for several gases and ions. By allowing the ionization to take place in a region containing a space-charge limited current of electrons, it has been possible to observe with great accuracy the formation of new positive ions because of the strong effect of positive ions on the electron current.[1] This method has the great advantage of being highly or completely insensitive to secondary electrons liberated from the walls and thus surmounts the greatest difficulty encountered in other methods.[2]

The electron space-charge was produced inside a cylindrical anode having an axially placed hot filament cathode. By using two such cylinders and filaments balanced against each other in a bridge circuit, with the filaments connected in parallel, and by allowing the ionization to take place in only one cylinder, all undesirable fluctuations in the space-charge current were eliminated. Extremely small changes in the electron current could thus be measured accurately.

Table I shows the results obtained (the numbers are the ion speeds in volts at which ionization was observed to start with the indicated ion and gas).

TABLE I.

TABLE I. *Ion energies in electron-volts at which ionization was observed to start.* (The dash indicates that no ionization was observed when ion speeds up to 530 volts were used.)

	Na^+	K^+	Rb^+	Cs^+
Ne	130 ± 2	86 ± 2	232 ± 4 ?	—
A	105 ± 3	82 ± 2	135 ± 3	340 ± 2
N_2	88 ± 2	79 ± 2	260 ?	—
CO_2	95 ?	75 ± 2		—

The manner of taking readings was to raise the speed of the beam and to observe the resulting deflection of the galvanometer measuring the difference of electron currents in the two cylinders. As the voltage was raised, the galvanometer showed no change in deflection (or very slight change) until the critical ionizing speed (see table) was reached. At this point the galvanometer started deflecting strongly, the deflection rising linearly with the voltage. The case of Cs^+ in A was the only exception. In this case the deflection started rather gradually, the range from 340 to 370 volts being covered before a linear deflection was reached.

The accuracies given are only approximate. They cover principally the errors that may arise from voltage gradients whose effect on the positive ion beam is unknown. Such gradients never spread over a range greater than five volts in the course of the measurements. Results could always be reproduced well within the errors given. A few results, as indicated, are doubtful because of the weakness of the ionization.

The results tend to be lower than those obtained by Beeck and Mouzon,[3] roughly by about the same multiplying factor. It is suggested that their results may have been higher because of space-charge retarding of the positive ion beam. The principal evidence for this is that at the higher speeds such as are necessary for Cs^+ in A the difference disappears. The higher voltages would naturally tend to eliminate space-charge effects.

If energy and momentum are to be conserved in a collision, the maximum energy available for outside work such as ionization or excitation is easily calculated to be the initial energy of the positive ion beam multiplied by the factor $m_2/(m_1+m_2)$, where m_2 is the mass of the gas atom and m_1 is the mass of the colliding ion. The application of this factor to the values in the table above gives the results in Table II.

TABLE II. *Reduced voltages available for ionization.*

	Na^+	K^+	Rb^+	Cs^+
Ne	60.8	29.3	44.7	—
A	66.8	41.9	43.0	78.5
N_2	48.3	33.0	64.2	—
CO_2	62.4	38.8		—

This reduction factor still does not bring the ionizing energies down to the true ionization potentials of the gases involved. There is no apparent correlation among the reduced voltages; the chief point of interest is that they are all fairly low (under 100 volts).

The writer wishes to express his gratitude to Professor L. B. Loeb for his suggestions and continual guidance throughout the work. The writer wishes also to thank Doctor C. H. Kunsman for his kindness in providing the catalysts used as alkali ion sources.

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July 14, 1934.

[1] K. H. Kingdon, Phys. Rev. **21**, 408 (1923).

[2] See, for example, R. M. Sutton and J. C. Mouzon, Phys. Rev. **35**, 695 (1930); T. J. Câmpan, Phys. Zeits. **30**, 858 (1929); O. Beeck, Ann. d. Physik [5] **6**, 1001 (1930); and others.

[3] O. Beeck, and J. C. Mouzon, Ann. d. Physik [5] **11**, 858 (1931).