tion line appears in the visible range, up to about 800°C.

As proved by repeated measurements under the described conditions, *no double refraction of the thallium vapor could be ascertained within the given limits of sensitivity up to 800°C.* It should be noted that in the above experiments the magnetic field was varied through a wide range. Thus the effect of space quantization could not have been accidentally masked by effects due to a small longitudinal component of the magnetic field since this effect varies with the field while the double refraction looked for is independent of it.

As a check on the proper functioning of the apparatus we observed the magneto-optic effect in the neighborhood of 5350A. The effect was largest at about 850°C corresponding to a vapor pressure of about 4 mm Hg. It should be emphasized that this effect is not in contradiction to the principle of spectroscopic stability being just another form of anomalous dispersion.

Electron Scattering by Atomic Electrons

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The velocity distributions of electrons scattered by helium atoms, at angles ranging from 10° to 60°, have been measured for electrons having energies of 800, 1000, and 1200 volts. The curves have a well-defined narrow maximum where the scattered electrons have the same velocity as the primary electrons, this being the well-known elastic scattering. In addition, for each angle of scattering, a single broad peak is superposed on the continuous distribution of velocities ranging from the maximum down to zero. This represents the inelastically scattered electrons. The position of each peak is such that the velocity corresponding to it, is given approximately by $v=u\cos\theta$, where u is the velocity of the electron before impact and θ the angle of scattering. This is the formula for the velocity of an electron when scattered through θ, by a free electron initially at rest. The inference is that we may associate these inelastic peaks with collisions between the incident electrons and the atomic electrons when the binding energy is small in comparison with the energy transfers during the collision. Jauncey's theory of the breadth of the modified line in the Compton effect is discussed in relation to the breadth of the inelastic peak, on replacing the photon by the incident electron.

INTRODUCTION

WHEN an electron is moving with a velocity u towards another electron, initially at rest, application of the principles of conservation of energy and momentum lead to the conclusion that if the first electron is deviated through an angle θ, its velocity will be given by $v_1=u\cos\theta$, while that of the second electron will be given by $v_2=u\sin\theta$, a result which implies that the two paths after collision are necessarily at right angles.[1] It is not possible to make a direct test of this result because of the difficulty of securing a sufficient density of free electrons. However, a close approximation to the ideal case is realized when a beam of beta-rays is passed through matter of low atomic number, a condition which insures that the binding energy of the atomic electrons may be neglected in comparison with the energy transfer involved during a collision. Experiments show that the beta-ray tracks in a Wilson cloud chamber are frequently forked and that the angle between the tracks is close to 90°.[2] This implies that the fork results from a collision between the beta-ray and an atomic electron, the nucleus playing no part. The beta-ray, because of its high speed, has to pass so close either to an atomic electron, or to the nucleus, in order to suffer an appreciable deflection, that the observed deflection can be attributed to a

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[1] This follows easily from Eqs. (3) and (4) (given later) on omitting the binding energy.

[2] Rutherford, Chadwick and Ellis, *Radiations from Radioactive Substances*, p. 238 (Macmillan).

collision either with one or the other. If the beta-ray passes close to the nucleus, it is deviated without loss of velocity, but if it passes close to an atomic electron, it may go off in any direction θ with the reduced speed $u \cos \theta$, while the atomic electron recoils with the speed $u \sin \theta$. Thus the distribution of velocities of electrons scattered by such an idealized atom should, in principle, consist of just two groups of electrons, one having the full original speed u, and the other having the speed $u \cos \theta$. (Recoil electrons, with speed $u \sin \theta$, will of course be present.) Experimentally, it is found that, for electrons of moderate speed (say 100 to 2000 volts), the scattered electrons have a continuous distribution of velocities from a maximum down to zero. Renninger[3] investigated the distribution of energy of electrons scattered by nitrogen, neon, and argon, and found that the distribution of energy curves had very weak broad maxima at the energy $V_1 = V_0 \cos^2 \theta$, when the energy of impact V_0 was 2000 volts. The maxima disappeared completely for $V_0 = 500$ volts. The explanation is that, as the energy of the incident electron is diminished, it is more and more inaccurate to regard the nucleus and the atomic electrons as independent scattering centers. It becomes increasingly necessary to consider the complete atom as the scattering agent. Thus the scattering effect of an individual atomic electron, which is clearly shown for fast electrons by the 90° beta-ray fork referred to, is to be recognized with difficulty in Renninger's 2000 volt curves, and disappears completely from his 500 volt curves.

The purpose of the present investigation is to ascertain whether, with electrons of energy of the order of 1000 volts, the curves for the distribution of energies of electrons scattered at any selected angle, indicate effects which can be attributed to collisions between the incident electrons and the atomic electrons. The criterion is the presence of a maximum, at the energy value $V_1 = V_0 \cos^2 \theta$, in the energy distribution curve for electrons scattered at θ. Helium was chosen as the working gas because the energy of binding of its atomic electrons is very small and one would expect its electrons to approximate more to free electrons than the electrons in the gases investigated by Renninger. (Because of its lower ionization potential, hydrogen would probably show the effect more prominently than helium.)

Experimental Method

The apparatus is shown diagrammatically in Fig. 1. An electron gun, G, with slits 1 mm by 6 mm and 14 mm apart, is mounted so that it can be rotated about the center of a bulb which forms the collision chamber. The electron gun is attached to a support (not shown in the diagram) which is carried on a tungsten pivot in a side tube joined to the bulb so that the axis of

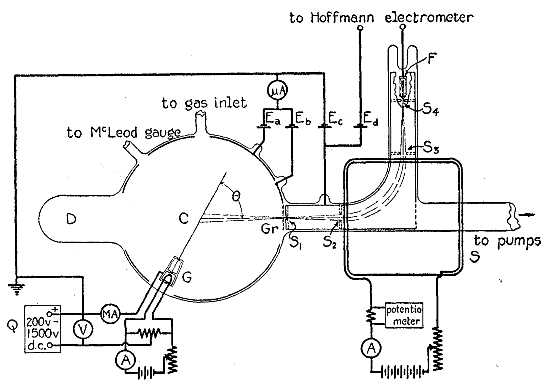

Fig. 1. Experimental apparatus.

rotation passes through C perpendicularly to the plane of the diagram. When the filament needs replacing, the electron gun is detached from its support and removed by cutting the side tube D. At the lower end of the support and sufficiently far removed from the collision center C, so as not to affect the electron paths, is a ring of soft iron. By means of an electromagnet which can be brought up from outside, the electron gun can be turned to any desired position. The bulb is lined with a layer of evaporated platinum, shown in the diagram by a thick line just inside the thin line representing the glass bulb. In front of the slit S_1 is a grid Gr made of fine platinum wires and connected to the outside by an independent lead. The angular width of the beam of electrons scattered at the collision center C,

[3] M. Renninger, Ann. d. Physik 9, 295 (1931); 10, 111 (1931).

into the analyzer, is defined by the slits S_1 and S_2 (0.20 mm by 6.0 mm, and 1.25 mm by 6.0 mm, respectively). The analyzer for separating out electrons of any desired velocity is of the new magnetic refocusing type recently described by Stephens.[4] For full details, the reader is referred to the paper cited. It is sufficient here to state that S represents the cross section of a square solenoid, and that S_1, S_2, S_3 and S_4 are the defining slits. The metal tube carrying the slits S_1 and S_2 fits the glass tube sufficiently well to insure that, because of the small dimensions of the slit S_1, a considerably lower pressure could be maintained in the analyzer than in the collision chamber. Helium, purified by passing it through charcoal in liquid air, is passed into the apparatus through a very fine capillary tube and pumped out continuously through the analyzer giving pressures of the order of 0.001 mm in the collision chamber. (On some occasions, when a fairly high pressure was needed, the gas was allowed to accumulate to the pressure desired.) As there were no greased ground glass joints in the apparatus, it was possible to bake out the whole apparatus at a temperature of 400°C.

As the electron gun, the platinum coating of the bulb, the grid *Gr*, and the slit S_1 with its carrying tube, were made of different materials, it was necessary to compensate for the various contact potentials to secure a really field-free collision region. This was secured by suitable voltages, provided by cell and potentiometer combinations represented by E_a, E_b and E_c. The correction was of course not so necessary for the high energies used in the investigation now reported as for the study of the much slower ejected electrons of energies below 5 or 10 volts, for which the apparatus was designed.

The high voltage d.c. supply unit, Q, which was used in place of B batteries had unusually good regulation. The ratio of the variation in the output voltage to that in the input voltage (about 110 volts) was 1 to 50, and the ratio of the variation in the output voltage to that in the load, at the load for which the unit was designed, was 1 to 1000. We are much indebted to Mr. O. H. A. Schmitt for designing the unit, an account of which will be published later.

[4] W. E. Stephens, Phys. Rev. **45**, 513 (1934).

The earth's magnetic field, in the vicinity of the apparatus was neutralized by a pair of Helmholtz coils. The current around the solenoid was measured by a potentiometer method, as in many cases a change of 0.1 percent in the magnetic field deflecting the electrons made a considerable change in the electron currents passing through the slit S_4 to the Faraday cylinder F. These electron currents were measured by a Hoffmann electrometer. (A battery, E_d, provided a retarding potential between F and S_4, when necessary for certain measurements.)

The experimental procedure was to set the electron gun so that the analyzer would accept the electrons scattered at a selected angle θ. Then by varying the deflecting magnetic field step by step, the distribution of velocities among the scattered electrons could be measured.

Results

The values of the electron currents to the collector in the analyzer are plotted as functions of the current, I_H, producing the deflecting magnetic field, for various angular settings of the electron gun. The values have been adjusted so that they all refer to the same gas pressure and to the same electron current from the gun. The usual correction for the change in length of the scattering path with θ has been made by multiplying the experimental values by the corresponding sin θ.

The curves obtained can be converted into true velocity distribution curves by dividing each ordinate by the numerical value of I_H. This comes about from the fact that the range of velocities, δv, accepted by the analyzer increases linearly with v (or I_H). It is well known in the technique of beta-ray analysis that this applies to the *continuous* spectrum of velocities, but does not apply to a *line* spectrum. For our present purpose, however, it is unnecessary to make this correction.

The various curves, assembled in Figs. 2, 3 and 4, are arranged so as to show the progressive shift in the positions of the inelastic peaks as the angle of scattering is increased from 11.9° to 31.9°. The magnitude of the scattered electron currents may be inferred from the scale alongside each curve. Figs. 5, 6 and 7, complete the results so far obtained. On account of the small scattered currents at these large angles the measurements were difficult. However these three

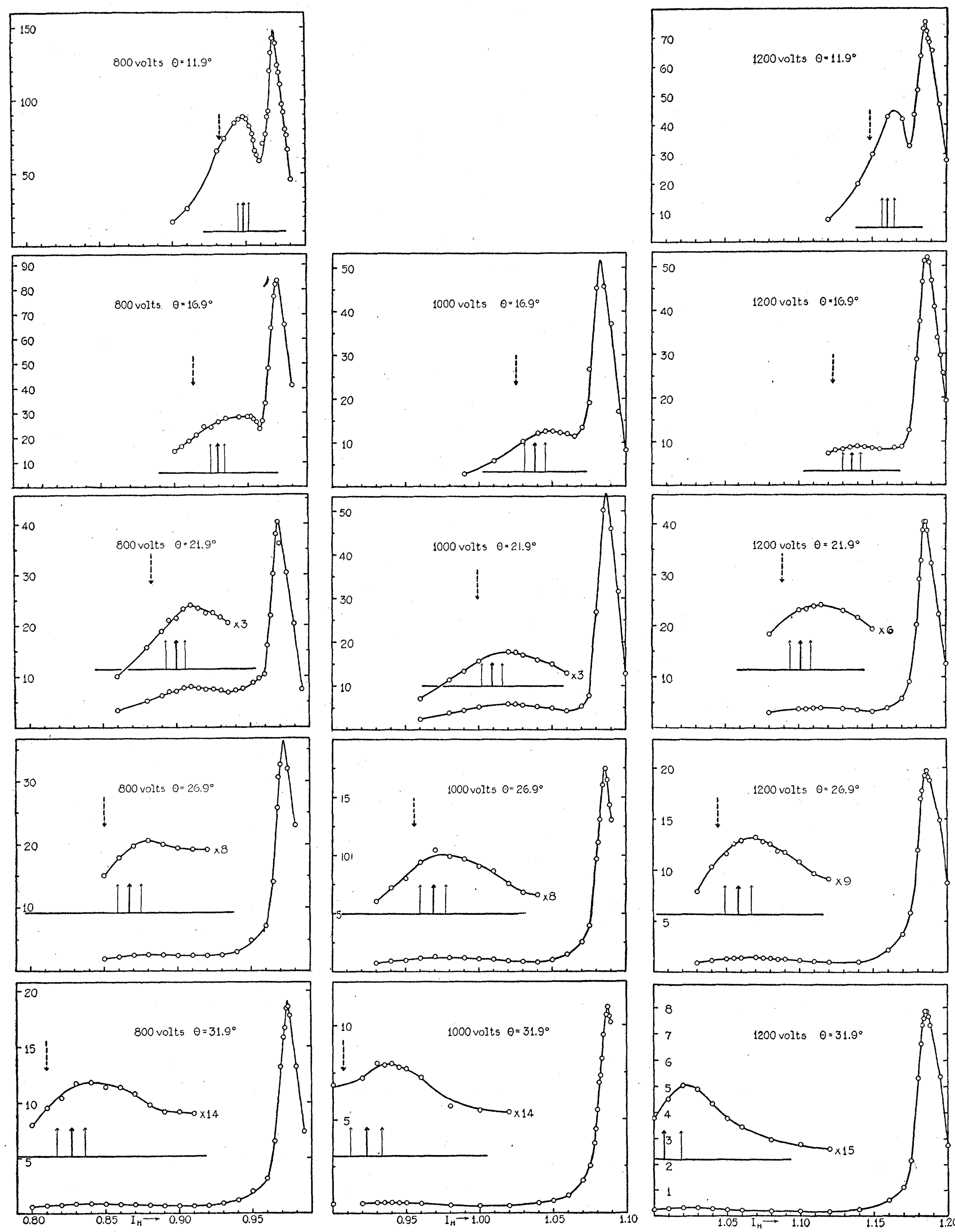

FIG. 2. FIG. 3. FIG. 4.

Velocity distribution curves for 800 to 1200 volt electrons scattered at angles between 11.9° and 31.9°. The I_H scale is identical for all curves in the same vertical column.

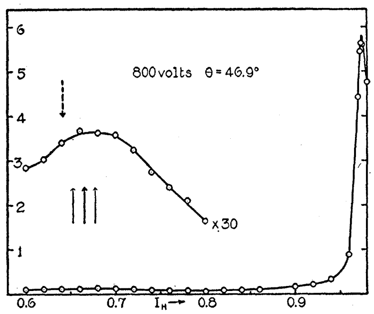

FIG. 5.

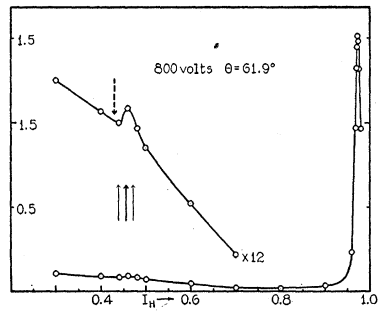

FIG. 6.

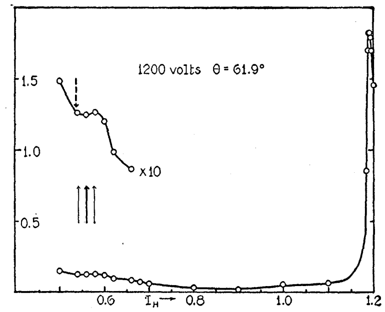

FIG. 7.

Velocity distribution curves for 800 volt electrons scattered at 46.9° and 61.9°, and for 1200 volt electrons at 61.9°.

curves were checked over and verified, and also it was shown that the background scattering with no gas in the apparatus was almost negligible.

Each curve has a sharp peak at the right-hand end, representing the elastically scattered electrons. The peaks are narrower than they appear to be at first glance because the zero for the I_H scale is far to the left of the origin of coordinates. The resolution of the apparatus is insufficient to separate the elastic peak from the lowest excitation peak, with electrons of energy above about 500 volts. In most of the graphs the inelastic peak is also plotted on a larger scale, the factor being indicated in each case. We have indicated by a heavy, unbroken arrow, the velocity (in units of I_H) that the electron should have if it were scattered through an angle θ, by a free electron initially at rest. (This is given by the change of velocity formula, $v_1 = u \cos \theta$.) Since the electron gun cannot be set to better than 0.5°, and since there may be an error of about 0.5° due to the electron beam not being collinear with the axis of the gun slits, we may assume that the effective angle is uncertain to $\pm 1°$. The corresponding "latitude" in the theoretical position of the peak is indicated by a pair of lighter arrows.

Discussion

As was stated in the introduction, the scattering of a beam of electrons by an atom in which the scattering may be done either by the nucleus or by one of the atomic electrons, but not by both, leads to a clear-cut distribution of velocities consisting of just two groups of electrons, those in the group scattered by the nucleus having the full velocity u, and those in the group scattered by an atomic electron, all having the same reduced velocity $u \cos \theta$. Experimentally we find that the positions of the inelastic peaks are in every case close to those given by the formula $v_1 = u \cos \theta$. There is however, in addition, a wide distribution of velocities among the inelastically scattered electrons. We can retain the idea of scattering by individual atomic electrons and still account for the spread in the velocity distribution, by taking over Jauncey's theory in which the finite width of the modified line in the Compton effect is accounted for. (It may not be superfluous to point out that there is a close analogy between the scattering of electrons and the Compton effect in x-ray scattering. The monochromatic unmodified line corresponds to the elastically scattered electrons, while the broad modified line corresponds to the distribution of velocities among the inelastically scattered electrons.) The simple derivation of the change of wave-length in the Compton effect assumes the atomic electron, with which the photon collides, to be at rest, and yields a monochromatic modified line. By assuming that the atomic electron is in motion, and may be moving in any direction whatsoever at the instant the photon strikes it, Jauncey[5] was able to explain the observed width of the modified line. To obtain the corresponding solution for the electron scattering problem, Professor Jauncey merely substitutes the energy and momentum of the incident electron for those of the photon in his theory.[6] The velocity

[5] G. E. M. Jauncey, Phys. Rev. **25**, 314, 723 (1925).

[6] We take pleasure in thanking Professor Jauncey for his interest in this problem.

w of the electron in the hydrogen atom is, according to the Bohr theory for circular orbits, given by $(1/2)mw^2 = Ve$, where V is the ionization potential. We may assume that this relation is true also for helium. The problem to be solved is that of finding the range in the velocity v_1 of the incident electron after collision with an atomic electron having a velocity w and moving in any direction whatsoever. The solution is

$$v_1 = u \cos\theta \pm [1 - 2(u/v_1)\cos\theta + (u/v_1)^2]^{\frac{1}{2}}, \quad (1)$$

where u is the velocity of the incident electron and θ the angle of scattering. When u is considerably larger than w this reduces to

$$v_1 = u \cos\theta [1 \pm (w/u)\sin\theta/\cos^2\theta]. \quad (2)$$

For w/u we may substitute the square root of the ionization potential (25 volts) divided by the energy of the incident electron expressed in volts, and obtain for 1200, 1000 and 800 volt electrons, the values, 0.102, 0.112 and 0.125. Then on substituting for θ we can get the "spread" of the energy distribution of electrons scattered by a free electron when this is moving in any direction with the assumed speed. The extent of this theoretical "spread" is indicated in the graphs (for scattering at angles up to 30°) by a horizontal line below the inelastic peak. It seems evident that the theory gives at least the order of magnitude of the observed spread. In view of the fact that formula (2) is only approximate in the range considered and that the peaks are evidently superposed on a considerable "background," it seems better to postpone further comments until results are available for the range 2000 to 10,000 volts, where we may expect the assumptions to hold better.

The v_1 in the expression $v_1 = u\cos\theta$, is obtained on the assumption that the atomic electron may be regarded as perfectly free. Following a procedure which was used in discussing the Compton effect shortly after its discovery, we may introduce the ionization potential of the atom, V, through the relation $Ve = (1/2)mw^2$ into the energy equation

$$(1/2)mu^2 = (1/2)mv_1^2 + (1/2)mv_2^2 + (1/2)mw^2, \quad (3)$$

where the last term takes care of the energy used for ionization, u and v_1 are the velocities of the incident electron before and after collision, and v_2 is the velocity of the recoil electron initially at rest. The momentum equations are

$$\begin{aligned} mu &= mv_1\cos\theta + mv_2\cos\phi \\ 0 &= mv_1\sin\theta + mv_2\sin\phi. \end{aligned} \quad (4)$$

(It will be noticed that the energy equation now takes notice of the fact that an atom is involved, although this is ignored in the momentum equation. The principal excuse for this simplification is that it enables a usable solution to be obtained.) The solutions are

$$\begin{aligned} v_1 &= u\cos\theta(1 - w^2/2u^2\cos^2\theta), \\ v_2 &= u\cos\phi(1 - w^2/2u^2\cos^2\phi), \end{aligned} \quad (5)$$

when w^2 is small compared to u^2. These equations state that the velocities of both the scattered electron and the recoil electron are reduced when the binding energy is taken into account. The values of the velocities of the scattered electrons, calculated in this way, are indicated in the graphs by a vertical *dotted* arrow. The "spread" of velocities about this new position calculated by Jauncey's formula would obviously be the same as before but displaced so as to be symmetrical about the vertical dotted arrow. The experimental results show that the peaks come closer to the theoretical position calculated when the binding energy is neglected. Too much importance should not be attached to this result as it may be due in part to an oversimplification of the problem leading to a slight displacement of the position of the peak, quite apart from considerations of the binding energy.

The expression[7] for the probability of finding an electron moving down unit solid angle, in a direction θ with the original direction of motion of a beam of electrons of velocity u approaching a free electron, initially at rest, is

$$(e^4/m^2u^4)4\cos\theta(1/\sin^4\theta + 1/\cos^4\theta). \quad (6)$$

This expression, because it includes also the probability of finding the recoil electron in the same solid angle as the scattered electron is useful in discussing experimental results since we cannot distinguish between the two electrons by

[7] O. Klemperer, *Einfuhrung in die Elektronik*, p. 206 (Springer). Originally given by Darwin in a somewhat different form.

an experiment. The expression has a very pronounced minimum at $\theta=45°$. This is in accord with the experimental fact that the scattered electron currents were exceptionally small at 45°.

It is to be recognized that there is a certain artificiality in simplifying the problem by assuming that the incident electron may be deviated either by the nucleus or by one of the atomic electrons, but the experimental results justify this as a rough approximation which should become better at higher speeds. Although the step from 800 to 1200 volts is relatively small, our results indicate that the peaks show a decided tendency to become sharper at the higher velocities.

Whenever classical methods of attacking a problem in atomic physics fail to give results in agreement with experiment it is generally found that better agreement may be obtained by the methods of wave mechanics, provided that a solution in a usable form can be found. We do not know of any explicit expression obtained by wave mechanics methods which can be used to check our results, but certain observations of Mott and Massey[8] have a qualitative bearing. The wave mechanics method, in which the atom is treated as a whole, leads to the result that while there is inelastic scattering at all angles with a wide range of velocities, the scattering is a maximum when the velocities and directions of the scattered and ejected electrons are such that momentum is conserved. They also state that interference effects may be expected when the ejected and scattered electrons have approximately the same energies, that is, in the neighborhood of 45°. Although, in accordance with Eq. (6) the scattering is very small near 45°, we have made careful measurements of the velocity distribution of the 800 volt electrons to see if any evidence of interference could be found. (Mott and Massey, of course, stated that the interference should be looked for in the angular distribution curve; they made no prediction as to what one should expect in a velocity distribution curve.) The curve obtained was smooth; no oscillations of appreciable magnitude were to be found (Fig. 5).

As the results of this investigation were being prepared for publication, a paper by Mohr and Nicoll[9] appeared on a related topic. Their method differed from ours in that they selected electrons with a certain loss of energy after collision and studied their angular distribution. They find definite evidence for collisions with the atomic electrons in hydrogen when the energy is as low as 100 volts. In the case of helium, the evidence is much less marked, appearing as very slight humps, and then only on the curves for the higher energy losses. Comparison with our results cannot be effected, as they did not use electrons with more than 300 volts energy.

The results already obtained suggest an extension of the investigation over a far wider range of electron velocities. It is desirable to investigate, whether or not, as we go to higher velocities the inelastically scattered electrons tend to crowd into a progressively narrowing peak at $v_1=u\cos\theta$, while the background becomes relatively less and less. Investigation with different gases should be made to test the view that atoms of high atomic number, and therefore of higher average binding energy, should have less pronounced peaks in the energy distribution curves than atoms of helium or hydrogen.

Had time permitted we should have liked to extend the scope of this investigation and to determine the shapes of the curves more accurately. However, the transfer of all the equipment of the department to a new physics laboratory will certainly involve a serious delay in the resumption of the work. This, together with the fact that one of the authors is unable to continue with the collaboration, is a logical reason for publishing the results as they are.

[8] Mott and Massey, *The Theory of Atomic Collisions*, pp. 167–172 (Oxford University Press).

[9] C. B. O. Mohr and F. H. Nicoll, Proc. Roy. Soc. **A144**, 596 (1934).