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## The Absorption of Monochromatic X-Rays of Short Wave-Length

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With a Société Genevoise double crystal spectrometer, the mass absorption coefficients of eight different elements extending from hydrogen (from paraffin and carbon) to lead were determined for a wave-length of 208.6 x-units. An attempt to make a similar series of measurements on the  $K\beta_1$  line of tungsten failed because of the low intensity of the x-rays. With the same instrument but with the first crystal removed as a single crystal spectrometer, measurements were extended down to 138 x-units. The atomic absorption coefficients determined from these measurements were found to be given accurately by the equation  $(\mu_a/Z) = C_k Z^{3.18} \lambda^q + (\sigma/\sigma_0)\sigma_0$ , where  $\sigma/\sigma_0$  is given by the

Klein-Nishina formula rather than the Breit and Dirac or Compton formulas for scattering. In the case of hydrogen however the atomic absorption coefficient (calculated from the difference between measurements on paraffin and carbon) was within experimental error equal to the Thomson scattering for the range of wave-lengths used. The constant  $q$  appears to have the value 2.50 for this region, and  $C_k$  the value  $5.81 \times 10^{-7}$ , if  $\lambda$  is measured in centimeters. In general the absorption coefficients agree fairly well with those of other observers except for a slight discrepancy in the case of copper; a peculiarity is pointed out in the case of silver.

### INTRODUCTION

IT is generally recognized that an accurate measurement of the x-ray absorption coefficients in the region of wave-lengths shorter than 0.250 Angstrom would be of considerable value, both in checking the validity of the empirical formula

$$\mu_a = C_k Z^p \lambda^q + K_1 Z \sigma_0 \quad (1)$$

for the total absorption and in giving additional information as to the proper values of the exponents  $p$  and  $q$ , but more especially since it is only for short wave-lengths that the last term representing the scattering is an appreciable part of the total absorption. The most direct test of the various formulas for the intensity of scattered x-rays as derived by A. H. Compton,<sup>1</sup> Breit<sup>2</sup> and Dirac,<sup>3</sup> and Klein and Nishina<sup>4</sup> is a comparison

with the experimental values of the intensity  $I_\phi$  scattered at any given angle  $\phi$ . Unfortunately it is very difficult to obtain experimental values of  $I_\phi$  which are sufficiently reliable for this test, particularly since the formula of Breit and Dirac and the Klein-Nishina formula give nearly the same result for the scattering at a given angle  $\phi$ . On the other hand, when the above formulas are applied to give the total scattering, the formulas of Compton and Breit and Dirac give about the same result, but the result given by the Klein-Nishina formula differs quite appreciably.

Unfortunately accurate absorption measurements are also difficult to make in this region. Hewlett,<sup>5</sup> Richtmyer,<sup>6</sup> Allen<sup>7</sup> and others have made such measurements, but there is considerable divergence in their results. One of the major difficulties in making absorption measurements

<sup>1</sup> A. H. Compton, *Phys. Rev.* **21**, 483 (1923).

<sup>2</sup> G. Breit, *Phys. Rev.* **27**, 242 (1926).

<sup>3</sup> P. A. M. Dirac, *Proc. Roy. Soc.* **A111**, 405 (1927).

<sup>4</sup> Klein and Nishina, *Zeits. f. Physik* **52**, 853 (1928).

<sup>5</sup> C. W. Hewlett, *Phys. Rev.* **17**, 284 (1921).

<sup>6</sup> F. K. Richtmyer, *Phys. Rev.* **18**, 13 (1921).

<sup>7</sup> S. J. M. Allen, *Phys. Rev.* **27**, 266 (1926).

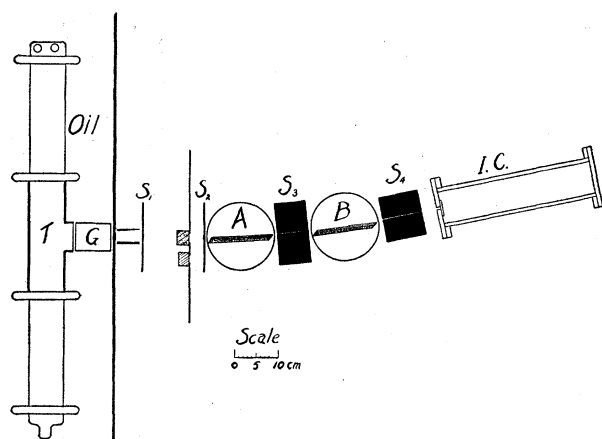


FIG. 1. Schematic diagram of apparatus.

for wave-lengths of 250 x-units and less is that in order to obtain sufficient intensity to make reliable measurements, the slit system must be so wide that  $\Delta\lambda$  is a large fraction of  $\lambda$  and the uncertainty in the effective wave-length is large. For this reason this investigation was undertaken to determine the feasibility of employing a double crystal spectrometer in the measure of absorption coefficients in this region.

#### APPARATUS

The instrument used was made by the Société Genevoise, and has been previously described by A. H. Compton.<sup>8</sup> A 140-kilovolt line-focus tube *T* (see Fig. 1) with a tungsten target was used as a source of x-rays. This tube was immersed in oil in a lead box mounted upon a lathe bed to permit the adjustment required with the Compton type of double crystal spectrometer. A short piece of glass tubing *G* covered at the ends with celluloid provided an air path for the x-rays in the box. The tube-target was cooled with kerosene circulated by a pump through a copper cooling coil immersed in running water. The high voltage was supplied by a transformer, rectified by 250 kilovolt valves and smoothed by condensers. The voltage was kept constant with the aid of a pre-reading voltmeter in the primary circuit and manually controlled rheostats. The tube current was read on a milliammeter in the aerial system reflecting a beam of light on a wall nine feet away. This current was kept constant

by a voltage stabilizer in the primary of the filament transformer circuit and an additional manually operated rheostat.

Two very large calcite crystals were used in the (1, 1) position according to the notation of Allison<sup>9</sup> to reflect the  $K\alpha_1$  line of tungsten into a large copper ionization chamber containing air at a pressure of thirty atmospheres. The ionization current was measured with a Compton electrometer operated at a sensitivity of about 3000 mm per volt. Especial care was taken to shield this electrical system from scattered x-rays, and dry air was passed slowly through the shields about the electrometer to ionization chamber connection. Various tests were made to insure that the x-radiation entering the ionization chamber was entirely the twice reflected  $K\alpha_1$  radiation. The materials investigated were all mounted upon a large disk between the first two spectrometer slits; by rotating this disk through a small angle a sample could be removed, and by an additional small rotation another substituted.

#### RESULTS

The materials investigated were paraffin, graphite, aluminum, copper, silver, tantalum, tungsten and lead. The paraffin was later analyzed for its carbon and hydrogen content, and the graphite and aluminum for small impurities. The other elements used were believed to be of a high degree of purity. The values obtained for the mass absorption coefficients of

TABLE I. Mass absorption coefficients for  $K\alpha'$  line of tungsten ( $\lambda = 208.6x$ ), with double crystal spectrometer.

Substance	No. Obs.	$\mu_m$	Prob. error
Lead	13	5.07	0.04
Tungsten	10	3.81	0.03
Tantalum	12	3.62	0.02
Silver	12	5.99	0.09
Copper	26	1.617	0.012
Aluminum	21	0.278	0.002
Carbon I	23	0.177	0.002
Carbon II	15	0.174	0.003
Paraffin	14	0.209	0.002

Hydrogen (calculated<sup>1</sup>) 0.40

<sup>1</sup> Value for hydrogen calculated from difference between measurements on carbon and paraffin.

<sup>9</sup> S. K. Allison and J. H. Williams, Phys. Rev. **35**, 149 (1930).

<sup>8</sup> A. H. Compton, R. S. I. **2**, 365 (1931).

the materials used are given in Table I. Each individual measurement consisted of three values of  $(I_0+I_b)$ , three values of  $(I+I_b)$ , and then three more values of  $(I_0+I_b)$ , where  $I$  and  $I_0$  are, respectively, the ionization currents due to the twice reflected x-ray beam with the absorber in the path and removed.  $I_b$ , the residual ionization current due to cosmic rays and local gamma radiation, was determined from an average of three or more readings taken at about two hour intervals. No ionization current determination was discarded unless an obvious experimental error had occurred during its reading.

Following this series of measurements the spectrometer was set on the  $K\beta_1$  line of tungsten and a few measurements taken. However, the intensity which could be obtained with this line was small in comparison with the residual ionization, and for this reason it was decided that it would be preferable to remove the first crystal and use the instrument as a single crystal spectrometer. There is, though, one interesting feature in connection with the measurements made with the two crystals set on this line. The wave-length of the  $K\beta_1$  line of tungsten is according to J. H. Williams<sup>10</sup> 184.0 x-units, while the  $K$  absorption edge of tantalum has been reported to be 183.6 x-units. The value obtained for the absorption coefficient of tantalum was  $3.00 \pm 0.09$ , a value which in spite of the rather large probable error seems to be somewhat too high to be on the long wave-length side of the  $K$  edge, and much too low to be on the short wave-length side.

With the instrument as a single crystal spectrometer the slit system was adjusted so that  $\Delta\lambda$  was 5 x-units. Absorption measurements were

TABLE II. Mass absorption and atomic absorption ( $\times 10^{25}$ ) coefficients.

Z	$\lambda=208.6x$			$\lambda=170x$			$\lambda=140x$		
	$\mu_m$	$\mu_a/Z$	$\tau_a/Z$	$\mu_m$	$\mu_a/Z$	$\tau_a/Z$	$\mu_m$	$\mu_a/Z$	$\tau_a/Z$
82	5.07	211.5	206.0						
74	3.81	156.3	150.8	0.36	384.0	378.8			
73	3.62	148.5	143.0	8.83	352.0	346.8			
47	5.99	229.0	223.5	3.62	138.4	133.1			
29	1.617	58.4	52.9	0.992	35.8	30.5	2.26	86.3	81.2
13	0.278	9.51	4.06	0.222	7.60	2.35	0.192	6.57	1.51
6	0.176	5.80	0.35	0.165	5.57	0.20	0.157	5.18	0.13
1	0.40 <sup>1</sup>	6.65	0.001 <sup>2</sup>	0.40 <sup>1</sup>	6.65	0.001 <sup>2</sup>	0.40 <sup>1</sup>	6.65	0.001 <sup>2</sup>
	$\sigma=5.45$			$\sigma=5.24$			$\sigma=5.05$		

<sup>1</sup> Calculated from carbon and paraffin.

<sup>2</sup> Obtained by an extension of straight lines of Fig. 3.

<sup>10</sup> John H. Williams, Phys. Rev. 40, 791 (1932).

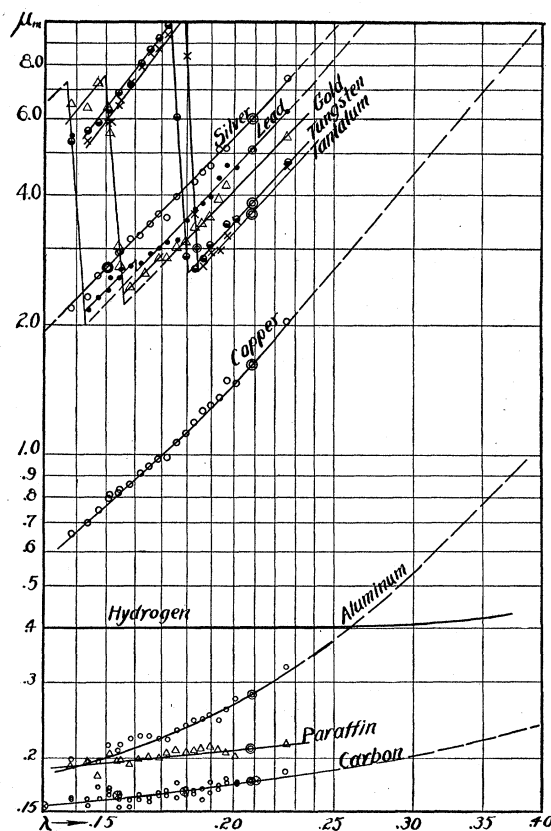


FIG. 2. Mass absorption curves for elements studied. Double circles at 0.2086A and 0.184A for tantalum represent measurements with double crystal spectrometer. Large circles with cross on carbon curve Hewlett's values for this region. Heavy circle at 0.151A for silver mean of fifteen separate readings with single crystal.

then carried down to 138 x-units. The results are shown in Fig. 2, where the solid lines represent the best average values of  $\mu_m$  from 0.140A up to 0.230A. The dotted lines represent extensions to fit the average values of other observers.

### DISCUSSION OF RESULTS

The values of  $\mu_m$  as measured at 208.6 x-units with the double crystal spectrometer, and taken from Fig. 2 at 170 and 140 x-units are given in Table II. In the second column is given the corresponding value of the atomic absorption coefficient divided by the atomic number of the element, and multiplied by a factor of  $10^{25}$ .

If the usual absorption equation

$$\mu_a = C_k Z^p \lambda^q + (\sigma/\sigma_0) Z \sigma_0 \quad (2)$$

is divided by the atomic number  $Z$ , there results

$$\mu_a/Z = C_k Z^{p-1} \lambda^q + (\sigma/\sigma_0)\sigma_0, \quad (3)$$

where  $\sigma_0$  is the classical expression for the scattering per electron, given by the equation

$$\sigma_0 = (8\pi/3)(e^4/m^2c^4) = 6.58 \times 10^{-25} \text{ cm}^2 \quad (4)$$

and  $(\sigma/\sigma_0)$  is a function of  $\lambda$ , given by the theoretical formula of either Compton, Breit and Dirac or Klein-Nishina. Since  $\sigma/\sigma_0$  is constant<sup>11</sup> for a given  $\lambda$ , it appears that if the proper constant term is subtracted from  $\mu_a/Z$  and the logarithm of the remainder,  $\tau_a/Z$ , plotted against the logarithm of  $Z$ , a straight line should result. The third column of Table II gives the resulting values of  $\tau_a$  after the values of  $\sigma$  as shown have been subtracted, and in Fig. 3 are plotted on logarithmic scales the values of  $\tau_a$  against  $Z$ . The proper values of  $\sigma$  to be used for each  $\lambda$  are accurately and easily obtained from such a graph, since a slight change in  $\sigma$  affects the larger values of  $\tau_a$  scarcely at all, but causes a large variation in the smaller values. There is one glaring discrepancy in Table II; if  $\mu_m$  is calculated for hydrogen in the usual manner from the absorption coefficients of carbon and paraffin, a constant value of 0.40 is obtained, giving a value of  $6.65 \times 10^{-25} \text{ cm}^2$  for  $\mu_a/Z$ , which in spite of the large probable error is almost exactly  $\sigma_0$ . The value of  $\tau_a$  for hydrogen is obtained by extending the straight lines of Fig. 3. There is another peculiarity which should be mentioned; the points for silver in each case fall just slightly under the straight lines, while the points for tantalum and tungsten for 170 x-units are considerably below. The slope of the lines is 3.18 except for the portion of the 208.6 line

TABLE III. Comparison of experimental and theoretical values of scattering per electron.

$\lambda$	$\sigma/\sigma_0$	Compton	Breit, Dirac	Klein-Nishina
208.6	0.828	0.811	0.813	0.825
170.0	0.796	0.778	0.782	0.794
140.0	0.768	0.743	0.746	0.768

$$\sigma_0 = 6.58 \times 10^{-25} \text{ cm}^2$$

<sup>11</sup> On both classical and quantum theories, this statement is valid only when there occurs no interference between the rays from the different electrons in the atoms, i.e., when no "excess scattering" occurs. The relation is thus more strictly valid for the light elements.

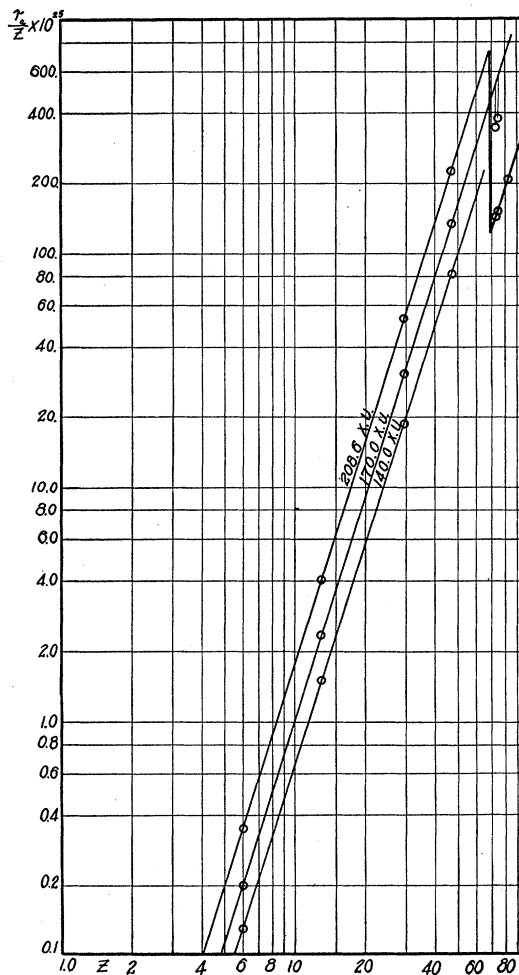


FIG. 3.  $\tau_a/Z$  plotted against  $Z$  for three wave-lengths.

beyond the  $K$  absorption edge, which has a slope of 3.10.

In Table III the experimental values of  $\sigma/\sigma_0$  are compared to the theoretical values. The agreement with the values calculated from the Klein-Nishina formula is seen to be exceptionally good. It is believed that this is really a significant test of these theories, since the difference between the predictions of the various theories seems to be greater than the experimental error. The exponent of  $\lambda$  indicated by these values of  $\tau_a$  is

TABLE IV. Exponent of  $\lambda$  in atomic absorption equation.

Pb	(Ta,W) <sup>1</sup>	(Ta,W) <sup>2</sup>	Ag	Cu	Al	(C) <sup>3</sup>
2.56	2.6	3.3	2.54	2.54	2.47	2.43

<sup>1</sup> Long wave-length side of  $K$  absorption edge.

<sup>2</sup> Short wave-length side of  $K$  absorption edge.

<sup>3</sup> Large probable error, since  $\tau_a$  is very small.

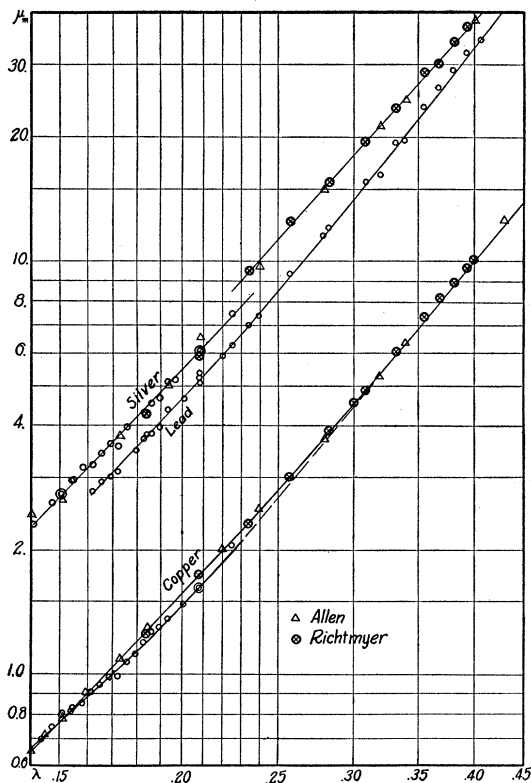


FIG. 4. Comparison of values of  $\mu_m$  for silver and copper with those of Richtmyer and Allen.

(for the four elements far removed from their  $K$  absorption edges) 2.50, with a large probable error.

Referring again to Fig. 2, it may be mentioned that the values of the  $K$  absorption discontinuities agree with the published values of other observers. It will be noticed that in the case of lead and gold the values of  $\mu_m$  on the long wavelength side of the  $K$  discontinuity may be interpreted as lying on a curve parallel to the main absorption curve but somewhat above it, up to a wave-length corresponding to an energy sufficient to remove an electron from the  $K$  to the  $L$  energy level. However, such a condition does not appear in the values obtained for tantalum and tungsten. Also the fit will be nearly as good for lead and gold if the absorption

curve is given an increased curvature in this region.

For the most part the values of  $\mu_m$  obtained agree fairly well with those of other observers. There are though two cases for which this is not true. In Fig. 4 the author's values are shown plotted with those of Allen<sup>7</sup> and Richtmyer<sup>6</sup> for silver, lead and copper. In the case of silver, all values agree quite well up to about 230 x-units. For longer wave-lengths than this their values lie on a higher curve as indicated. For the case of lead no such anomaly occurs, and all values lie on a smooth curve; this curve was included for comparison with that of silver. In the case of copper the author's values diverge considerably from those of others in this wave-length region.

With a value of 2.50 for the exponent of  $\lambda$ , the constant  $C_k$  in Eq. (3) has a value of  $5.8 \times 10^{-7}$ . It should be noted however that a very small variation in the exponent of  $\lambda$  will produce large changes in the constant  $C_k$ , since  $\lambda$  in this equation is to be measured in centimeters. The fact that the range of wave-lengths used was rather limited together with the fact that  $\lambda$  is small prohibits a really good test of the dependence of  $\tau_a$  on  $\lambda$ . Nevertheless the values shown in Table IV, with the exception of the high value for tungsten and tantalum on the short wave-length side of the  $K$  absorption edge, seem to indicate a value of the exponent of  $\lambda$  considerably smaller than previously reported for this region. It is perhaps interesting to compare this value with Messner's<sup>12</sup> recently reported value of 2.50 for the extremely long wave-length region, and the value of 2.56 obtained by Patten<sup>13</sup> in the study of the  $L$  absorption edges of the heavy elements.

It is a great pleasure to express my appreciation to Professor A. H. Compton for help and encouragement throughout the course of this work.

<sup>12</sup> R. H. Messner, *Zeits. f. Physik* **85**, 727 (1933).

<sup>13</sup> C. G. Patten, *Phys. Rev.* **45**, 662 (1934).