mediate field for this line. The strong field case is plotted in Fig. 3(a) for comparison. ($E=180{,}000$ volts/cm.)

It seems probable that for some field between 100 and 400 volts/cm, H_β show a smaller doublet separation, behaving similar to H_α at 500 volts/cm; however fields of this order of magnitude will be too strong for lines higher in the Balmer series, since the splitting due to the Stark effect increases approximately as the square of the total quantum number n. It is to be expected, therefore, that these higher series lines will be even better indicators of the field present in the discharge tube.

In conclusion I wish to express my appreciation to Professor G. Breit, who suggested the problem, and has given many helpful suggestions throughout the progress of the work. I also wish to thank Dr. I. Lowen who helped with some of the computations on H_β.

DECEMBER 15, 1934 PHYSICAL REVIEW VOLUME 46

The Production of Intense Beams of Positive Ions

R. Dudley Fowler and G. E. Gibson, *Radiation Laboratory, Department of Physics and Department of Chemistry, University of California*

(Received September 13, 1934)

The theory of the extraction of intense beams of positive ions from the low voltage arc is discussed, and the develops ment of a requisite apparatus described, with which beam- of several milliamperes of ions from H_2, He, N_2 and CO_2 have been produced. The influence of the gas pressure and current density in the arc, and of the extractor voltage and geometry on the density of the beams has been studied. The electrostatic divergence, and the magnetic convergence of the beams is discussed, in connection with the observed spreading. Magnetic analyses of the beams from hydrogen have shown them to consist of H_3^+, H_{3-1}^+, H_{3-2}^+, H_2^+, H_{2-1}^+ and H_1^+, the relative composition being a function of pressure. At a pressure of 27 bars the beams consist almost entirely of H_3^+, and at 96 bars pressure, entirely of protons. Magnetic analyses of the beams from N_2, O_2, and CO_2 have also been made. The secondary emission from a monel target has been measured for H_1^+, H_2^+, H_3^+, He^+, N_2^+, N_{2-1}^+ and CO^+, and found to be 1 to 2 per incident ion in all cases. The observed target heating has been used as a rough check on the current measurements.

Very rapid strides in the study of the atomic nucleus have been made recently, as a consequence of the use of electrically produced beams of high velocity ions[1] to effect nuclear transmutations. Currents of from one-tenth to one hundred microamperes of protons and deutons having energies equivalent to falling through fields of from thirty thousand to three million volts have been used very fruitfully by the various workers in the field. The employment of more intense beams will no doubt enable many new phenomena to be noted, and perhaps permit transmutations on a spectroscopically, if not chemically observable scale. A source of positive ions giving beams of several milliamperes has been developed and will be herein described.

Cockroft and Walton, and their collaborators at Cambridge, use a canal-ray tube as a source of ions. A discharge of twenty to fifty milliamperes is maintained between a perforated iron cathode and the inside of a meter-long iron anode, by a potential difference of from twenty to fifty kilovolts. The discharge tube and its accompanying source of rectified power is maintained at from thirty to nine hundred kilovolts above ground potential, by transformers. A cylindrical tube at ground potential placed coaxially beneath the hole in the cathode accelerates the ions emerging therefrom to a velocity corresponding to the potential difference between ground and the source. The ions are then magnetically deflected through ninety degrees onto the desired target. This method produces working beams up to one hundred microamperes, and has the advantage of operating at low gas pressures.

[1] Cockroft and Walton, Proc. Roy. Soc. **A129**, 477 (1930); Lawrence and Livingston, Phys. Rev. **40**, 19 (1932).

Lawrence and his co-workers derive their fast ions from a cloud produced at the center of the acceleration apparatus by the impact of electrons from a hot tungsten filament. Ions from the cloud are accelerated in a series of twenty thousand volt increments, finally emerging from the accelerators with velocities up to that corresponding to a direct fall through a three million volt field. Beams up to fourteen microamperes are thus produced.

In each type of source, the ions are produced by electron impact and then move towards a negative electrode. The greater number of ions strike the electrode, and are neutralized; a few pass through an orifice of some sort in the electrode, and of these, yet fewer are accelerated to the desired speed.

In the low voltage arc, the current is practically limited only by the electron emissivity of the hot cathode, and the potential drop across the arc, as its name implies, is small—generally not over two hundred volts. A ten-ampere discharge therefore would necessitate the expenditure of two kilowatts of power. If a negative, perforated electrode at fifty thousand volts, inserted in the arc, withdraws to and through itself a total of twenty milliamperes of positive ions, the power expended is one thousand watts. A few watts will be required to heat the arc cathode, so that the aggregate power required is about three and a half kilowatts.

In view therefore, of the comparative ease with which large numbers of positive ions can be produced in the low voltage arc, it was deemed the best source from which to derive more intense ion beams.

The theory of the complicated phenomena occurring during electrical discharges through gases has been treated by Thomson[2] and notably by Langmuir and Compton.[3] In the low voltage arc, ions are generated uniformly throughout most of the space between the cathode and the anode. This region, called the plasma, contains approximately equal numbers of positive ions and electrons, and is therefore relatively field-free, in contradistinction to the regions of high field, contiguous to anode, cathode, and arc walls, called sheaths. Because of the higher mobility of the plasma electrons, the walls of the discharge tube, or floating probes inserted in the plasma, become negatively charged, repelling the advance towards them of electrons, and attracting positive ions. This results in the formation of a region called a positive ion sheath in which the electron concentration is lower than that of the positive ions, and which appears "darker" than the plasma, because fewer atomic excitations occur in it. The anode, or a positively charged probe, similarly repels positive ions, and attracts electrons, with the formation of an electron sheath—the anode dark space.

I. Extraction of Positive Ions From a Plasma

A. The effect of collector voltage

When a probe is made negative with respect to the plasma, a positive ion current of density I_p flows to it, and electrons are repelled; and if the negative potential is made sufficiently great, no electrons reach the probe. The classic experiments of Langmuir and Mott-Smith[4] with probes in discharges in mercury vapor have shown that this occurs when the probe voltage reaches -5 to -10 volts with respect to the surrounding plasma, and that the current flowing to a plane collector is practically independent of applied voltages greater than these values, even up to negative voltages of 1200. The density of the collector current depends only on the number of ions crossing each square centimeter of the surface of the ion sheath, that is, on the random current density I_p of ions passing per second per unit area across any imaginary plane within the plasma.

The surface of a plane sheath is considered by Langmuir[3] as a plane emitter of positive ions, at zero potential. The flow of current between this emitter and a plane collector at potential V is

$$I_p = 5.462 \times 10^{-8} M^{-\frac{1}{2}} V^{\frac{3}{2}} X^{-2} \text{ amp./cm}^2, \quad (1)$$

where M is the "molecular weight" (O=16) of the ion, X is the thickness of the sheath in cm, and V is expressed in volts.

[2] Thomson, *Conduction of Electricity through Gases*, 3rd Ed., Cambridge Univ. Press, 1933.

[3] Langmuir and Compton, (a) Rev. Mod. Phys. **2**, 123 (1930); (b) Rev. Mod. Phys. **3**, 191 (1931); (c) Langmuir, J. Frank. Inst. **214**, 275 (1932).

[4] Langmuir and Mott-Smith, G. E. Rev. **27**, 449, 538, 616, 762 (1924).

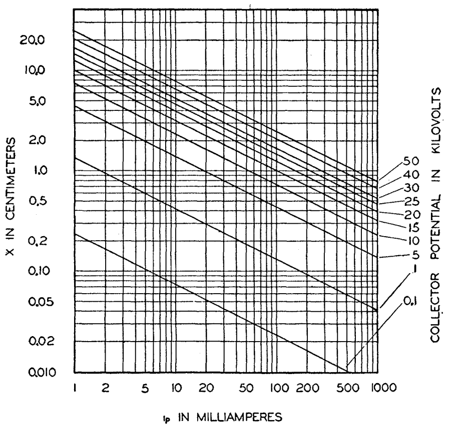

FIG. 1. Sheath thickness and proton current.

In a unipolar discharge, such as the electron flow between a plane cathode and a plane parallel anode, X is constant, and the current between the electrodes is limited, at any prescribed voltage, by the space charge, according to Eq. (1). However, when a plasma serves as an emitter of ions, I_p is predetermined, and X will assume such a value, according to the collector voltage, that Eq. (1) is fulfilled.

Sheath thicknesses as functions of I_p, at various voltages, for protons, are given in Fig. 1. From these curves the sheath thickness for any ionic species may be obtained, by multiplying the value given by the fourth root of the ratio of the ionic charge to its "molecular weight."

B. Effect of the density of the arc current

It would be expected that the random ion current density I_p should be proportional to I_x, the drift current density. (I_x = the current between anode and cathode divided by the cross section of the arc chamber.) The experiments of Langmuir and Mott-Smith,[4] have shown this to be very approximately so, for discharges through mercury vapor at low pressures. The limiting value of I_p will be attained when all the gas molecules become ionized. If the pressure of the gas is p bars, then crossing any unit plane area in the gas there will be

$$n = p/2mv \text{ molecules per sec. per cm}^2, \quad (2)$$

where m is the mass of the molecule, and v its velocity. Now

$$v^2 = 3RT/Nm, \quad (3)$$

where N = Avogadro's number, and R is the gas constant. Whence, substituting $m = M/N$,

$$n = pN(1/12RTM)^{\frac{1}{2}}. \quad (4)$$

If these n molecules all form univalent ions,

$$I_p = ne = pNe(1/12RTM)^{\frac{1}{2}} \quad (5)$$

or evaluating $e = 1.591 \times 10^{-19}$ coulombs, and $R = 8.32 \times 10^7$ ergs per °K per mol,

$$I_p = 3.06p/(MT)^{\frac{1}{2}} \text{ amp./cm}^2. \quad (6)$$

Limiting values of I_p for various ions, at one bar, and at temperatures of 1000°K and 10,000°K, calculated by means of Eq. (6), are given in Table I.

TABLE I. *Limiting values of I_p (amp. per cm²) for various ions at pressure of one bar.*

	I_p (amp./cm²)							
T°K	H_1^+	H_2^+	H_3^+	He^+	CO_2^+	CO^+	N_2^+	N^+
1000	0.0968	0.0684	0.0559	0.0484	0.0146	0.0183	0.0183	0.0259
10000	.0306	.0214	.0177	.0154	.0046	.0058	.0058	.0082

C. Effect of gas pressure

The low voltage arc functions over a limited range of gas pressures. The lower limit occurs when the mean free path for electrons is approximately equal to the distance between the electrodes, and the upper, when the mean free path is so short that the electrons cannot acquire the ionization velocity between collisions.

Over this range it is to be expected that for a constant value of the drift current I_x, I_p should vary linearly with the gas pressure; for the mean free paths of electrons in the plasma varies inversely as the pressure, and the shorter the mean free path, the greater the number of collisions resulting in ionization. The work of Langmuir and Mott-Smith previously cited has shown this to be true for discharges in mercury vapor.

The random ion current density I_p produced in any gas by a predetermined value of I_x, is in general a complicated function of I_x, depending upon the probabilities of ionization by electron, positive ion, and photon impact, at any given pressure.[5]

[5] See reference 3a, pp. 124–135.

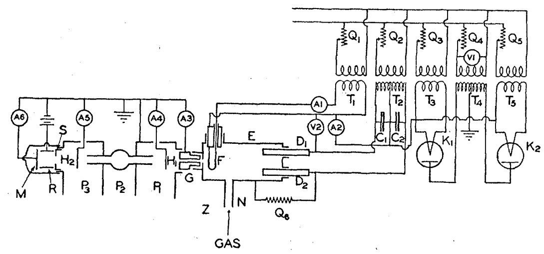

FIG. 2. Schematic diagram of apparatus.

II. Experimental

A. Design and operation of apparatus

Divers forms of apparatus were tried, the most satisfactory of which is schematically represented in Fig. 2. An arc is maintained in the chamber E, between the hot cathode F and anodes D_1 and D_2, and ions from the resulting plasma are extracted by a negatively charged, perforated electrode G. For convenience in measuring, and to eliminate long-path discharges, G is maintained at ground potential and the arc and its power supply at high voltage. The power for the filament and arc currents is obtained from the 220 volt commercial supply by the use of transformers T_1 and T_2, which are insulated between windings for 30,000 volts.

The hot cathode F is heated by a current of 50–75 amperes from T_1 (220 to 10 volts), the current being determined by resistance Q_1, and measured by ammeter A_1. The arc current is supplied by T_2 (220 to 440 volts) and measured by A_2; its value is limited either by resistance Q_2 or by the saturation emission from F. Two anodes, and center-tapping of T_2 allow the arc to run during both halves of the alternating-current cycle. The 500 ohm resistance Q_6 between the metal walls of E and one of the anodes, makes the arc self-starting. The voltage drop across the arc is measured by V_2.

The arc system is maintained at from 10,000 to 22,000 volts (r.m.s. value) positive with respect to G, by T_4 (220 to 44,000 volts) and a full-wave rectifier consisting of kenotrons K_1 and K_2. T_3 and T_5 (220 to 10 volts) supply heating current for the kenotrons. The high voltage d.c. output is adjusted by resistance Q_4, and estimated by voltmeter V_1 across the primary of T_4.

The desired gas is admitted to E, at N, through a variable capillary "leak," from a commercial cylinder fitted with a reducing valve, and removed through G, by pumps at P_1, P_2 and P_3. There pumps are of the Apiezon oil type (designed by Mr. David Sloan, of this laboratory), and have a speed of 80 liters of air per second, each. The gas flow is limited by the channel through G, and by the small connecting tubes between the pumps. This differential pumping system maintains the pressure at P_3 at 0.05 bar, when the arc pressure is 50 bars.

The current of ions striking G is measured by the milliameter A_3, while that passing through G can strike the removable targets H_1 or H_2, and be measured by A_4 or A_5, or pass through a collimating diaphragm S, to a target M, to a magnetic "analyzer," or to high voltage accelerators, as desired.

The actual construction of the apparatus is shown in cross section and to scale by Fig. 3. To insure proper cooling, all metal parts of the arc, and the extractor G, are made of copper. The anodes are insulated from the arc chamber E by glass cylinders M and C; to one of these is sealed a connection to the gas supply, and the other is connected through a stopcock and liquid air trap, to a McLeod gauge.

The cathode F consists of a 50 mil tungsten wire, held by set-screws S_1 and S_2 to the holder D, and the lead P. P is insulated from D, and D from E, by the glass cylinders J and I, respectively. This arrangement greatly facilitates the renewal of the filament.

Water-cooling is effected by three parallel feed systems, consisting of the anodes in series, the two cathode branches in series, and the two arc branches in series. Each system is connected to supply and drain through several feet of heavy-walled rubber tubing. This insulation is quite sufficient, resulting in a power loss of less than two hundred watts.

G is insulated from E by the glass cylinder K, and from the grounded pump system by the hard rubber ring R.

All metal-to-metal connections are either soft or silver soldered, and all metal-to-glass joints made with picein wax.

Targets H_1 and H_2 are made of monel metal, and can be turned in and out of the ion beam by

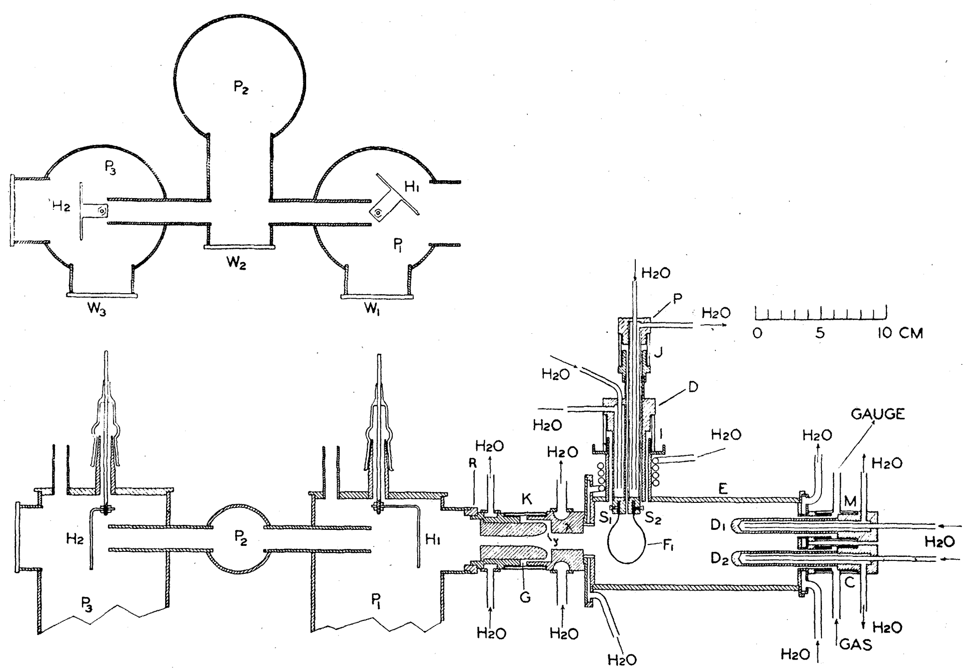

FIG. 3. Detail of apparatus.

means of greased ground joints. Their arrangement, and that of the pumps, is more clearly seen in the inset of Fig. 3, which also shows the glass windows W_1, W_2 and W_3. These windows enable visual observation of the ion beam, and the targets.

The variable gas "leak"[6] is extremely simple and rugged, and allows pressures from one to 100 bars to be maintained in the arc chamber, when the gas cylinder reducing valve is set at 5 to 15 lbs. per sq. in. It consists of a German silver tube, a portion of which is nearly closed by flattening, bent into a U-shaped spring. Flow variation is accomplished by opening or closing the spring with a screw.

Considerable difficulty was at first experienced in the operation of the arc. At certain pressures and currents, the windings of the arc transformer (T_2, Fig. 2) would become shorted. In no case had the transformer been overloaded. Inspection revealed small punctures in the between-turn insulation of both primary and secondary windings, such as would be caused by very high potentials between them. Inasmuch as these punctures occurred even when there was no high voltage applied, they could not have been due to transients from the latter source. Consequently it was assumed that at certain pressures and currents the discharge in the arc chamber became intermittent, the interruptions occurring with very high frequency; this would result in very high values of Ldi/dt in both windings of the arc transformer and so cause between-turn punctures. Intermittent discharge in the electric arc under constant potential has been noted by various workers,[7] and Langmuir[8] and Thomson.[9] Frequencies from several thousand to 10^9 per second have been observed.

Condensers in parallel with the cathode and anodes of the arc prevent di/dt from becoming great in the transformer windings. For this purpose two 2 mfd. condensers, C_1 and C_2 (Fig. 2) were inserted across the terminals of the arc transformer.

[6] Described in Rev. Sci. Inst., January, 1935.

[7] Appleton and West, Phil. Mag. **45**, 879 (1923); Newman, Phil. Mag. **47**, 939 (1924); Clay, Phil. Mag. **1**, 985 (1925); Thomson, Phil. Mag. **11**, 697 (1929); Tonks and Langmuir, Phys. Rev. **33**, 195 (1929); Wood, Phys. Rev. **35**, 673 (1930).

[8] Compton and Langmuir, reference 3a, p. 239.

[9] Thomson, reference 2, p. 447.

B. Results

Typical experimental data are plotted in the curves of Figs. 4–12. The arc current is denoted by I_x, the total positive ion current striking and passing through the extractor by I_p, and the current striking the target H_1 (Fig. 2), by I_u. In each case the extractor voltage was 19,000 r.m.s. and the arc drop was 210 volts.

(*a*) *Effect of arc current* (I_x) *on total positive ion current* (I_p). From Figs. 4–13 it will be seen that the total positive ion current I_p is very nearly linear with I_x, the drift current; the relation is actually of the form

$$I_p = aI_x^b, \qquad (7)$$

where $1 < b < 1.5$. Langmuir and Mott-Smith[4] found a similar relation with discharges in mercury vapor.

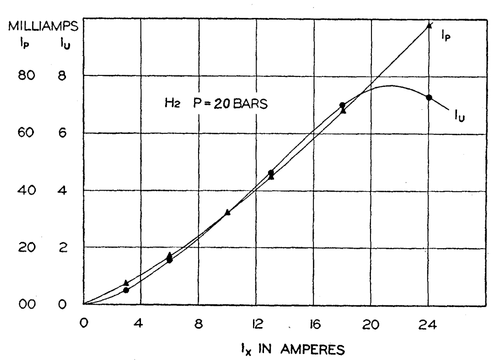

FIG. 4.

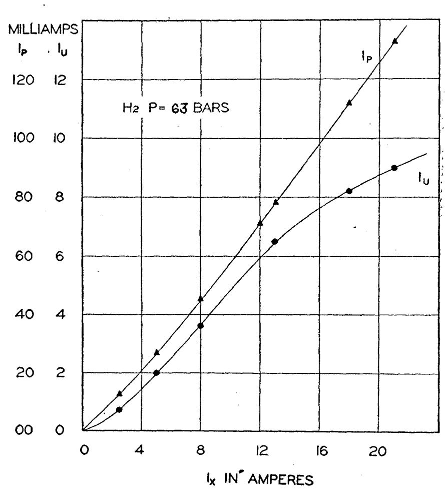

FIG. 6.

(*b*) *Effect of pressure on* I_p. The effect of pressure, in the case of hydrogen, is shown in Fig. 14, plotted from the data of Figs. 4, 5 and 6. The dotted portions of the curves represent probable extrapolations to zero pressure. Actually the arc became very unstable at 18 bars, and ceased to function at 16–17 bars. Over the range measured, I_p is linear with pressure. The same is also true for the other gases, bearing out the statements of Part I–C.

(*c*) *Relation between total* (I_p) *and "useful"* (I_u) *positive ion currents*. It will be noticed from the curves of Figs. 4–13 and particularly from those of Figs. 7, 8 and 9, that I_u is not linear with I_p, as

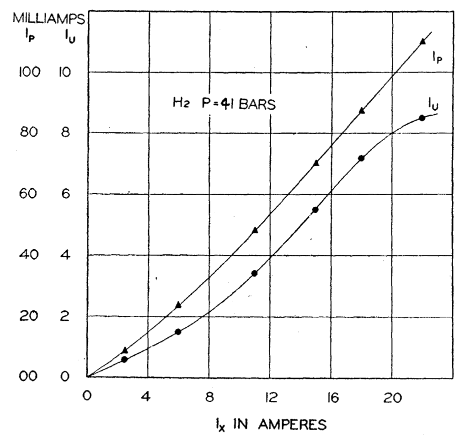

FIG. 5.

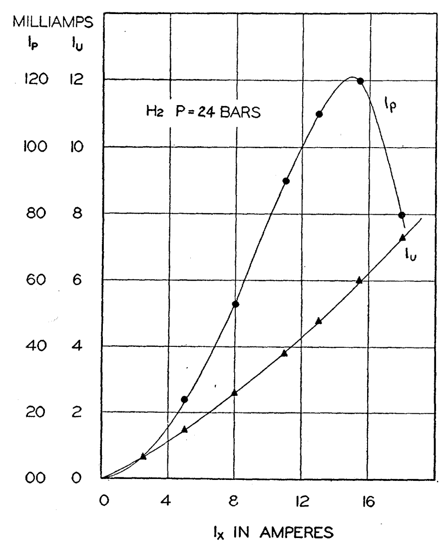

FIG. 7.

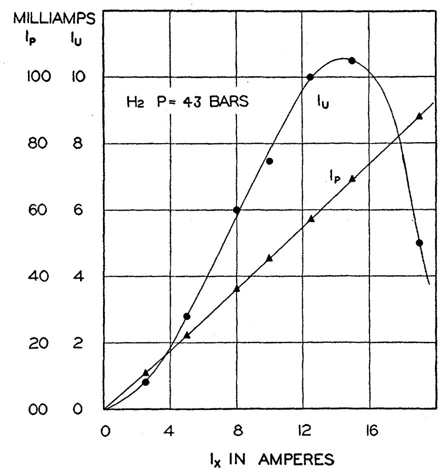

FIG. 8.

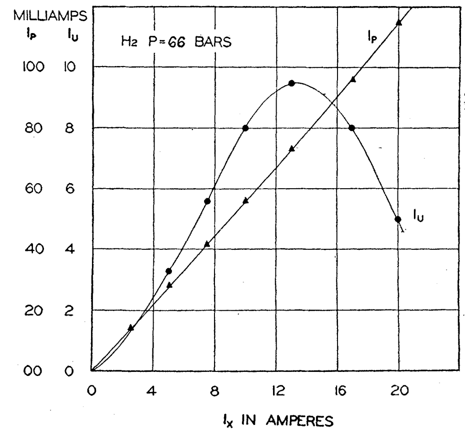

FIG. 9.

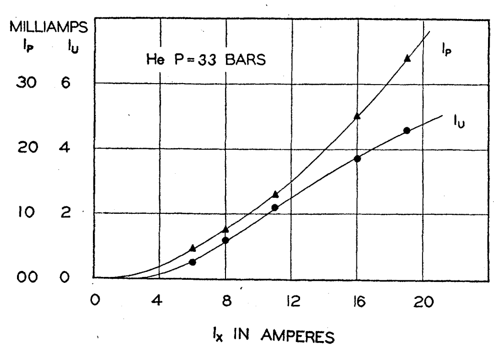

FIG. 10.

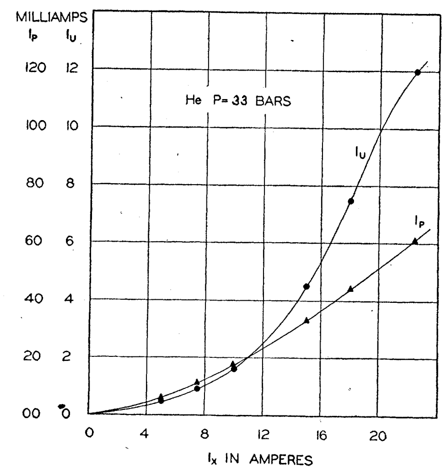

FIG. 11.

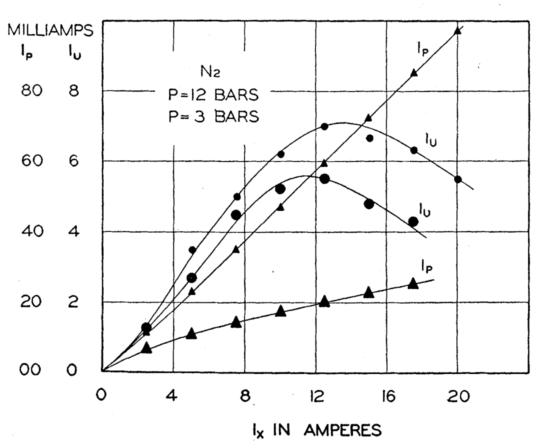

FIG. 12.

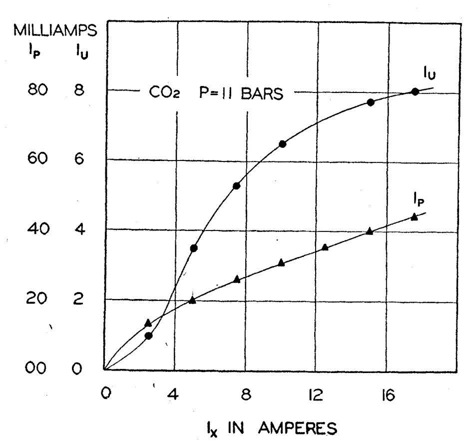

FIG. 13.

FIGS. 4–13. Arc current I_x, total positive ion and useful ion current at various pressures.

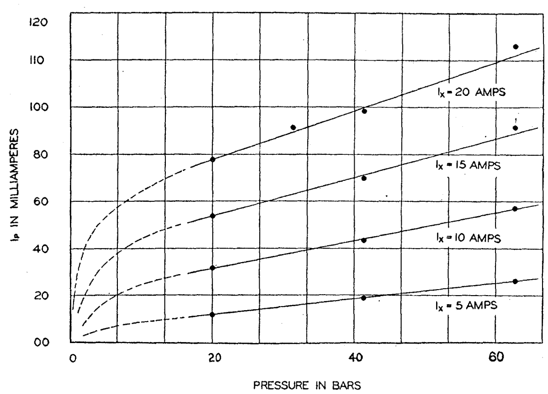

FIG. 14. Arc current I_x *vs.* pressure. (Data of Figs. 4, 5 and 6.)

might at first be expected, but goes through a maximum. That I_u/I_p is not independent of I_p is accounted for by the change with I_p of the shape and thickness of the positive ion sheath extending from G (Fig. 3). The exact functional relation between I_u/I_p and I_p is very difficult to determine theoretically, on a physical basis, as the potential distribution between G (Fig. 3) and its surroundings is quite complicated. Qualitatively, the variation of I_u/I_p with I_p may be explained as follows: At very low values of I_p, the sheath is relatively thick, and its surface so curved that a certain "effective" fraction of the lines of force therefrom to G have curvatures resulting in the passage of ions through G. As I_p increases, the sheath thins, and its curvature changes in such a way that the effective fraction of force lines increases, with consequent increase in I_u/I_p. Further increase in I_p thins the sheath still more, even to its recession into the hole through G, giving rise to such curvature that very few of the force lines are effective, and therefore bringing about a decrease in I_u/I_p.

At any given value of I_p, I_u/I_p is dependent on the geometry of the extractor G and its environs. The arrangement shown in Fig. 3 was found to be the most generally satisfactory. The hole through X has a cross section of 1 cm^{-2} and is 2.5 cm long (I_p is therefore the random ion current density). With a hole through G of the same diameter as that through X, a still higher ratio of I_u to I_p was obtained, I_u even exceeding $I_p - I_u$ at low values of I_p. (Because of the limited speed of the pumping system, the diameter of the hole shown, 7.5 mm, was the largest consistent with pressures over P_3 sufficiently low for high voltage acceleration of the ions.) When X was omitted, and G placed directly into the arc chamber, I_u/I_p became very much smaller.

The ratio I_u/I_p is also dependent on y, the distance between G and X (Fig. 3). This is shown very markedly by Figs. 4, 5, 6 and 10, the data for which were obtained with $y = 2$ mm, and Figs. 7, 8, 9 and 11, where the data were obtained with $y = 8$ mm. With the larger value of y, I_u/I_p is much higher. Presumably the effect results from extension of the straight portion of the lines of force between G and the sheath surface, which results in a greater number of effective lines.

The difficulties in the way of a systematic study of the influence of geometry on I_u/I_p are obvious.

(*d*) *Effect of extractor voltage on I_p and I_u.* Table II contains typical data showing the variation of I_p and I_u with voltage. In I–A it was stated that I_p should be independent of the extractor voltage. Table II shows this to be very far from true. This may be accounted for by the fact that the discharge in the arc chamber is concentrated in a cylindrical region between the cathode and the anodes, and although the plasma fills the entire arc chamber (and extends out through the hole through X), the ion density therein diminishes as the distance from the cylindrical region increases. Increasing the voltage on the extractor G extends the sheath, bringing it nearer to the region of maximum ion density, and therefore increases I_p.

TABLE II. *Typical data showing the variation of I_p and I_u with voltage in hydrogen. Arc drop 210 volts.*

(a) Distance between G and X, 2 mm. Pressure, 31 bars. $I_x = 18$ amp.				(b) Distance between G and X, 8 mm. Pressure, 50 bars. $I_x = 10$ amp.			
Volts (r.m.s.)	I_p (m.a.)	I_u (m.a.)	I_u/I_p	Volts (r.m.s.)	I_p (m.a.)	I_u (m.a.)	I_u/I_p
10,200	47.5	3.5	0.0736	12,800	28.0	6.0	0.214
13,000	60.0	4.5	.0750	14,000	30.0	6.1	.203
14,000	65.0	5.0	.0767	15,300	33.0	6.3	.190
15,000	70.0	5.5	.0785	16,200	35.0	6.5	.185
16,000	75.0	6.0	.0800	17,600	37.5	6.7	.181
17,000	81.0	6.5	.0803	18,600	42.0	7.0	.166
18,200	87.5	7.5	.0857	21,000	47.5	7.3	.153

In (a) of Table II, I_u/I_p increases with voltage, whereas in (b), it decreases. This may be accounted for by considerations similar to those of the previous section. In obtaining the data of (a), I_x was 18 amperes, and at the lowest voltage the sheath was of less than the optimum thickness for maximum I_u/I_p,; thus increasing the voltage, and therefore the sheath thickness, increases I_u/I_p. The data of (b), however, were taken at the lower value of 10 amperes for I_x, and at the lowest voltage the sheath was already thicker than the optimum value for maximum I_u/I_p, so increase in voltage brought about a decrease in I_u/I_p.

(*e*) *Divergence of the ion beam*

(1) *Natural divergence of an initially parallel beam.* The ions of an originally parallel beam are subject to two forces, at right angles to the initial direction of motion, which deflect them from their parallel courses: (1) the electrostatic force exerted on the particles by the electrostatic field set up by them, resulting in divergence of the beam, and (2), the force exerted on the moving particles by the electromagnetic field caused by them, resulting in convergence of the beam.

The electrostatic field at the periphery of a circular beam of charged particles is easily shown[10] to be

$$E=2\rho/r \text{ e.s.u.}, \tag{8}$$

where ρ is the density of particles per unit length of a beam of radius r.

Now
$$I=\rho v, \tag{9}$$

where I is the total charge per second crossing any imaginary plane perpendicularly cutting the beam, and v is the velocity of the ions. Whence the outward force acting on an ion at the periphery of the beam is

$$Ee=m(d^2r/dt^2)_{\text{out}}=2Ie/vr \tag{10}$$

whence
$$(d^2r/dt^2)_{\text{out}}=2Ie/vrm. \tag{11}$$

The circular electromagnetic field at the periphery of a beam of charged particles is easily shown to be

$$H=2I/r \text{ e.m.u.} \tag{12}$$

The force exerted on a moving charged particle by the field H is

$$F=Hev \tag{13}$$

and is perpendicular to H; whence the force acting to converge the beam is

$$m(d^2r/dt^2)_{\text{in}}=2Iev/r, \tag{14}$$

whence
$$(d^2r/dt^2)_{\text{in}}=2Iev/rm. \tag{15}$$

Expressing I in amperes, (15) becomes

$$(d^2r/dt^2)_{\text{in}}=Iv/mr\ 2\times10^{-1}\times1.591\times10^{-20} \tag{16}$$

and (11) becomes

$$(d^2r/dt^2)_{\text{out}}=I/rvm\ 3\times10^{9}\times4.774\times10^{-10} \tag{17}$$

whence
$$\frac{(d^2r/dt^2)_{\text{in}}}{(d^2r/dt^2)_{\text{out}}}=1/9\times10^{-20}v^2. \tag{18}$$

It is thus evident that when the particles of the beam move with the velocity of light, the outward and inward accelerations are equal and opposite; that is, an initially parallel beam will remain such. At all lesser velocities, the outward acceleration is greater, and the beam will be divergent.

Since
$$v=2Ve/m \tag{19}$$

$$\frac{(d^2r/dt^2)_{\text{in}}}{(d^2r/dt^2)_{\text{out}}}=2/9\times10^{-20}\frac{Ve}{m}. \tag{20}$$

For protons, this becomes

$$\frac{(d^2r/dt^2)_{\text{in}}}{(d^2r/dt^2)_{\text{out}}}=2.13\times10^{-9}V \tag{21}$$

when V is expressed in volts.

Consequently, for 20,000 volt protons, the inward acceleration is only 4.26×10^{-5} times the outward acceleration, and since the latter will be shown to result in a comparatively small divergence, the former may be neglected.

The distances from the origin of an initially parallel beam at which it has doubled, trebled and quadrupled in radius, due to electrostatic

[10] Watson, Phil. Mag. **3**, 849 (1927); Zworykin, J. Frank. Inst. **215**, 546 (1933).

divergence can be shown[11] from (11) to be

$$S_{r=2a}=0.00188A;\quad S_{r=3a}=0.00280A;$$

$$S_{r=4a}=0.00358A;$$

where $$A=aV^{\frac{3}{4}}n^{\frac{1}{4}}/M^{\frac{1}{4}}I^{\frac{1}{2}},$$

where a is the initial radius of the beam, in cm, V is expressed in volts, n is the valence of the ions, M is the "molecular weight" of the ions, and I is expressed in amperes.

For a 5-milliampere beam of 20,000 volt protons, initially of radius 0.375 cm,

$$S_{r=0.75}=16.8\text{ cm};\quad S_{r=1.025}=24.9\text{ cm};$$

$$S_{r=1.5}=31.9\text{ cm}.$$

(2) *Divergence due to non-parallel extraction of the ions.* The current to the target H_2 (Fig. 2) enables the actual spread of the beam to be calculated. In a typical experiment with hydrogen, the current to the target H_1 (Fig. 2) was 5 m.a., and the extractor voltage 20,000, peak value. The diameter of the beam on striking H_2 was 1.5 cm, and the current was 1.25 m.a. Had not the beam been circumscribed to this diameter by the pumping tube between P_2 and P_3 (Fig. 2), its original 5 m.a. would all have struck H_2, in a circular spot of diameter 3.00 cm. This increase in diameter is closely equal to that due to the electrostatic divergence; the ion beam on emerging from the extractor is therefore very nearly parallel.

(*f*) *Magnetic resolution of the beams into their component ions.* For analyses of the beams, the target M (Fig. 2) was removed, and the ions permitted to traverse a 30 cm long glass tube to a rectangular slit. This cut out a 0.1×1.0 cm section of the beam, and allowed it to pass between the poles of a magnet, undeflected to one target, or, bent through an arc of radius 117 cm, to another.

In order to increase the sharpness of the ion "peaks," the extractor voltage was made more nearly uniform by the insertion of a one microfarad condenser between G and the arc chamber (Fig. 3).

Typical data have been plotted in Figs. 15 and 16. In each case the extractor voltage was 16,900, considered as d.c. (12,000 volts, r.m.s. value). The theoretical values of the magnetic field for each ionic species are indicated on the curves.

The curves of Fig. 15 show the constitution of the beams from hydrogen, as a function of the gas pressure. Six peaks have been found: corresponding to H_3^+, H_2^+, H_1^+, H_{3-1}^+ (H_2^+ resulting from the dissociation of H_3^+ after extraction) and H_{2-1}^+ and H_{3-2}^+ (H_1^+ resulting from the dissociation of H_2^+ and H_3^+, respectively, after extraction). It will be noticed that the fields required to deflect the H_{3-2}^+ and H_{2-1}^+ ions were stronger than the calculated fields. This may be accounted for on the assumption that dissociation

[11] We are indebted to Mr. J. C. Oxtoby, of the Department of Mathematics, for the following solution of Eq. (11), made at the instigation of Professor Lawrence.

$$d^2r/dt^2=2Ie/vrm=2I/(r2Vm/e).$$

Let
$$4I/(2Vm/e)=b^2$$
then
$$d^2r/dt^2=b^2/2r.$$

Multiplying by $2(dr/dt)dt$, and integrating
$$(dr/dt)^2=b^2\log_e r+c.$$

If the initial radius of the beam is a, $dr/dt=0$, for $r=a$. Therefore
$$(dr/dt)^2=b^2\log_e r/a$$
and
$$dr/dt=b(\log_e r/a)^{1/2}.$$
Now let $y=(\log_e r/a)^{1/2}$; then $y^2=\log_e r/a$ and
$$r=ae^{y^2}$$
and
$$dr/dt=(dr/dy)dy/dt=ae^{y^2}2ydy/dt.$$
Therefore
$$2aye^{y^2}dy/dt=by;\qquad (b/2a)dt=e^{y^2}dy$$
and, since when $t=0$, $y=0$,
$$(b/2a)t=\int_0^y e^{y^2}dy.$$
Now, $S=vt$; whence $bs/2av=\int_0^y e^{y^2}dy$ or
$$\frac{I^{1/2}}{a(2V)^{3/4}(e/m)^{1/4}}S=\int_0^y e^{y^2}dy.$$
This is integrable, in the form of a series
$$\frac{I^{1/2}}{a(2V)^{3/4}(e/m)^{1/4}}S=y+\frac{y^3}{3}+\frac{y^5}{5\times2!}+\frac{y^7}{7\times3!}+\frac{y^9}{9\times4!}+\cdots.$$

It is convenient to express the divergence in terms of the distance from the origin of the beam, at which it has doubled, trebled, and quadrupled, that is, when $r=2a$, $3a$ and $4a$; or when $y=\log_e 2$, $y=\log_e 3$, $y=\log_e 4$. The summation of the first five terms of the above series are as follows:

$$\Sigma^5{}_{r=2a}=1.07241;\qquad \Sigma^5{}_{r=3a}=1.5972;\qquad \Sigma^5{}_{r=4a}=2.0425.$$

Whence, expressing V in volts, I in amperes, m in terms of the "molecular weight" of the ions, and e in terms of n, the number of charges in the ion

$$S_{r=2a}=0.00188aV^{3/4}n^{1/4}/M^{1/4}I^{1/2}$$
$$S_{r=3a}=0.00280aV^{3/4}n^{1/4}/M^{1/4}I^{1/2}$$
$$S_{r=4a}=0.00358aV^{3/4}n^{1/4}/M^{1/4}I^{1/2}.$$

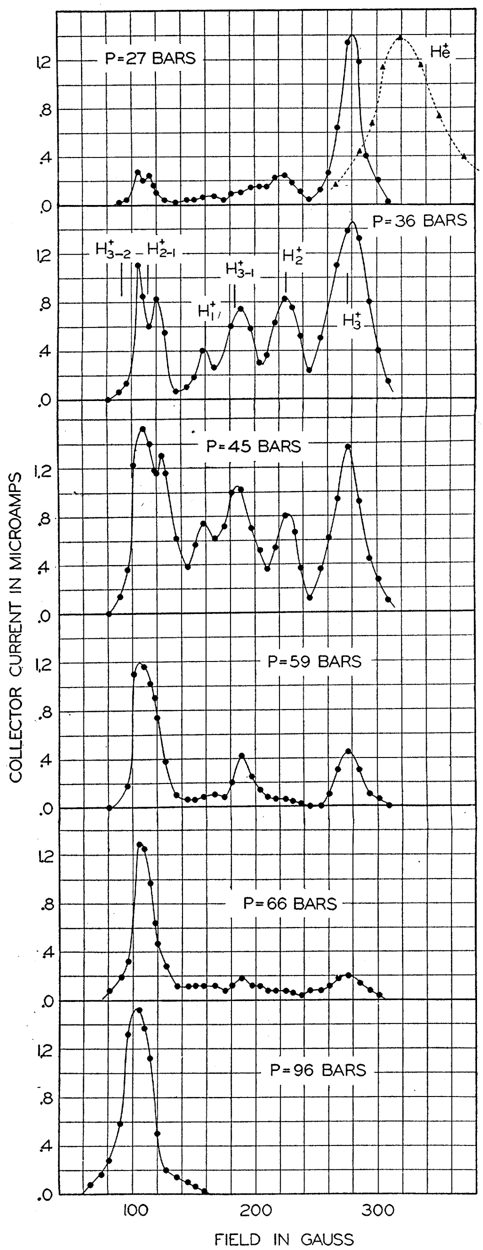

FIG. 15. Current to collector *vs.* magnetic field. The peaks correspond to different ions as indicated in the upper curves.

of the H_3^+ and H_2^+ took place in the sheath, before the molecular ions had acquired their full 16,900 volts equivalent velocities, so that they fell through a part of the field as H_1^+ ions and consequently acquired higher velocities than they would have as H_2^+ or H_3^+.

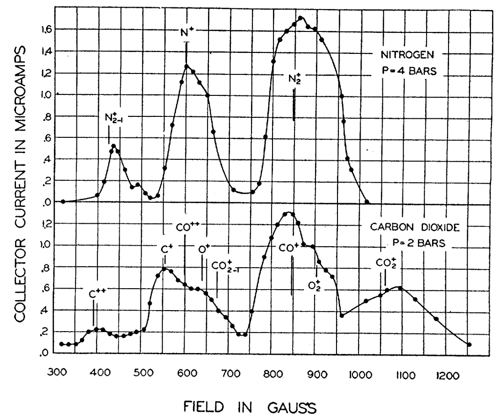

FIG. 16. Current to collector *vs.* magnetic field when N_2 and CO_2 were in the arc space.

The curves of Fig. 15 show that at 27 bars the hydrogen beams consist practically entirely of H_3^+, and at 96 bars, entirely of H_1^+ (H_{3-2}^+, actually, but for nuclear work, involving the further high voltage acceleration of the protons, this is obviously of no significance).

Fig. 16 gives typical data for nitrogen and CO_2. The nitrogen used was obtained from a commercial cylinder, and the curve shows evidence of the presence of oxygen (O^+ and O_2^+). The CO_2 beams contain many different ions, the most likely of which are indicated on the curve.

Experiments were also run on oxygen (pressure, 5 bars, $I_x=10$ amp., $I_u=8$ m.a.), in which peaks corresponding to O^+ and O_2^+ were obtained.

(*g*) *Secondary electron formation.* To estimate the number of secondary electrons emitted from a target by the impact of a positive ion of a given kind, the target used in the magnetic deflection experiments was provided with a guard ring, similar to *R* of Fig. 2. Ions were prevented from striking this ring by a shield similar to *S* of Fig. 2. Some typical data are given in Table III.

From these data it is seen that when floating, the guard ring becomes so negatively charged as to stop the flow of secondary electrons, and that the emitted secondaries have velocities less than 22.5 equivalent volts.

The target H_1 (Figs. 2 and 3) was enclosed by the grounded pumphead P_1, so the conditions in making the measurements of I_u in Figs. 4–13 were similar to those in the previous experiment

TABLE III. *Typical data on the emission of secondary electrons from a monel metal target bombarded with positive ions of various kinds. Voltage of ions 16,900 (condenser between extractor and arc chamber.)*

Ion	Electron current from ground to target, in microamperes						Secondaries per incident ion
	Potential of guard ring with respect to ground, volts						
	−45	−22.5	Floating	Grounded	+22.5	+45	
H_1^+	0.28	0.28	0.28	0.36	0.48	0.48	0.71
H_2^+	.28	.28	.28	.44	.62	.62	1.21
H_3^+	.28	.28	.28	.46	.70	.70	1.50
He^+	.34	.34	.34	.46	.66	.66	0.94
N_2^+	.38	.38	.38	.50	.84	.84	1.21
N_1^+	.32	.32	.32	.44	.72	.72	1.25
CO^+	.28	.28	.28	.40	.72	.72	1.57

with the guard ring grounded. Whence the actual positive ion currents for hydrogen were $I_u \times 28/36$ at the higher pressures (60 bars and above), and $I_u \times 28/46$ at the lower pressures (20–27 bars), and, for helium, $I_u \times 34/46$.

(*h*) *Heating of the targets.* The target H_1 (Fig. 2) was a square of monel metal, $0.05 \times 3 \times 3$ cm. In one experiment with hydrogen at 63 bars arc pressure and $V = 19{,}000$ r.m.s., when I_u measured 11 m.a., a hole 1.5 cm in diameter was melted in this target in a very few seconds (M.P. of monel 1633°K). A measured current of 2 m.a. was enough to heat a 1.5 cm spot on target H_2 (monel, $0.05 \times 4 \times 4$ cm) to a dull red.

Because of the complex relation between the measured and peak values of the ion current and the voltage, the observed heating can only serve as a rough check on the measured secondary formation. Thus, assuming that with the 11 m.a. beam of H_1^+ ions the temperature of the target H_1 (Fig. 2) was 1400°K, and that the total emissivity of monel at this temperature is 35 percent of that from a perfect black-body, the radiation is 128 watts. If the actual ion current (from Table III) is $11 \times 28/36$ m.a., and $V = 19{,}000$ volts (r.m.s.) the power is 162 watts, in rough agreement with the above value.

(*i*) *The further intensification of the beams.* It seems entirely possible that still more intense beams may be obtained, by modifying the apparatus along the following lines:

(1) Increasing the diameter of the orifices through G and X (Fig. 3). Doubling these, for example, should increase I_u to four times the present amount for given values of I_x and V. However, this would necessitate increasing the speed of the pump p_1 (Fig. 3) by a factor of 8, i.e., to 640 *e*/sec., in order to maintain the necessary low pressure for high voltage acceleration.

(2) Increasing the extractor voltage. From the discussion of (d), it is seen that I_u/I_p is an increasing function of the voltage when I_p is of such a value that extension of the sheath results in closer approach to the optimum thickness for maximum I_u/I_p. Increasing I_p, by increasing I_x, results in thinning of the sheath to less than the optimum thickness, and increasing the voltage again extends it to the optimum value. It should therefore be entirely possible to greatly intensify the beams, by increasing the extractor voltage to 50,000, which could be done by replacing the glass cylinder K (Fig. 3) by a lavite insulator of such shape and size as to prevent flash-over between G and the arc chamber.

In conclusion, our sincere appreciation is expressed to Professor Gilbert N. Lewis for his generous support, and kindly interest; to Professor Ernest O. Lawrence, for the generous donation of a room in the Radiation Laboratory, the loan of equipment that helped in no small part to make possible this research, and, more particularly, for his friendly interest; and to David H. Sloan, for the benefit of many helpful suggestions and discussions.