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Counter Calibration and Cosmic-Ray Intensity

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An absolute calibration of a set of counters has been obtained by determining their efficiencies and effective volumes. From observations recorded with this calibrated set, the number of ionizing cosmic rays is 0.80 ± 0.028 per cm^2 per minute from unit solid angle at the vertical at sea level. The number of such rays from all directions as obtained by integration over an angular distribution curve is 1.48 ± 0.055 cm^{-2} $min.^{-1}$. This combined with Millikan's ionization chamber measurements gives an average specific ionization of 100 ± 3.7 ion pairs per cm path of a ray. Comparison with previous measurements is made.

INTRODUCTION

THE study of cosmic radiation with ionization chambers gives a measure of the ionization produced in a closed vessel under the conditions of the experiment. Millikan[1] has made a careful study of his apparatus and as a result has been able to give an absolute measurement of the number of ions produced in air under standard conditions at sea level. Many observers have used Geiger-Müller counters to obtain relative values for the number of ionizing rays passing through a region defined by their particular arrangements of counters. To relate the two methods it is desirable to determine the absolute intensity of ionizing rays; i.e., the number per cm^2 per min. per unit solid angle. Johnson[2] has expressed his results on the angular distribution in this way but has pointed out that they are not strictly equal to the absolute intensities because of uncertainties in the efficiencies and end corrections of his counters. In the following paragraphs we discuss briefly our procedure for the experimental determination of the efficiencies and sensitive volumes of a set of counters. Accidental coincidences and the effects of showers are also considered. With the aid of these results we obtain a value for the vertical ray intensity which in conjunction with the angular distribution curve of Johnson and Stevenson[3] and Millikan's ionization data leads to a determination of the average ionizing power of the corpuscles.

ACCIDENTAL COUNTS

In a coincidence counting circuit it is important to reduce the number of accidental coincidences as far as possible. We have used a circuit similar to that described by Johnson and Street.[4] It was shown there that the resolving time τ could be determined approximately from the circuit constants, and the double and triple accidental counting rates determined from the

[1] R. A. Millikan, Phys. Rev. **39**, 397 (1932).
[2] T. H. Johnson, Phys. Rev. **43**, 307 (1933).
[3] T. H. Johnson and E. C. Stevenson, Phys. Rev. **43**, 583 (1933).
[4] T. H. Johnson and J. C. Street, J. Frank. Inst. **215**, 239 (1933).

approximate[5] relations

$$A_{12}=2N_1N_2\tau, \tag{1a}$$

$$A_{123}=2(N_{23}N_1+N_{13}N_2+N_{12}N_3)\tau, \tag{1b}$$

where the N's with single subscripts are the individual counting rates of the designated counters and those with double subscripts the coincidence counting rates of two counters. We have made a more precise measurement of the accidental counts by direct measurement with the arrangement of counters shown in Fig. 1. The

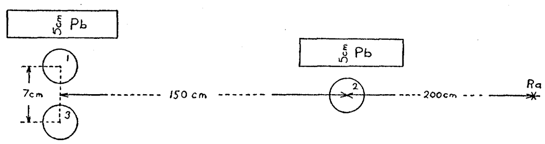

FIG. 1. Arrangement of counters for the measurement of accidental counts.

lead shields greatly reduce the true coincidence rate due to showers. The various observed rates for the normal case and when stimulated by a radium capsule are given in the first part of Table I. To a first approximation the rates N_{12} and N_{23} for the stimulated case are accidental coincidences because of the relatively small normal rates. With this assumption approximate determinations of τ_{12} and τ_{23} were obtained by use of (1a). These τ's were used to find the accidental rates A_{12} and A_{23} for the normal case and from these the real rates $[R_{12}=N_{12}-A_{12}$ and $R_{23}=N_{23}-A_{23}]$ due to showers and to horizontal rays were obtained. R_{12} and R_{23} are reduced by radium stimulation on account of the lower counter efficiency for that condition. This has been determined (see section of this paper on efficiency) to give the real rates for the stimulated case. The accidental rates for the stimulated case were then found by subtracting the real from the observed rates and these in turn were used to obtain more accurate values of the resolving times. The accidental and real rates and the resolving times are tabulated in the latter part of the table. The circuits are symmetrical and we have taken the average of τ_{12} and τ_{23} as the value to be used in subsequent calculations.

TABLE I. *Counter rates, normal and when stimulated by radium capsule.*

	Normal	Stimulated		Normal	Stimulated
N_1	148 min.$^{-1}$	333 min.$^{-1}$	A_{13}	0.028 min.$^{-1}$	0.133 min.$^{-1}$
N_2	138	551	A_{23}	.027	.219
N_3	142	306	A_{123}	.0014	.0050
N_{12}	0.046	0.258	R_{12}	.017	.013
N_{13}	7.69	6.58	R_{13}	7.66	6.45
N_{23}	.047	.234	R_{23}	.020	.015
N_{123}	.003	.006	R_{123}	.001	.001
A_{12}	.029	.245	τ_{12}	6.7×10^{-7} min.	
			τ_{23}	6.5×10^{-7} min.	

EFFICIENCIES

For three counters placed in line in a vertical plane as shown in Fig. 2, a triple coincidence may represent the passage of a single ray through the train of counters or the simultaneous excitation of the three counters by several rays from a shower. Likewise a double coincidence between the two extreme counters may be due to a single ray passing through them (and the central counter) or it may be due to shower particles passing through the extreme counters. If the observed triple and double coincidence rates are corrected for accidental counts and the latter rate for those showers with no ray through the central counter, the resulting rates will represent occurrences of simultaneous excitation in all three counters. The two rates should be equal provided the central counter produces a discharge whenever a ray passes through it. Since the

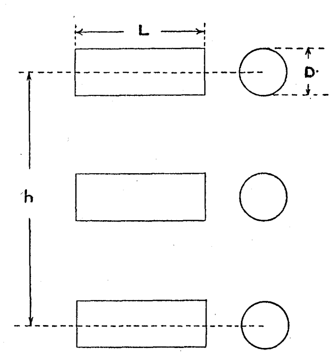

FIG. 2. Arrangement of counters for the measurement of the efficiency of the central counter.

[5] *Note:* The expressions are applicable if the double rates are small in comparison with the individual rates in (1a) and if the triple rate is small in comparison with the double rates in Eq. (1b). When these conditions are not fulfilled the following relations are more accurate

$A_{12}=2(N_1-R_{12})(N_2-R_{12})\tau$

$A_{123}=2[N_{23}N_1+N_{13}N_2+N_{12}N_3-R_{123}(N_1+N_2+N_3)]\tau,$

where R_{12} and R_{123} are the real double and triple rates.

counters are not perfectly efficient in producing discharges the corrected rates are not equal and the efficiency of the central counter is given by the ratio of the triple rate to double. The results of a series of efficiency measurements are given in Table II.

TABLE II. *Efficiency (E) of counters.*

Central counter	h	c_t/T	c_d/T	N_t	N_d	N_t/N_d	E
1	20	2.43	2.66 ±0.018	2.43	2.64	0.922	0.938±0.0096
2	20	2.38	2.58 ± .016	2.38	2.56	.930	.948± .0088
3	20	2.48	2.70 ± .023	2.48	2.68	.926	.943± .0120
3	40	0.631	0.743± .011	0.628	0.719	.873	.943± .0209

The first column gives the number of the central counter the efficiency of which is being measured and the second the distance in cm between extreme counters. The numbers of counts per min. c_t/T and c_d/T have been corrected for barometric fluctuations[6] and reduced to a pressure of 76 cm of mercury. The probable errors have been estimated from the residuals. From these rates the accidental counts have been subtracted to give N_t and N_d. Showers were reduced to a minimum by carrying out the experiment under a light roof (one inch of wood about two meters above the counter set). To determine the correction due to the remaining showers we have made use of the data for the two separations of the end counters as given in the lower two lines of the table. Here it is apparent that the counting rate decreases rapidly with increasing separation whereas it is reasonable to assume that the shower coincidences do not vary appreciably. The apparent efficiency N_t/N_d with no correction for showers is considerably less for the greater spacing. The true efficiency must be independent of the spacing and this condition is realized if we subtract from all the double rates a constant 0.052 count per min. which we attribute to showers. The final corrected efficiencies are given in the last column.

The efficiencies have also been determined by a second method from an independent set of data. The counters were placed in line and the individual counting rates controlled by means of a radium capsule. It is assumed that immediately following a discharge of a counter there is an interval of time σ during which it is insensitive, probably due to the fact that the potential difference between the two electrodes is insufficient to produce another discharge. This period of inactivity together with the individual counting rate limits the efficiency of the counter.

$$E_1=1-N_1\sigma_1 \quad (2a); \qquad E_1'=1-N_1'\sigma_1 \quad (2b)$$

where the primed symbols refer to values obtained when the counters were stimulated by a radioactive source. The observed double counting rates are

$$N_{12}=E_1E_2n_{12}, \tag{3a}$$

$$N_{12}'=(E_1-\Delta E_1)(E_2-\Delta E_2)n_{12} \tag{3b}$$

and similar expressions for the other pairs, where ΔE is the difference between efficiencies for the normal and stimulated condition and n_{12} is the actual number of ionizing rays passing through the two counters. The difference between normal and stimulated double counting rates is obtained from Eq. (3).

$$\Delta N_{12}/N_{12}=\Delta E_1/E_1+\Delta E_2/E_2-\Delta E_1\Delta E_2/E_1E_2, \tag{4}$$

$$\begin{aligned}\Delta N_{12}/N_{12}=\Delta N_1\sigma_1/(1-N_1\sigma_1)\\ +\Delta N_2\sigma_2/(1-N_2\sigma_2)\\ -\sigma_1\sigma_2\Delta N_1\Delta N_2/(1-N_1\sigma_1)(1-N_2\sigma_2).\end{aligned} \tag{5}$$

In this and two similar equations for $\Delta N_{13}/N_{13}$ and $\Delta N_{23}/N_{23}$ all quantities but the three σ's are observed experimentally. The observed rates (corrected for barometric fluctuations and for accidental counts) for the three counters in line are tabulated in Table III together with the σ's

TABLE III.

	Normal	Stimulated
N_1	145 min.$^{-1}$	572 min.$^{-1}$
N_2	133	621
N_3	132	568
N_{12}	3.70	2.59
N_{13}	10.59	7.17
N_{23}	10.77	6.58
σ_1	2.7 $\times 10^{-4}$ min.	
σ_2	4.0 $\times 10^{-4}$ min.	
σ_3	5.0 $\times 10^{-4}$ min.	
E_1	0.960	
E_2	.947	
E_3	.934	

[6] E. C. Stevenson and T. H. Johnson, Washington Meeting of Am. Phys. Soc., April, 1934; Phys. Rev. **45**, 758A (1934).

and E's found from the solution of the equation. The conditions of this test were exactly similar to the arrangement which gave the data in Table II. The agreement between the two independent determinations of the efficiencies is good evidence that the assumptions employed are valid.

Effective Dimensions of a Counter

In order to determine the effective diameter of a counter, the three counters were arranged as shown in Fig. 3 and the ratios of the triple to double counting rates were measured for various displacements of the central counter. Assuming that the counters behave as uniformly sensitive rectangular areas, it is possible to derive approximate expressions for the ratio as a function of the displacement s and the diameter D.

$$\text{For } s<D/2, \quad N_t/N_d=K[1-2(s/D)^2], \tag{6a}$$

$$\text{For } s>D/2, \quad N_t/N_d=2K(1-s/D)^2. \tag{6b}$$

The theoretical curve is shown in Fig. 3 with the experimental points. The values of K and D were determined from the maximum and half values of N_t/N_d and a constant triple counting rate 0.066 per min. due to showers was subtracted from the experimental observation. The curve is in good agreement with the experimental points, indicating that the counter is uniformly sensitive over its diameter. The effective diameter is found to be the same as the internal geometric diameter, 3.82 cm.

The effective length L was measured by turning the central counter through 90° as

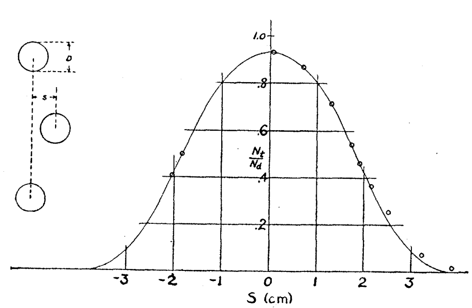

FIG. 3. Arrangement of counters for the determination of the effective diameter of a counter. The ratio of the triple to double counting rate is plotted as a function of the displacement of the central counter. Observations are shown as points on the theoretical curve.

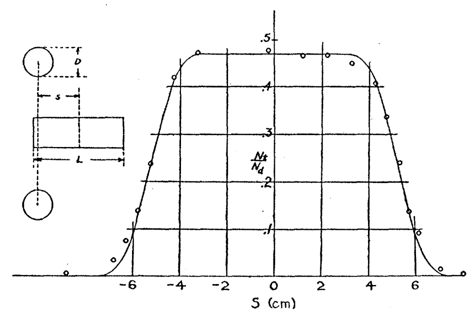

FIG. 4. Arrangement of counters for the determination of the effective length of a counter. Observed values of N_t/N_d are shown as points on the theoretical curve.

shown in Fig. 4. The theoretical curve was obtained from formulas (6) and has been plotted in the figure. Under the conditions of operation, the effective length was found to be 10.5 cm as compared with the geometric length of 13 cm. This difference is probably dependent on the threshold and operating potentials as well as on the geometry. The threshold and operating potentials were in all our measurements 1200 and 1500 volts, respectively. The central wire in the counters was three mil tungsten.

The effective diameter and length have been measured for only a single counter, but the relative cross-sectional areas have been computed from the counting rates of the three arrangements in Table II.

Integration over Counters

With these values for the lengths, diameters and efficiencies, $j(0)$ the intensity in rays per unit area from unit solid angle at the vertical can be obtained by integration over the counters. Employing the approximations used by Johnson[2] that for the small angles along the counter lengths the intensity varies as the square of the cosine of the angle from the vertical and that the variation over the diameter is negligible, the vertical triple counting rate is given by

$$N_t=E_1E_2E_3j(0)D^2h^4\int_0^L\int_0^L\frac{dl_1dl_2}{[h^2+(l_1-l_2)^2]^3}, \tag{7a}$$

$$N_t=\tfrac{1}{4}E_1E_2E_3j(0)D^2\left[\frac{L^2}{L^2+h^2}+\frac{3L}{h}\tan^{-1}\frac{L}{h}\right]. \tag{7b}$$

Since the computation was made for plane surfaces rather than for cylinders, there is an additive correction for the circular ends. The upper counter as seen from the lower is a rectangle with half ellipses on the ends. The number of rays passing through one counter and a half ellipse of the other is

$$N=\tfrac{1}{8}j(0)\frac{\pi}{h}\frac{D^3}{4}\left[1-\frac{h^4}{(h^2+L^2)^2}\right]. \tag{8}$$

The total correction is four times this amount which combined with Eq. (7b) gives

$$N_t=\tfrac{1}{4}j(0)E_1E_2E_3D^2\left\{\frac{\pi D}{2h}\left[1-\frac{h^4}{(h^2+L^2)^2}\right]+\frac{L^2}{L^2+h^2}+\frac{3L}{h}\tan^{-1}\frac{L}{h}\right\}. \tag{9}$$

It is also necessary to take account of the fact that the width as well as the length needs correction due to the cylindrical form of the counters. This correction is a 0.7 percent increase in diameter for each counter for the 20 cm spacing and is a fourth as much for the 40 cm spacing.

Substitution of the values of Table II gives 0.750 for $j(0)$ for the 20 cm spacing and 0.732 for 40 cm. The agreement between the two values tends to verify Eq. (9). The three counters which were used in this work were selected because of their remarkable constancy. In over two years of continuous operation no detectable change in their sensitivity in a coincidence counting unit has been observed. The counter walls, however, are fairly thick, approximately 2.5 mm nonex glass plus 0.3 mm copper. On this account, the value of $j(0)$ cited above includes only those rays with sufficient energy to penetrate four counter walls; i.e., 1 cm glass plus 1.2 mm copper. We have not counted the upper wall of the top counter for we have reason to believe from Schindler's[7] curves that as many rays are produced as are absorbed in this wall. To obtain $j(0)$ for rays of lesser penetrating power we arranged a double coincidence set of thin walled Pyrex glass counters (0.7 mm glass plus 0.13 mm copper per single wall) and measured the fractional decrease in counting rate when two of the heavy-walled counters were introduced as absorber between the Pyrex counters. This decrease was found to be 7 percent of the total. Thus $j(0)$ becomes 0.80 ± 0.028 ionizing ray per cm² per unit solid angle per minute.

The total intensity from all directions of ionizing rays per cm² considering the unit of area always perpendicular to the ray is

$$J=2\pi\int_0^{\pi/2} j(\theta)\sin\theta d\theta. \tag{10}$$

The integration has been performed graphically by using the experimental curve of Johnson and Stevenson[3] and gives the result of 1.48 ±0.055 rays per cm² per min.

If we assume that all rays have the same ionizing power, the volume ionization Q in ion pairs per cm³ per minute is related to J by the expression

$$Q=JI, \tag{11}$$

where I represents the average number of ion pairs along one cm of path of a ray under the conditions for which Q is determined. Putting $Q=2.48$ ions cm^{-3} sec.$^{-1}$ in air under standard conditions as given by Millikan[1] we obtain 100 ± 3.7 ions cm^{-1} for I.

Previous results are 135 ± 13 ion pairs per cm by Köhlhorster and Tuwin[8] with a counter method, 60 cm^{-1} from W. F. G. Swann's[9] direct measurement with a linear amplifier, an upper limit of 70 cm^{-1} by Evans and Neher[10] from fluctuation measurements and an estimate of 122 cm^{-1} by Anderson[11] from energy loss measurements with a cloud chamber. We are confident of the accuracy of our ray intensity measurement but it is possible that a fraction of the residual ionization in a closed vessel is due to some form of radiation which does not contribute to the coincidence rates. This may occur in three ways none of which have definitely been shown to be inconsistent with other experiments. These are (1) shower groups of particles which pass together through the train of counters and there-

[7] H. Schindler, Zeits. f. Physik **72**, 625 (1931).

[8] Köhlhorster and Tuwin, Zeits. f. Physik **73**, 130 (1931).

[9] W. F. G. Swann, Phys. Rev. **44**, 961 (1933).

[10] Evans and Neher, Phys. Rev. **45**, 144 (1934).

[11] C. Anderson, Phys. Rev. **44**, 406 (1933).

fore register as one particle, (2) recoil nuclei[12] from neutron collisions, and (3) low energy (less than 10^6 volt) electrons in equilibrium with the primary radiation which contribute to the ionization because of the equilibrium condition but not to the coincidence rate because they do not penetrate the intermediate counter walls. Since our measurements were taken under conditions which would be expected to give few if any showers from nearby material it seems unlikely that (1) is an important contribution. If we adopt the *extreme* view that we have as many shower groups passing through the very small solid angle subtended by our counters as are observed by Anderson[13] in his cloud chamber which is practically surrounded by dense material we find that our value of J is increased to 1.87 rays cm^{-2} $min.^{-1}$ and I therefore reduced to 79 ion pairs per cm. There is no existing evidence which permits an estimate of the importance of (2) and (3). Should it be found that they are important our value for I would be lowered to approach the results of Swann and Evans and Neher but no correction will raise our value unless there is a considerable error in the volume ionization.

[12] G. L. Locher, Phys. Rev. **44**, 779 (1933).

[13] Anderson, Millikan, Neddermeyer and Pickering, Phys. Rev. **45**, 352 (1934).

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Self-Consistent Field and Some X-Ray Terms of the Sodium Atom*

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Self-consistent field functions were found for neutral sodium, for which they are tabulated, and for the six internally ionized states $1s$, $2p$, $1s2s$, $2p2s$, $1s2p$, $(2p)^2$, these figures denoting in each case the quantum symbols of the missing electrons. With these functions the corresponding atomic energies were computed by the method of Fock and Slater. Applying a first-order correction for relativity and spin-orbit interaction, a close check is obtained between the observed and calculated frequency of Na $K\alpha$, the deviation being of the order of 0.1 percent. Finally the energy parameters of the Fock equations were computed to the first order in the exchange for the $2s$ and $2p$ electrons in the singly ionized states and are compared with the corresponding ionization energies, which they exceed by about 10 percent.

INCIDENTAL to an inquiry into the origin of the $K\alpha$-satellites[1] it became necessary to compute energies by the self-consistent field method[2] for the states belonging to the configurations $1s$, $2p$, $1s2s$, $2p2s$, $1s2p$, $(2p)^2$, denoting configurations in every case by the quantum symbols of the electrons missing from the neutral atom. The results of this work including the computation for the normal sodium atom, are reported in this note.

The equations employed in calculating the electron wave functions differed from those used by Hartree in the manner suggested by Fock,[3] i.e., in retaining higher order Coulomb terms and exchange terms for electrons in the same subshell. They differ from Fock's equations by the omission of the non-diagonal (exchange) terms, and their solutions may be regarded as a first approximation to the solutions of Fock's equations. Averaging the potential over the sphere by using as weight function the angular distribution function, $|Y_l^m(\vartheta, \varphi)|^2$, for the electronic state (n, l, m, μ) whose wave function we wish to find, the equation for the normalized radial part of this wave function, $f(n, l, m, \mu; r)/r$, becomes:

* This research was supported by a grant from the Heckscher Foundation established at Cornell University by August Heckscher.

[1] See succeeding paper in this issue.

[2] D. R. Hartree, Proc. Cam. Phil. Soc. **24**, 89, 111 (1928).

[3] V. Fock, Zeits. f. Physik **61**, 126 (1930).