twice that in curve a. This increase of C has increased the time constant of the curve but decreased the initial ordinate. The former change tends to increase τ while the latter tends to diminish it. In this particular case, doubling C has increased τ by about 27 percent. It seems quite possible that in some cases increasing C may actually decrease τ.

If a pulse occurs before the v of a preceding one has fallen to zero, it will be smaller than the preceding pulse and have a smaller τ. So for very rapid counting rates one has a statistical distribution of τ's.

For higher values of V or C, the discharges may take the form shown in Fig. 3. This is a photograph of a single sweep during which two discharges occurred. (Some 60 cycle induction is present.) Here v rises to $V-V_0$ and remains there for a time T before decaying exponentially. The time τ is of course given by T and may vary widely in successive pulses. The average T is an increasing function of C and decreases as R increases.

The observations described here are quite uniform throughout a set of counters made in the same "batch." The wire in all counters was 3 mil tungsten and the cylinders were oxidized copper. The pressure in all cases was 7 or 8 cm. The qualitative phenomena are independent of counter size, of whether the gas is air or argon and oxygen, and of change in R between 5×10^8 and 5×10^9 ohms.

We hope to submit a report of quantitative nature in the near future.

It is a pleasure to acknowledge the helpful support of Dr. W. F. G. Swann.

W. E. Danforth

The Bartol Research Foundation
of the Franklin Institute,
Swarthmore, Pennsylvania,
November 6, 1934.

On the Statistical Theory of Errors

Professor R. A. Fisher has most kindly responded to our request for criticisms of the article that appeared under the above title.[1] The material in his letter is much too valuable to be filed away, so with his consent we here present the substance of his comments, together with some additions here and there of our own.

It is doubtful if on page 135 it was made sufficiently clear that in the absence of a reliable estimate of σ, the u test cannot be used, and that the z test (which is equivalent to Fisher and Student's t test) is the only recourse. (By a *reliable estimate of* σ we mean an estimate that is considerably more reliable than can be obtained from the single sample under test.) The z test is not inherently misleading; it tests objectively a proposed value of z, and for this purpose it is of course perfectly valid (as we say). Like any statistical test, the z test lays down and accepts a perfectly definite hazard. Misinterpretations of the z test may be common, but the blame should be placed, not on the test itself, but on misunderstandings of the nature that we point out on page 135. What is more to be feared than over-confidence in the z test is the use of the normal probability integral (the u test) with an estimate of σ based *on the single sample* under test.

The separation of the parameters of the parent population from *estimates* of these parameters has been a gradual process. Many writers have been extremely careless in confusing that which is estimated with an estimate of it. Thinking to avoid any such looseness, we systematically used Greek and Latin letters to distinguish the two classes of quantity. It is perhaps well to go even further and use distinguishing names for the two classes. For this purpose there are in use today the terms "parameter" of the parent population and "statistic" of the sample, the work "statistic" having been introduced by Fisher (footnote 4 of our article) in 1921 to fill the need of a term antithetical to "parameter." A parent population is completely specified by its one or two or more parameters, but a sample of n would require n different statistics for its specification. To each of these statistics there corresponds a particular parameter or parametric function toward which the value of the statistic tends as the sample is indefinitely increased; but to each parameter there "corresponds," in this sense, as many statistics as there can be of samples from a given parent population, to which number there is no limit. For these reasons it would doubtless have been better to have written "corresponding statistic of a sample" on page 142, 7 lines below section (*3e*), to avoid giving the impression that there is a one to one correspondence between the two quantities s and σ.

In further connection with fiducial probability it should be mentioned that fiducial values can be taken only from distributions of statistics that contain the whole of the information that can be obtained from the sample. The distribution of s fulfills this requirement, and our discussion of fiducially related values of σ and s is therefore valid, but it is worth while to note that the distribution of, for example, the arithmetic mean deviation, from which Peters' formula (see any text on least squares) is derived, could not be so used. There is not room here, and neither was there in the original article, to discuss the criteria of "efficiency" and "sufficiency," but they might at least be mentioned with a reference. The reader will find them discussed in the papers cited in footnotes 4 and 31.

W. Edwards Deming,
Bureau of Chemistry and Soils,
Washington, D. C.
Raymond T. Birge,
University of California
Berkeley.
November 9, 1934.

[1] Deming and Birge, Rev. Mod. Phys. **6**, 119–161 (1934).

A Low-Power Positive-Ion Source of High Intensity

Numerous tests have been made in this laboratory of various low voltage and high voltage ion-sources, all of which presented serious limitations in use. Dr. F. L. Mohler of the National Bureau of Standards kindly described to us last summer some experiments having a similar objective, in which with low power he had obtained very large current-densities to a negative probe inserted from one side into a hot cathode, low voltage arc *confined in a quartz capillary*. We duplicated this result, but rapid deterioration of the quartz and the difficulty of its replacement with the necessary probe-canal accurately in alignment led us to consider other materials for the arc-capillary. Since every positive ion diffusing to the quartz wall is neutralized on contact it appeared that a *metal capillary*, allowed to "float" in potential, should serve equally well. This was promptly confirmed, and a series of subsequent experiments, concerned chiefly with the empirical space-charge problem of focussing such high ion-densities into a beam, led to the ion-source shown in Fig. 1. Difficulty has been encountered in brief efforts to focus these intense ion-beams on the target of the high voltage tube, but description of the ion-source may be of immediate value to other workers.

The important features of the ion-source are shown to scale in Fig. 1. The steel arc-body A is bored out, forming the arc-capillary B joining the arc-spaces which contain the oxide-coated filament C and anode D (110 volts with resistance). A side volume E for ions diffusing to the probe F is provided with the important "focussing-diaphragm" G which gives very high current-density to the small canal H in the center of the probe. Conical insert J provides flat gas-seal faces. The probe is supported on the Lavite insulator K, and is held at 500 to 7000 volts filtered direct current by a Variac-controlled '66 rectifier-set. Focussing electrode L is held at 2 to 15 kilovolts direct current by a ZP-85 kenotron set with condensers. Protective resistors of high value (50,000) are essential to prevent probe- and focus-voltages from "blowing out" the arc (possibly by oscillations). A self-striking "auxiliary arc" (0.1 ampere) to the steel arc-body is usually used to strike the main arc through the capillary (0.2 to 2 amperes); otherwise the application of a small spark-coil to the (floating) arc-body starts the capillary-arc. Pressures are roughly 2×10^{-2} in the arc and 2×10^{-5} in the tube; speed of Apiezon pumping system is about 30 liters/sec. (air); hydrogen flow is perhaps 10 cc per hour at N P T, allowing the use of deuterium gas without recovery at nominal cost. The ratio of atomic to molecular ions is high and is unimportant since magnetic analysis is used at the target.

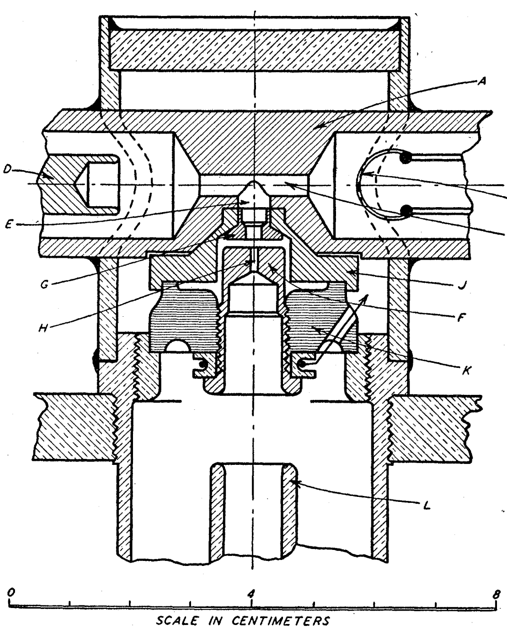

FIG. 1. Ion-source detail. (Department of Terrestrial Magnetism, Carnegie Institution of Washington, No. 3274.)

With a total power-consumption at maximum under 250 watts, and this gas flow which can be handled by modest pumping arrangements, true ion-currents (no secondary electrons) up to 1.5 *milliamperes* are readily obtained through the probe-canal 1 mm in diameter and 4 mm long shown in Fig. 1 (probe-voltage 7 kilovolts; probe-current 3 milliamperes; arc 2 amperes). Higher ion-currents undoubtedly can be obtained by using a larger probe-canal and higher current-density in the capillary. Application of 6 kilovolts to the focussing electrodes shown (we have made tests on various electrode-arrangements) will completely focus the probe-canal output-current of 25 microamperes through a 5-mm diameter hole at a distance of 11 cm (position of next electrode-gap) when the probe is held at 700 volts; 9 kilovolts are required with the probe at 1000 volts (80 microamperes), and 15 kilovolts are required to focus the probe-output of 250 microamperes at 1500 volts. The simple "lens" FL does not bring the whole ion-beam into good focus at large distances without proper intermediate focussing-gaps.

M. A. TUVE
O. DAHL
C. M. VAN ATTA[1]

Department of Terrestrial Magnetism,
Carnegie Institution of Washington,
Washington, D. C.,
November 14, 1934.

[1] National Research Fellow, Massachusetts Institute of Technology guest investigator at the Department of Terrestrial Magnetism.