

lines up to the point where these bands run into the heads of 8-0 and 8-1, respectively. The succeeding points deviate from these lines and follow lines with a considerably greater slope. This might be the result of a perturbation as these authors believed, but since it might also indicate that they failed to follow the branches of 10-1 and 10-2 through the regions where there is a great overlapping of lines, the data on these regions were carefully examined. It was found that the measured lines can be reassigned to the various branches in a very plausible way so that the deviation just mentioned vanishes and we obtain a set of combinations which agree for the bands 8-1, 9-1 and 10-1, and are slightly different for 8-0 and 10-2. This, together with the fact that the rotational constants obtained from these combinations are what we should expect for the ground level, is rather conclusive evidence that these combinations are $\Delta_2 F''$ rather than $\Delta_2 F'$ as Naudé and Christy believed.

A new set of combinations was then made to find the true $\Delta_2 F'$ values. From these one finds that the rotational constants vary in the vibrational states of the upper electronic level in such a fashion that it is difficult to make an extrapolation to the vibrationless state which has great significance. Furthermore some perturbations are found in the level $v'=8$. These duplicate in the bands 8-0 and 8-1 and are additional confirmation of the correctness of the new combinations. Large perturbations are to be expected in the upper state since it has been observed⁴ that many of the band heads are considerably displaced from their expected positions.

The results of the new analysis are given in Table I. An important feature is that it shows that the *even* rotational

TABLE I. Constants of the sulfur molecule.

| | $B(\text{cm}^{-1})$ | $D_e(\text{calc}), \alpha_e, B_e(\text{cm}^{-1})$ | r_e |
|-----------|---------------------|---|-------------------------|
| $v''=0$ | 0.309 | $D_e'' = -2.2 \times 10^{-7}$ | $r_e'' = 1.840\text{A}$ |
| $v''=1$ | 0.307 | $\alpha_e'' = 0.0016$ | |
| $v''=2$ | 0.306 | $B_e'' = 0.309_6$ | |
| $v' = 8$ | 0.230 | $D_e' = -2.4 \times 10^{-7}$ | |
| $v' = 9$ | 0.229 | (0.211 for $K > 21$) | |
| $v' = 10$ | 0.239 | | |

levels are present in the ground state of the sulfur molecule instead of the odd levels as found in oxygen. Since the electronic states of these two molecules are presumably of similar character, this shows that if the oxygen molecule is symmetric in the nuclei, as has been generally accepted, the sulfur molecule is antisymmetric. However, in case the character of the electronic states has been incorrectly assigned, it is possible that the symmetry character should be reversed in both cases.

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¹ S. M. Naudé and A. Christy, Phys. Rev. **37**, 490 (1931).

² R. M. Badger, J. Chem. Phys. **2**, 128 (1934).

³ The v' numbering of Naudé and Christy should be increased by one according to the results of Fowler and Vaidya, Proc. Roy. Soc. **A132**, 310 (1931).

⁴ A. Christy and S. M. Naudé, Phys. Rev. **37**, 903 (1931).

Concerning the Recovery Time of Geiger-Mueller Counters

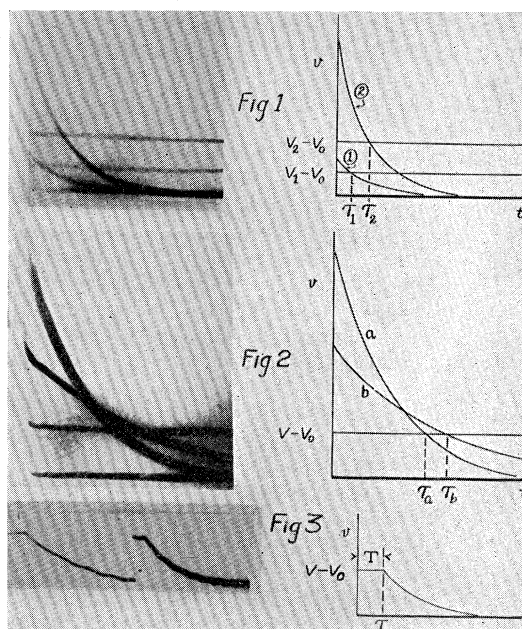
We have equipped a Braun tube with a timing "sweep" which is initiated by the phenomenon to be studied. Thus, pulses recurring at random will give retracing curves on the fluorescent screen and so may be photographed. Figs. 1-3 show three oscillograms with a schematic diagram at the right of each.

The ordinates of the curves represent the negative potential v of the wire of a Geiger-Mueller counter connected in the usual way, namely, with the outer cylinder at constant (negative) potential V , and the wire grounded through a high resistance R . Figs. 1 and 2 show that when a discharge occurs, v rises suddenly and then falls following an exponential curve. As one would expect its time constant is equal to RC where C is the total capacity between the wire and ground. Curves 1 and 2 in Fig. 1 show discharges for two values V_1 and V_2 of the applied voltage. The horizontal lines $V_1 - V_0$ and $V_2 - V_0$ show the excess of V over the starting potential V_0 . In other words $V - V_0$ represents that value of v for which the voltage across the counter equals the starting voltage.

By suddenly applying x-radiation we have seen that a counter will discharge after v has fallen below $V - V_0$ but not before. Therefore, the recovery times for the respective systems are defined by the associated time intervals denoted by the τ 's in the oscillograms.

In Fig. 1 V_2 is greater than V_1 , so one sees that the excess of the initial value of v over $V - V_0$ increases with increasing V . Therefore the time τ is larger for higher operating voltages.

The two curves in Fig. 2 were taken with the same applied voltage V . In curve *b* the value of C was about



FIGS. 1-3.

twice that in curve *a*. This increase of *C* has increased the time constant of the curve but decreased the initial ordinate. The former change tends to increase τ while the latter tends to diminish it. In this particular case, doubling *C* has increased τ by about 27 percent. It seems quite possible that in some cases increasing *C* may actually decrease τ .

If a pulse occurs before the *v* of a preceding one has fallen to zero, it will be smaller than the preceding pulse and have a smaller τ . So for very rapid counting rates one has a statistical distribution of τ 's.

For higher values of *V* or *C*, the discharges may take the form shown in Fig. 3. This is a photograph of a single sweep during which two discharges occurred. (Some 60 cycle induction is present.) Here *v* rises to $V - V_0$ and remains there for a time *T* before decaying exponentially. The time τ is of course given by *T* and may vary widely in successive pulses. The average *T* is an increasing function of *C* and decreases as *R* increases.

The observations described here are quite uniform throughout a set of counters made in the same "batch." The wire in all counters was 3 mil tungsten and the cylinders were oxidized copper. The pressure in all cases was 7 or 8 cm. The qualitative phenomena are independent of counter size, of whether the gas is air or argon and oxygen, and of change in *R* between 5×10^8 and 5×10^9 ohms.

We hope to submit a report of quantitative nature in the near future.

It is a pleasure to acknowledge the helpful support of Dr. W. F. G. Swann.

W. E. DANFORTH

The Bartol Research Foundation
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November 6, 1934.

On the Statistical Theory of Errors

Professor R. A. Fisher has most kindly responded to our request for criticisms of the article that appeared under the above title.¹ The material in his letter is much too valuable to be filed away, so with his consent we here present the substance of his comments, together with some additions here and there of our own.

It is doubtful if on page 135 it was made sufficiently clear that in the absence of a reliable estimate of σ , the *u* test cannot be used, and that the *z* test (which is equivalent to Fisher and Student's *t* test) is the only recourse. (By a *reliable estimate of σ* we mean an estimate that is considerably more reliable than can be obtained from the single sample under test.) The *z* test is not inherently misleading; it tests objectively a proposed value of *z*, and for this purpose it is of course perfectly valid (as we say). Like any statistical test, the *z* test lays down and accepts a perfectly definite hazard. Misinterpretations of the *z* test may be common, but the blame should be placed, not on the test itself, but on misunderstandings of the nature that we point out on page 135. What is more to be feared than over-confidence in the *z* test is the use of the normal prob-

ability integral (the *u* test) with an estimate of σ based on the *single sample* under test.

The separation of the parameters of the parent population from *estimates* of these parameters has been a gradual process. Many writers have been extremely careless in confusing that which is estimated with an estimate of it. Thinking to avoid any such looseness, we systematically used Greek and Latin letters to distinguish the two classes of quantity. It is perhaps well to go even further and use distinguishing names for the two classes. For this purpose there are in use today the terms "parameter" of the parent population and "statistic" of the sample, the work "statistic" having been introduced by Fisher (footnote 4 of our article) in 1921 to fill the need of a term antithetical to "parameter." A parent population is completely specified by its one or two or more parameters, but a sample of *n* would require *n* different statistics for its specification. To each of these statistics there corresponds a particular parameter or parametric function toward which the value of the statistic tends as the sample is indefinitely increased; but to each parameter there "corresponds," in this sense, as many statistics as there can be of samples from a given parent population, to which number there is no limit. For these reasons it would doubtless have been better to have written "corresponding statistic of a sample" on page 142, 7 lines below section (3e), to avoid giving the impression that there is a one to one correspondence between the two quantities *s* and σ .

In further connection with fiducial probability it should be mentioned that fiducial values can be taken only from distributions of statistics that contain the whole of the information that can be obtained from the sample. The distribution of *s* fulfills this requirement, and our discussion of fiducially related values of σ and *s* is therefore valid, but it is worth while to note that the distribution of, for example, the arithmetic mean deviation, from which Peters' formula (see any text on least squares) is derived, could not be so used. There is not room here, and neither was there in the original article, to discuss the criteria of "efficiency" and "sufficiency," but they might at least be mentioned with a reference. The reader will find them discussed in the papers cited in footnotes 4 and 31.

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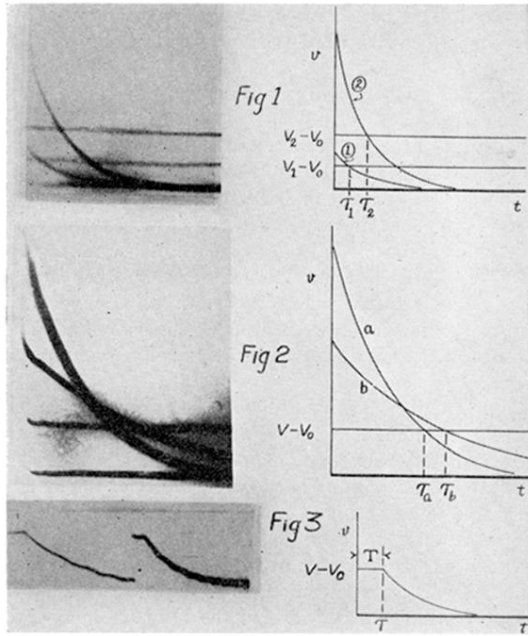
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November 9, 1934.

¹ Deming and Birge, *Rev. Mod. Phys.* 6, 119-161 (1934).

A Low-Power Positive-Ion Source of High Intensity

Numerous tests have been made in this laboratory of various low voltage and high voltage ion-sources, all of which presented serious limitations in use. Dr. F. L. Mohler of the National Bureau of Standards kindly described to us last summer some experiments having a similar objective, in which with low power he had obtained



FIGS. 1-3.