Magnetically Self-Focussing Streams

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(Received January 13, 1933)

Streams of fast electrons which can accumulate positive ions in sufficient quantity to have a linear density of positives about equal to the linear density of electrons, along the stream, become magnetically self-focussing when the current exceeds a value which can be calculated from the initial stream conditions. Focussing conditions obtain when breakdown occurs in cold emission. The characteristic features of breakdown are explained by the theory. Failure of high voltage tubes is also discussed.

INTRODUCTION

ALTHOUGH the focussing effect which residual gas can have on low voltage electron streams due to the fact that the ions have much smaller velocities than the electrons freed by ionization of the residual gas, has been discussed,[1] not much attention seems to have been given to the focussing effect on electron streams due to the effect of magnetic attractions between the parts of the stream. Calculation shows that such focussing can be very important at high voltages and the result explains some phenomena which occur in high voltage tubes, and which have never been satisfactorily explained. The result has a bearing on the phenomenon of breakdown in cold emission.

How such focussing can take place can be seen qualitatively by considering a stream of high velocity electrons with velocity, u, all moving parallel to a direction which we may choose as the Z-axis, and positive ions moving with velocity, v, in the opposite direction. The density of electrons, positives, and residual gas are assumed to be small and collisions infrequent. The force acting between any two electrons has components[2]

$$F_x=(1-u^2/c^2)E_x;\quad F_y=(1-u^2/c^2)E_y;\quad F_z=E_z;\quad (1)$$

where E_x, E_y and E_z are the components of the force which would exist if the electrons were not moving, i.e., the familiar Coulomb force. The force acting between any two positives has components

$$F_x'=(1-v^2/c^2)E_x';\ F_y'=(1-v^2/c^2)E_y';\ F_z'=E_z';\quad (2)$$

and the force between any electron and positive

$$F_x''=(1+uv/c^2)E_x'';\ F_y''=(1+uv/c^2)E_y'';\ F_z''=E_z''.\quad (3)$$

If the density of positives everywhere equals the density of electrons, the static attractions and repulsions cancel, but the additional magnetic force in every case is an attraction so that the charge in every element of volume attracts the charge in every other element of volume. In a cylindrically symmetric stream which is of uniform composition along its length, and which is long compared with its diameter, the attraction between each small element of volume and any long thin element of the stream parallel to the axis, varies inversely as the radial distance from the first small element to the axis of the long thin element, and so the potential energy of the first small element is proportional to the integral of the logarithm of the radial distance to the various long thin elements of volume.

The stream could not spread out indefinitely because in so doing, the potential energy of the stream would approach a logarithmic infinity. No single electron or positive could leave the stream indefinitely far because its individual potential energy would approach a logarithmic infinity. Hence positives in such a stream could

[1] Johnson, J.O.S.A. and R.S.I. **6**, 701 (1922) and Buchta, J.O.S.A. and R.S.I. **10**, 581 (1925).
[2] See Mason and Weaver, *Electromagnetic Field*, p. 299.

not leave it except by moving out of one of the ends, unless there were more positives than electrons, in which case, only enough would leave the stream to equalize the numbers of each kind of particle per unit length of stream.

The hypothesis of positives at the same density as electrons is not so improbable of realization as might have appeared at first sight, both because the velocity of a positive is much smaller than that of an electron formed by the same collision, and also because the positives once formed in the stream have to pass down the stream clear to the end. An electron freed by the same collision that formed a positive, is acted upon by an attractive force which is much smaller than that acting on the high velocity electrons in the stream, as may be seen from the form of expressions (1). The force between the slow electron the Z-component of whose velocity is u', and each fast electron with velocity u, has components

$$F_x'''=(1-uu'/c^2)E_x'''; \quad F_y'''=(1-uu'/c^2)E_y'''; \quad F_z'''=E_z'''.$$

Since u' is much smaller than u, the attractive force which is proportional to $u\cdot u'/c^2$ is much smaller than the attractive force on a fast electron at the same position. A strong selection thus acts to allow the slow electrons so freed to leave the stream radially with whatever radial velocity they had after the collisions, rather than to allow them to expel the high velocity electrons. Especially in the case of high voltage streams in regions of low field in the direction of the stream will the stream tend to collect positives at high density. This hypothesis will be treated more in detail, later, after a much more definite and quantitative treatment has been given the problem.

Equilibrium Distribution

It will be convenient for the later discussion of actual streams to describe first a special distribution of the particles which is in dynamic equilibrium and which can be designated as the equilibrium distribution. This is to be a cylindrically symmetric stream having the Z-axis as the axis of symmetry. The stream consists of electrons with velocity, u, in the positive Z-direction, and positive ions having velocity, v, in the opposite direction. Superimposed on the relatively large velocity, u, the electrons are supposed to have a Maxwellian distribution of X- and Y-components of velocity so that to an observer moving parallel to the stream with a velocity, u, the number of electrons per unit length of stream with X-components of velocity between U and $U+dU$, and with Y-components between V and $V+dV$, is

$$\lambda_1\left(\frac{m}{2\pi kT_1}\right)\exp\left[-\frac{m}{2kT_1}(U^2+V^2)\right]dU\cdot dV$$

where λ_1 is the number of all electrons per unit length of stream, m is the mass of an electron, and T_1 is a constant which would be called the temperature if the Z-components of velocity were also being considered. It can be called the temperature of the electrons if it is kept in mind that temperature in this discussion means the two-dimensional temperature so defined. The distribution of Z-components of velocity does not make any difference in this problem because small variations in the kinetic energies due to Z-components of velocity cannot affect appreciably the radial distribution of the particles in the stream. Similarly, the positives are supposed to have a Maxwellian distribution of X- and Y-components of velocity superimposed on the velocity, v, in the negative Z-direction, so that to an observer moving with a velocity, v, the number of positives with X-components between U and $U+dU$, and Y-components between V and $V+dV$, is

$$\lambda_2\left(\frac{M}{2\pi kT_2}\right)\exp\left[-\frac{\mu}{2kT_2}(U^2+V^2)\right]dU\cdot dV$$

where λ_2 is the number of all positives per unit length of stream, M is the mass of a positive, and T_2 is the (two-dimensional) temperature of the positives.

Although collisions are assumed to be infrequent, and hence no thermodynamical equilibrium is being approached, it can be shown[3] that a system of particles moving in a field where the potential energy of a particle is $X(x, y)$, and having a distribution such that the number of particles between x and $x+dx$, y and $y+dy$,

[3] See Jeans, *Dynamical Theory of Gases*, p. 89.

z and $z+dz$ and with velocities between U and $U+dU$, V and $V+dV$, is

$$\rho_0\left(\frac{m}{2\pi kT}\right)\exp\left[-\frac{X(x,y)}{kT}\right]$$
$$\exp\left[-\frac{m}{2kT}(U^2+V^2)\right]dx\cdot dy\cdot dz\cdot dU\cdot dV$$

will retain this distribution. The familiar derivation of this expression[4] shows that this kind of distribution is retained in spite of collisions and so it is obvious that such a distribution is retained when collisions are negligible.

A solution will now be obtained for a distribution of electrons and positives each of which has this kind of distribution, and this will be called the equilibrium distribution. This name need not include any ideas concerning how this distribution was arrived at, provided this distribution will be retained by the stream.

To an observer moving along with the electrons, the density of electrons is

$$\rho_{11}=\rho_{110}\cdot e^{-X_{11}(r)/kT_{11}}, \tag{4}$$

where ρ_{110} is the density of electrons at a radial distance where the potential energy of an electron is zero. Similarly, to an observer moving along with the positives, the density of positives is

$$\rho_{22}=\rho_{220}\cdot e^{-X_{22}(r)/kT_{22}}. \tag{5}$$

By applying Poisson's equation in the first instance, since

$$X_{11}=-e\cdot V_1$$

where V_1 is the electric potential in the first system of coordinates,

$$\nabla^2 V_1=-4\pi[-e\cdot\rho_{11}+e\cdot\rho_{21}],$$

where ρ_{11} is the density of electrons in the first system of coordinates, and ρ_{21} is the density of positives in the first system of coordinates. Substituting from Eq. (4),

$$\nabla^2 \log \rho_{11}=4\pi e^2[\rho_{11}-\rho_{21}]/kT_{11}. \tag{6}$$

Analogously, on a second system of coordinates moving with a velocity, v, with the positives,

$$\nabla^2 V_{22}=-4\pi[-e\cdot\rho_{12}+e\cdot\rho_{22}]$$

and substituting from Eq. (5)

$$\nabla^2 \log \rho_{22}=4\pi e^2[\rho_{22}-\rho_{12}]/kT_{22}, \tag{7}$$

where ρ_{22} is the density of positives in the second system of coordinates and ρ_{12} is the density of electrons in this system. Writing

$$4\pi e^2/kT_{11}=\alpha_1 \quad\text{and}\quad 4\pi e^2/kT_{22}=\alpha_2$$

the equations can be transformed to equations in the rest system of coordinates[5] by writing

$$\rho_{11}=\rho_1/\beta_1;\quad \rho_{12}=\beta_2[1+uv/c^2]\cdot\rho_1;\quad \rho_{21}=\beta_1[1+uv/c^2]\cdot\rho_2;\quad \rho_{22}=\rho_2/\beta_2,$$

where

$$\beta_1=(1-u^2/c^2)^{-\frac{1}{2}};\quad \beta_2=(1-v^2/c^2)^{-\frac{1}{2}}$$

and Eqs. (6) and (7) become

$$\nabla^2 \log \rho_1=\alpha_1\beta_1[1-u^2/c^2]\cdot\rho_1-\alpha_1\beta_1[1+uv/c^2]\cdot\rho_2, \tag{8}$$
$$\nabla^2 \log \rho_2=\alpha_2\beta_2[1-v^2/c^2]\cdot\rho_2-\alpha_2\beta_2[1+uv/c^2]\cdot\rho_1. \tag{9}$$

These are exactly the same equations as are arrived at, if, instead of using Poisson's equation in each of two moving systems of coordinates, we calculate the potential energy of each of the two kinds of particle in the rest system of coordinates, using expressions (1), (2) and (3) for the forces between particles given in the Introduction, *viz.*,

$$X_1=\int_R^r 2e\left[1+\frac{uv}{c^2}\right]\frac{d\lambda}{\lambda}\int_0^\lambda e\cdot\rho_2(\xi)\cdot 2\pi\xi\cdot d\xi-\int_R^r 2e\left[1-\frac{u^2}{c^2}\right]\frac{d\lambda}{\lambda}\int_0^\lambda e\cdot\rho_1(\xi)\cdot 2\pi\xi\cdot d\xi,$$

$$X_2=\int_R^r 2e\left[1+\frac{uv}{c^2}\right]\frac{d\lambda}{\lambda}\int_0^\lambda e\cdot\rho_1(\xi)\cdot 2\pi\xi\cdot d\xi-\int_R^r 2e\left[1-\frac{v^2}{c^2}\right]\frac{d\lambda}{\lambda}\int_0^\lambda e\cdot\rho_2(\xi)\cdot 2\pi\xi\cdot d\xi,$$

[4] The derivation usually includes the distribution in the Z-component of velocity, too, but since the potential function is not a function of z, the distribution in the Z-component of velocity cannot affect the density.

[5] A discussion of the relativistic transformation of density can be found in Eddington's *Mathematical Theory of Relativity*, p. 33, *et seq.*

where R is the radial distance from the axis at which the potential energy is zero. These can be substituted in the Boltzmann expressions (4) and (5) (taking logarithms and derivatives to put the equations into the form of (8) and (9)) if we keep in mind that the temperatures in the rest system must be relativistically transformed from those in the moving systems by

$$T_1=\beta_1\cdot T_{11};\quad T_2=\beta_2\cdot T_{22}$$

because the two-dimensional temperatures used here are proportional to the transverse kinetic energies in which the mass and transverse velocities must be transformed.

Since exactly the same equations are obtained regardless of which method of approach to the problem is used, even to the extent of being relativistically invariant, we are justified in using expressions (1), (2) and (3) for the forces on the particles in the later treatment of non-equilibrium cases.

A particular solution of (8) and (9) is

$$\rho_1=\rho_0/[1+b\rho_0r^2]^2$$
$$\rho_2=\frac{\alpha_1\beta_1[1-u^2/c^2]+\alpha_2\beta_2[1+uv/c^2]}{\alpha_2\beta_2[1-v^2/c^2]+\alpha_1\beta_1[1+uv/c^2]}\cdot\rho_1, \tag{10}$$

where

$$b=\frac{\alpha_1\alpha_2\beta_1\beta_2}{8c^2}\cdot\frac{(u+v)^2}{\alpha_1\beta_1[1+uv/c^2]+\alpha_2\beta_2[1-v^2/c^2]}.$$

This solution is of course relativistically invariant.

If the ratio of the density of electrons to the density of positives in each of any two elements of volume in the stream, swept out by passing two small elements of cross-sectional area along the entire stream parallel to the axis, is greater than

$$\frac{1+v/c}{1-u/c}\quad\text{or less than}\quad\frac{1-v/c}{1+u/c}$$

it is seen from the application of expressions (1), (2) and (3) given for the forces between particles, in the Introduction, that the charge in any two such elements would repel and an equilibrium distribution with such ratios present is impossible. Thus, although expressions (10), are only a particular solution, they are a very good approximation to the complete solution for the distribution when the velocities u and v are small compared with the velocity of light, c, and in fact no serious error will be introduced by setting the coefficient in Eqs. (10) equal to unity. This makes the approximate solution

$$\rho_1=\rho_2=\rho_0/[1+b\rho_0r^2]^2,$$

where

$$b=\frac{\pi e^2}{2kc^2}\cdot\frac{(u+v)^2}{T_1+T_2}. \tag{11}$$

The number of either kind of particle per unit length of stream is

$$\lambda_0=[c^2\cdot 2K(T_1+T_2)]/e^2(u+v)^2 \tag{12}$$

and the current is

$$i_0=[c^2\cdot 2K(T_1+T_2)]/e(u+v),$$

which is rigidly fixed by the values of u, v, T_1 and T_2. Thus an equilibrium distribution of the kind just described can exist only for one special value of the current. This will be designated as the critical current.

Sources and Distributions of Positives

At electron velocities corresponding to potential drops of the order of 5000 volts and larger, the probability that a collision with a neutral molecule will result in ionization is to a good approximation (except when the velocities approach that of light) inversely proportional to the voltage of the primary electron.[6] The number of ions formed by each electron per centimeter of path from gases more apt to be present in tubes at moderately high vacuum, can be esti-

[6] See J. J. Thomson, *Conduction of Electricity through Gases*, Vol. 2, 3rd Ed., p. 96, *et seq.*

mated at least for order of magnitude as $200p/V$, where p is the pressure in mm of mercury, and V is the potential of the electrons in e.s.u. If the current in e.s.u. in a stream of fast electrons is i, the number of ions formed per centimeter length of stream, per second, is $200pi/Ve$. If it is supposed that these ions are formed in a region having a uniform field intensity, E, the number of ions per centimeter length of stream coming from along the stream back a distance, d, is

$$200\frac{p}{V}\cdot\frac{i}{e}\cdot\left(\frac{2Md}{Ee}\right)^{\frac{1}{2}},$$

where M is the mass of an ion. For this to be equal to or greater than the number of fast electrons per centimeter length of stream

$$\frac{i}{e}\left(\frac{m}{2Ve}\right)^{\frac{1}{2}}\leq 200\frac{p}{V}\cdot\frac{i}{e}\left(\frac{2Md}{Ee}\right)^{\frac{1}{2}}$$

and so the pressure of the gas must be, approximately

$$p\geq 2\cdot 10^{-7}(VE/Wd)^{\frac{1}{2}}, \qquad (13)$$

where V and E are in volts, and volts per centimeter, respectively, and W is the molecular weight of the gas from which the ions are formed. If the actual gas pressure exceeds the value calculated from expression (13) for the conditions at any part of the stream, the number of positives per unit length of stream will exceed the number of fast electrons at that part of the stream.

In addition to the residual gas as a source of ions, we must keep in mind that it has been found in numerous cases[7] that large amounts of the anode material can be removed from the anode by high voltage streams. This necessarily increases the density of matter in the vicinity of the stream while large electron currents are passing.

In order to see what is to be expected to be the kinetic energy of the positives after they are formed, some experiments by Ishino[8] can be considered in which it was found that when fast electrons are passed through a rarefied gas, the electrons freed in ionizations have transverse kinetic energies distributed so that about ninety percent of the electrons have energies less than forty volts and about ten percent have energies greater than this value. Since the rapidly moving ionizing electron acts on both the atomic electron and the rest of the atom during the same interval of time, the momentum given the two must be equal and opposite so at least ninety percent of the ions must have transverse energies less than $0.022/W$ electron-volts, where W is the molecular weight. If the ions are formed from residual gas at room temperature, their velocities are not greatly affected by ionization and the average kinetic energy is approximately the energy corresponding to that temperature.

If positive ions are present at densities greater than those of the high velocity electrons, only enough of the low velocity electrons freed in the formation of positives will move directly out of the stream, to leave an average charge density only slightly positive in the stream. The static focussing field which can be obtained in this way is a small field (order of magnitude of a few volts per centimeter) similar in nature to the focussing field characteristic of the low voltage streams in the presence of gas at much higher pressure, mentioned earlier.[1]

If the positive ions have velocities much smaller than the fast electrons, as they do have except in extreme proximity to the cathode, it is apparent from the form of expressions (1), (2) and (3) that the excess of positive ions over the fast electrons, together with the neutralizing space charge of slow electrons, do not appreciably affect the motions of the fast electrons magnetically and consequently have a negligible effect on the stream. Thus in any section where there are at least as many positives as fast electrons, we are concerned with the dynamics of a stream consisting of equal numbers of each kind of particle per unit length of stream.

Stability

In order to extend this treatment to streams under conditions apt to occur experimentally, a function whose value will increase as the particles spread out from the axis can be used as a criterion of how much the stream has spread:

$$F=\int_0^\infty r^2\cdot\rho\cdot 2\pi r\cdot dr.$$

[7] Hull and Burger, Phys. Rev. **31**, 1121 (1928); Snoddy, Phys. Rev. **37**, 1878 (1931); Newman, Phil. Mag. **14**, 712 (1932); Bennett, Phys. Rev. **37**, 590 (1931).
[8] Ishino, Phil. Mag. **32**, 202 (1916).

A maximum possible value for this function can be calculated subject to the restriction that the potential energy of the stream per unit length of stream shall not exceed a value, H, which is the initial potential energy plus the sum of the kinetic energies due to components of velocity transverse to the axis, of all the particles.

Although the density of positives will not in general equal the density of electrons at all radial distances from the axis, the maximum spread occurs when the two densities are everywhere equal, so that in order to find the maximum value of F, we can write the restriction as

$$\tfrac{1}{2}\cdot(2e^2(u+v)^2/c^2)\int_0^\infty \log(r/R)\cdot 2\pi r\cdot\rho(r)\cdot dr\int_0^r \rho(\xi)\cdot 2\pi\xi\cdot d\xi = H.$$

This calculation can be simplified by using as the independent variable, n, the total number of either kind of particle from the axis out to a radial distance, r, and letting the distance, r, be dependent on n. Then

$$F=\int_0^N r^2\cdot dn \quad\text{and}\quad H=g\int_0^N n\cdot\log(r/R)dn,$$

where $g=e^2(u+v)^2/c^2$ and N is the total number of either kind of particle per unit length of stream. In order to find the maximum spread, set $\delta\{F+\lambda H\}=0$, where λ is an arbitrary multiplier. This gives a distribution with uniform density ρ' from the axis out to a radial distance, r_0, and zero beyond, where

$$\pi r_0^2\rho'=N;\quad \pi r^2\rho'=n;\quad r=r_0(n/N)^{\frac{1}{2}}.$$

If the stream is initially distributed uniformly inside a radial distance from the axis, s_0, and has a total kinetic energy due to transverse components of velocity, K, which can be related to another constant, T, by $K=2N\cdot kT$, the maximum spread is a uniform distribution out to a radial distance, r_0, given by

$$g\int_0^N n\left(\log\frac{r_0}{R}\cdot\left(\frac{n}{N}\right)^{\frac{1}{2}}\right)\cdot dn = g\int_0^N n\left(\log\frac{s_0}{R}\left(\frac{n}{N}\right)^{\frac{1}{2}}\right)\cdot dn+2N\cdot kT,$$

i.e.,

$$r_0=s_0\cdot\exp\left[\frac{c^2\cdot 2\cdot 2kT}{e^2(u+v)^2}\cdot\frac{1}{N}\right]. \tag{14}$$

More than half of the particles must always lie nearer the axis than $1/2^{\frac{1}{2}}$ times this value of r_0.

If the kinetic energy of the particles due to transverse components of velocity could be redistributed among the particles so that the transverse components of velocity would have Maxwellian distributions, the (two-dimensional) temperature would be T given by $K=2N\cdot kT$ and the linear density λ_0 of the stream which could have an equilibrium distribution such as given by Eqs. (12), can be substituted in expression (14). Thus more than half the particles in the stream must always lie nearer the axis than a radial distance given by

$$(s_0/2^{\frac{1}{2}})\cdot e^{\lambda_0/N}=(s_0/2^{\frac{1}{2}})\cdot e^{i_0/I}, \tag{15}$$

where i_0 is the critical current (expression (12)) and I is the actual current.

If half of the particles get outside this radial distance, all of the transverse velocities would be simultaneously reduced to zero which is obviously impossible. Thus this expression is an exaggerated outside limit of the radial distance inside which at least half of the particles must always remain. In fact, calculation of the directions in which most of the particles would begin to drift for several special distributions show that the ratio i_0/I distinguishes between streams which begin to contract towards the axis, and those which begin to expand, the contraction occurring when $I>i_0$.

From the form of expression (15), it is seen that if the actual current is one-tenth the critical current, the radial distance inside which at least half the particles must always remain, is liable to increase to more than 10,000 times the initial value for the outer radius of the stream, which for most experimental cases means no observable focussing action. In this sense, it may be said that the critical current given by expression (12) rather sharply distinguishes between streams which focus and those which do not. This critical current may be written as

$$i_0 = 2.5 \cdot 10^{-3} T / V^{\frac{1}{2}}, \qquad (16)$$

where i_0 is in amperes, V is the potential through which the fast electrons have fallen, in volts (except in extreme proximity to the cathode), and T is in equivalent degrees absolute, and is a constant proportional to the total transverse energy of the stream per unit length.

Applications

In an earlier communication,[9] evidence was presented supporting the contention that breakdown in cold emission is caused by positive ions liberated at the anode by small currents, and travelling back along the electron stream to the cathode. A difficulty with this idea is that even if positives do bombard the cathode in this way, still at such low densities they might be expected to eliminate any emitting areas where they strike, instead of producing them. Now this is just what has been observed to occur except when the current rises to above the order of 10^{-2} amperes. It was not clear then why this order of magnitude of the current should be so critical in changing from elimination of emitting areas to severe rupture of the cathode surface.

Focussing the positives on a small area of the cathode cannot be produced in the analogous way to the focussing of low voltage electron streams in gases at relatively higher pressures[1] by interchanging the rôles of positives and electrons, because the electrons are not produced by the positives from the residual gas as would be required by that kind of explanation.

On the other hand, if the current is momentarily large enough to give magnetic self-focussing over any considerable part of the stream, the electrons towards the anode are brought nearer the axis by this focussing action, and then since the initial total energy of positives in any part of the stream depends on the position with respect to the axis at which they were formed, and since bringing electrons nearer the axis in parts of the stream towards the anode reacts to decrease the total transverse energy of the stream per unit length in the central part where self-focussing conditions obtain, the effect of the magnetic self-focussing is to bring all particles nearer the axis over an increasingly long segment of the stream. In an analogous way, bringing the positives nearer the axis of the stream very near the cathode increases the emission from the part of the cathode delivering electrons along the axis, and this serves to decrease the total transverse energy per unit length in the central part of the stream.

This kind of process can go on indefinitely provided only that conditions for magnetic self-focussing obtain over any segment of the stream long compared with its diameter. This kind of focussing results in concentrating the positive ion current at the cathode until the current density of the positive ions is a far higher order of magnitude than the value which is known to eliminate emitting areas instead of producing them. At these higher positive ion current densities, the positives would certainly be expected to dig out the small craters which it is well known are produced during breakdown. In this way, we have an explanation of why the order of magnitude of current given by expression (16) distinguishes between breakdown and no breakdown.

It seems probable that the entire process just described takes place in a very small interval of time (order of 10^{-4} or 10^{-5} seconds) after which the streams are no longer self-focussing, because the transverse energies of the electrons arising at the ragged edges of the crater are too great, or that the density of matter near the cathode rises to so high a value that collisions can no longer be neglected with respect to the dynamics of the stream. The point of the discussion has been to show that structural defects in the cathode surface are not necessary for breakdown, but that rather, we must look to stream conditions to explain the many characteristic features of breakdown.

Any explanation of breakdown in cold emission to agree with experimental fact, should explain

[9] Bennett, Phys. Rev. **40**, 416 (1932).

the following general observations: (1) breakdown is much more abrupt when the electrodes are unoutgassed than when they are outgassed,[10] i.e., the rise of current to the maximum value permitted by the resistance in series with the source of potential is from an initially much lower value in the former than in the latter case; (2) the fineness of the polish on the surface of the cathode does not affect the value of the field intensity at which breakdown occurs[11] but only affects the fields at which the first small currents pass; (3) with unoutgassed electrodes, after breakdown has occurred, if a field intensity a little less than the field at which breakdown occurred is applied to the cathode, large sudden surges of current pass through the tube, similar in every observable respect to the surge of current which accompanied the original breakdown, but the frequency of occurrence of these surges decreases with time; (4) breakdown can be prevented, or at least its severity greatly diminished either by outgassing the electrodes (both are necessary) or by increasing the resistance in series with the source of potential[12] to a value which will limit the maximum value to which the current can rise, to the order of 10^{-3} or 10^{-4} or less; (5) before breakdown, the current is very erratic, rising abruptly, and dropping sometimes abruptly, and sometimes gradually; these erratic fluctuations while the current is small occur at values of field much higher than the values of the field for the same order of magnitude of current after breakdown; but once the value of the current has increased to more than the order of 10^{-2} amperes, the resistance of the tube decreases and the surface of the cathode becomes permanently altered to give much higher order currents at a given field than before; (6) when breakdown occurs, small flashes of light are observed both at the anode and at the cathode.

[10] Millikan and Shackelford, Phys. Rev. **15**, 239 (1920); Eyring, Mackeown and Millikan, Phys. Rev. **31**, 900 (1928); Bennett, Phys. Rev. **37**, 582 (1931).

[11] Only superficial significance was given this fact on p. 584 of the paper in Phys. Rev. **37**, 582 (1931).

[12] Unpublished work. See R. W. Mebs, Phys. Rev. **43**, 1058 (1933).

The explanation which writers have usually used in the past, *viz.*, that a particle of impurity or a piece of the metal of the cathode is torn bodily out of the cathode by the field, fails to explain the above observations, but their explanation in terms of the idea of magnetically self-focussing streams follows easily from the discussion of streams already given.

The theory of streams also explains the fact that even after electrodes have been fatigued or "conditioned" and are giving steady field-current characteristics, if the residual gas pressure is allowed to rise to the order of magnitude corresponding to expression (13) the current becomes unsteady, and the surges of current which pass can be prevented from entirely disfiguring the cathode only by increasing the resistance in series with the source of potential.

The fact that the field intensity for breakdown in mercury obtained recently by Beams[13] agrees so well with the results of the writer for other metals with a similar arrangement of electrodes shows that surface impurities can at most only serve to give the first small electron currents, but that an entirely different explanation is necessary in order to explain breakdown.

The theory of streams explains the fact that the experience with tubes to which extremely high potentials are applied[14] has been that destructive cold emission occurs at fields of the order of 100,000 volts per centimeter, but that the application of fields of the order of 1,000,000 volts per centimeter to surfaces prepared in the same way, but held at much shorter distances from the anode were found by the writer to be necessary to produce measurable emission. In the case of high voltage tubes, the longer distances and the higher voltages both aid in the formation of self-focussing streams.

In conclusion, the writer wishes to thank Professor L. H. Thomas of this laboratory, for the very generous assistance which he has given in discussions of mathematical methods.

[13] J. W. Beams, Phys. Rev. **44**, 803 (1933); Bennett, Phys. Rev. **37**, 582 (1931).

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