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The Analysis of the Cosmic-Ray Absorption Curve

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A method of solving the integral equation of absorption

$$f(s)=\int_0^\infty \phi(\mu)\exp(-\mu s)d\mu$$

is developed which gives the function ϕ as a series of Laguerre orthogonal functions, whose coefficients are determined from the power series expansion of a function $F(x)$. This in turn is simply related to the given function $f(s)$; in case the latter is given numerically, the expansion may be accomplished by the method of least squares. The function ϕ (called the absorption coefficient spectrum of the radiation) is determined for cosmic rays at several altitudes and latitudes. In each case there are two maxima of intensity, one at $\mu=0.06$ and one at $\mu=0.6$ (meters water)$^{-1}$. The minimum of intensity occurs at $\mu=0.25$, or $1/\mu=4$ m water $=30$ cm mercury. This is also the range of the rays responsible for the inflection points of the Compton-Stephenson high altitude curve, but no consistent explanation of the correlation is found. The spectrum also shows a range of negative intensity, indicating the presence of secondary radiations. In no case is there any evidence for a line structure of the spectrum. No basis is found for restricting the assumption of exponential absorption to apply only to equatorial data; neither is any evidence discovered which makes the assumption a necessary one at any latitude. Only data on the absorption in air and water are considered; several points are indicated at which the existing data require extension.

I. INTRODUCTION

THE cosmic-ray absorption curve has been analyzed by Millikan and his co-workers,[1] by Kulenkampff,[2] Lenz[3] and others. These interpretations have been based on various assumptions, some of which are common to all, while others are specific with one writer. The most common assumption is that a unidirectional beam of homogeneous rays is absorbed according to an exponential law, which corresponds roughly to supposing that the rays are electromagnetic in nature. This second form of the assumption cannot be maintained unqualifiedly, since it is known, both from the existence of a latitude effect and from experiments with expansion chambers, that there are charged particles among the rays. However, the assumption of exponential absorption has had a certain measure of success and the results of different workers show a consistency which is remarkable in view of the serious objections advanced by L. D. Weld against the method of calculation.[4] This method consists in introducing the second assumption that the rays have a line spectrum and then guessing both the absorption coefficient and intensity of each line; from this guess a synthetic curve can be constructed and compared with the observed absorption curve. When a sufficiently close fit is obtained, the guess is supposed to be

[1] R. A. Millikan and G. H. Cameron, Phys. Rev. **37**, 240 (1931). I. Bowen, R. A. Millikan and V. H. Neher, Phys. Rev. **44**, 246 (1933).
[2] V. H. Kulenkampff, Phys. Zeits. **30**, 561 (1929).
[3] E. Lenz. Zeits. f. Physik **83**, 194 (1933).
[4] L. D. Weld, Phys. Rev. **40**, 713 (1932).

correct. This is, perhaps, not an entirely fair description of the procedure, since there are certain features of the observed curve which can be used as aids in determining the trial values of the coefficients,[3] but in its essentials it is correct. Weld devised a procedure for correcting the first trial values by the method of least squares[5] and found the corrections to be inadmissably large, from which he concluded that the results were meaningless.

Since it is probable that few writers have considered the assumption of a line spectrum as other than a simplification, it becomes desirable to eliminate it. This is one purpose of the present paper; a second is to apply the assumption of exponential absorption in a uniform manner to data of various types, which has not hitherto been possible because of the method of calculation.

II. Theory of Absorption Curves

Consider first a beam of unidirectional but inhomogeneous radiation, so that the intensity of that portion of the radiation whose absorption coefficient lies between μ and $\mu+d\mu$ is $\phi(\mu)d\mu$. The function $\phi(\mu)$ may be called the absorption coefficient spectrum of the radiation, or simply its spectrum if no confusion can result from the omission of the modifier. After passing through a thickness s of matter, the total intensity will be

$$f(s)=\int_0^\infty \phi(\mu)\exp(-\mu s)d\mu. \tag{1}$$

If, in addition to being inhomogeneous, the radiation is incident upon the surface $s=0$ from all directions with equal intensity, the total intensity at s will be ($\sigma=s/\cos\theta$)

$$I(s)=\int_0^{\pi/2} f(s/\cos\theta)\sin\theta d\theta = s\int_s^\infty [f(\sigma)/\sigma^2]d\sigma. \tag{2}$$

As Gross[6] has shown, this relation may be solved for f:

$$f=I-s(dI/ds)=I\{1-[d(\log I)/d(\log s)]\}. \tag{3}$$

The first form of Eq. (3) yields a simple graphical method of constructing the f-curve from the I-curve: at abscissa s draw the tangent to the I-curve, continuing it until it intersects the I-axis. The projection of this point on the ordinate at s will be a point on the f-curve. This graphical method becomes very inaccurate for large values of s, and the logarithmic form is preferrable for numerical calculation.

The function $I(s)$ may also be written

$$I(s)=\int_0^\infty \theta(\mu)\exp(-\mu s)d\mu; \tag{4}$$

substitution of Eq. (1) in the second form of Eq. (2) yields

$$\theta(\mu)=(1/\mu^2)\int_0^\mu \mu\phi(\mu)d\mu, \tag{5}$$

or

$$\phi=2\theta+\mu(d\theta/d\mu). \tag{6}$$

The problem presented by the cosmic-ray data is the determination of $\phi(\mu)$ from a knowledge of $I(s)$; two procedures are available: (1) f may be calculated from Eq. (3) and Eq. (1) then solved for ϕ; or (2) Eq. (4) may be solved for θ, and ϕ calculated from Eq. (6). Both methods involve the solution of an integral equation of the same form (Eqs. (1) and (4)); for reasons which will appear, the first procedure will be adopted, and $f(s)$ treated as a known function.

In Section VIII it will be important to be free of the assumption of uniform incidence from all directions; it will be noted that this assumption does not enter into Eq. (4), which is valid for any directional distribution whatever. It does enter, through Eq. (2), into Eqs. (5) and (6).

III. Solution of Eq. (1)

Eq. (1) arises in numerous physical problems, e.g., the determination of the "luminosity-depth" function of a thick x-ray target from the observed angular distribution of its radiation. If s is a complex number, the equation may also be made the basis of the theory of electric circuits.[7] In the particular cases that s is a pure imaginary or that f and ϕ are analytic functions, it may be solved by Fourier's theorem. Unfortunately, neither of these special cases is realized in the cosmic-ray problem. Carson[7] has based a treatment of

[5] An alternative method to Weld's is K. Pearson's *Method of False Position*, Phil. Mag. **5**, 658 (1903).

[6] B. Gross, Zeits. f. Physik **83**, 214 (1933).

[7] J. R. Carson, *Electric Circuit Theory and the Operational Calculus*, McGraw-Hill (1926). Eq. (29), p. 21 is identical with Eq. (1) above, except for notation.

TABLE I. *Values of the Laguerre functions* $u_n(\alpha)$.

α	u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
.1	0.95123	0.85611	0.76574	0.67997	0.59864	0.52161	0.44872	0.37984
.2	.90484	.72387	.56100	.41502	.28478	.16920	.06725	−.02207
.3	.86071	.60250	.38302	.19839	.04505	−.08033	−.18080	−.25916
.4	.81873	.49124	.22924	.02402	−.13231	−.24678	−.32572	−.37478
.5	.77880	.38940	.09735	−.11357	−.25757	−.34701	−.39263	−.40368
.6	.74082	.29633	−.01482	−.21928	−.33974	−.39534	−.40213	−.37349
.7	.70469	.21141	−.10923	−.29750	−.38664	−.40382	−.37103	−.30581
.8	.67032	.13406	−.18769	−.35214	−.40505	−.38257	−.31283	−.21730
.9	.63763	.06376	−.25186	−.38672	−.40085	−.34000	−.23830	−.12048
1.0	.60653	.00000	−.30327	−.40435	−.37908	−.28305	−.15584	−.02455
1.1	.57695	−.05770	−.34329	−.40781	−.34405	−.21736	−.07193	.06403
1.2	.54881	−.10976	−.37319	−.39953	−.29943	−.14749	.00863	.14097
1.3	.52205	−.15661	−.39414	−.38170	−.24832	−.07705	.08237	.20372
1.4	.49659	−.19863	−.40720	−.35622	−.19330	−.00885	.14693	.25107
1.5	.47237	−.23618	−.41332	−.32475	−.13654	.05499	.20085	.28283
1.7	.42741	−.29919	−.40818	−.24954	−.02450	.16386	.27440	.30251
2.0	.36788	−.36788	−.36788	−.12263	.12263	.26978	.30248	.24409
2.2	.33287	−.39945	−.32621	−.03817	.19886	.30098	.27572	.16742
2.5	.28650	−.42976	−.25069	.07760	.27531	.29583	.18967	.03093
2.7	.25924	−.44071	−.19573	.14375	.30132	.26467	.11502	−.05761
3.0	.22313	−.44626	−.11157	.22313	.30680	.18966	−.00279	−.16655
3.5	.17377	−.43443	.02172	.30048	.24663	.03091	−.16689	−.25299
4.0	.13534	−.40601	.13534	.31578	.13534	−.11729	−.24962	−.22040
4.5	.10540	−.36890	.22397	.28326	.00906	−.21846	−.24421	−.10929
5.0	.08209	−.32834	.28730	.21889	−.10603	−.25994	−.17158	.02671
5.5	.06393	−.28768	.32763	.13718	−.19428	−.24574	−.06336	.14275
6.0	.04979	−.24894	.34851	.04979	−.24894	−.18919	.04979	.21195
7.0	.03020	−.18118	.34727	−.11072	−.26045	−.01560	.20664	.19050
8.0	.01832	−.12821	.31137	−.22589	−.17705	.14530	.22019	.03274
9.0	.01111	−.08887	.26106	−.28883	−.05138	.23107	.11984	−.12958
10.0	.00674	−.06064	.20888	−.30770	.07412	.23134	−.02321	−.20823
11.0	.00409	−.04087	.16143	−.29561	.17454	.16667	−.14545	−.18442
12.0	.00248	−.02727	.12146	−.26523	.24044	.06792	−.21169	−.08846
13.0	.00150	−.01804	.08945	−.22652	.27269	−.03693	−.21493	.03166
14.0	.00091	−.01185	.06474	−.18633	.27752	−.12845	−.16704	.13397
15.0	.00055	−.00774	.04618	−.14878	.26292	−.19648	−.08811	.19359
17.0	.00020	−.00326	.02269	−.08858	.20443	−.25622	.17127	.12175
20.0	.00005	−.00086	.00731	−.03597	.11143	−.21636	.23169	−.04623
25.0	.00000	−.00009	.00098	−.00649	.02845	−.08586	.17664	−.22921
30.0	.00000	−.00001	.00012	−.00099	.00561	−.02276	.06740	−.14417

Heaviside's operational methods on the equation, practically without restriction on the complex nature of s, so that Heaviside's solutions are available. However, they are at best convergent power series and at worst asymptotic series; in either case they are valuable only for restricted ranges of μ and s. The following solution is valid for wider ranges of the variables, and has not previously appeared in the literature, to the writer's knowledge.

It is convenient to introduce the dimensionless variables

$$\xi = s/s_0, \qquad \alpha = 2s_0\mu, \tag{7}$$

where s_0 is any convenient unit of length. Eq. (1) then becomes

$$f(\xi) = (1/2)\int_0^\infty \Phi(\alpha)\exp(-\alpha\xi/2)d\alpha, \tag{8}$$

where

$$\Phi(\alpha) = \phi(\mu)/s_0. \tag{9}$$

This function may be expanded as a series of Laguerre orthogonal functions:[8]

$$\Phi(\alpha) = \Sigma a_n u_n(\alpha), \qquad u_n(\alpha) = (1/n!)\exp(-\alpha/2)L_n(\alpha), \tag{10}$$

where L_n is the nth Laguerre polynomial. On substituting Eq. (10) into Eq. (8), the latter becomes

$$f(\xi) = \Sigma a_n(\xi-1)^n(\xi+1)^{-n-1}, \tag{11}$$

since it is readily shown that (cf. Sommerfeld, reference 8, page 94)

$$(1/2n!)\int_0^\infty L_n(\alpha)\exp[-\alpha(\xi+1)/2]d\alpha = (\xi-1)^n(\xi+1)^{-n-1}. \tag{12}$$

[8] Courant-Hilbert, *Methoden der Mathematischen Physik*, 1st Ed., p. 79; A. Sommerfeld, *Ergänzungsband*, p. 75.

On introducing the abbreviations

$$x=(\xi-1)/(\xi+1), \quad F(x)=(\xi+1)f(\xi), \quad (13)$$

Eq. (11) becomes

$$F(x)=\Sigma a_n x^n. \quad (14)$$

Thus, in order to obtain the coefficients of Eq. (10), it is merely necessary to calculate $F(x)$ from $f(s)$ and then approximate it by a polynomial. This may be done by the method of least squares, or any other available procedure. To plot the function $\Phi(\alpha)$, the numerical values of the Laguerre functions are required; these are given in Table I[9] for $n=0\cdots 7$.

IV. The "Electrometer Zero" and the "Low Absorption Limit"

The data obtained from the ionization chamber measurements of Millikan and Cameron[1] and of Regener[10] are the values of $I(s)$+const., the constant being the "electrometer zero." This is not determined directly, but is obtained by extrapolation on the assumption that $I(s)\rightarrow a \exp(-\mu_0 s)$ for large values of s, or alternatively, that $I(s)\rightarrow aG(\mu_0 s)$ (G being the Gold function). Both of these procedures are based on the idea that there is a "low absorption limit" to the cosmic-ray spectrum analogous to the short wave limit of an x-ray spectrum. This idea is an obvious one if the cosmic rays are electromagnetic radiation, but its verification would be remarkable if this assumption is not correct. It is important to clarify the issues on this point before proceeding further, since they are also related to Weld's results.

If the idea of a low absorption limit is to be adopted, it leads naturally to replacing Eq. (4) by

$$I(s)=\int_{\mu_0}^{\infty}\theta(\mu)\exp(-\mu s)d\mu, \quad (4.1)$$

μ_0 being the smallest value of μ for which $\theta(\mu)$ is not identically zero. If this integral be subjected to repeated partial integration, it results in

$$I(s)=\exp(-\mu_0 s)\{\theta(\mu_0)/s + \theta'(\mu_0)/s^2+O(1/s^3)\}, \quad (4.2)$$

θ' being the derivative of θ. If it be assumed that θ approaches zero in a continuous manner as μ approaches μ_0 from above, $\theta(\mu_0)$ vanishes and $I\rightarrow\theta'(\mu_0)[\exp(-\mu_0 s)]/s^2$. An examination of Regener's data from this point of view leads only to the conclusion that $\theta'(\mu_0)$ is very large, but does not yield any value for μ_0. The data would have to be extended to enormously greater depths before they could be used to determine the low absorption limit on this basis. Alternatively, one may suppose that $\theta(\mu_0)$ is not equal to zero which is equivalent to supposing that $I\rightarrow aG(\mu_0 s)$; on this basis it is possible to obtain values of all three quantities, μ_0, $\theta(\mu_0)$ and the electrometer zero, but μ_0 is only slightly greater than its estimated probable error—the data would even tolerate a small negative value. This is one of Weld's results, in essence.

Under these circumstances, the only reasonable tentative assumption to make is that μ_0 and $\theta(\mu_0)$ are both zero, which will be done in the following. This point can be satisfactorily cleared up only by a direct determination of the electrometer zero, which will increase the accuracy of the μ_0 calculation. This assumption leads to a value of 0.73 v/hr for the zero of Regener's instrument, as compared to the value 0.78 diffidently adopted by him. If the zero of the Millikan-Cameron instrument be determined so that their data agree with Regener's, a value of 0.95 ion/cm³/sec. is obtained, as compared to the value 1.2±0.2 adopted by them.

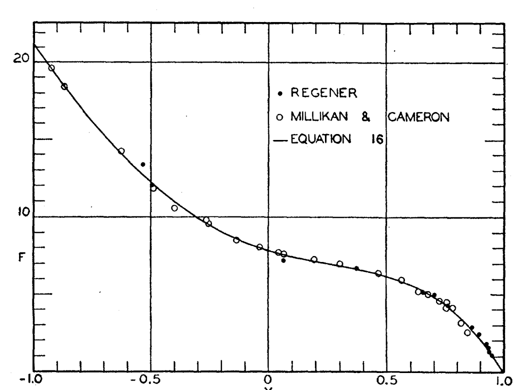

Fig. 1. Graph of $F(x)$, numerical data as in Eq. (15).

[9] Table I was prepared by Mrs. G. S. Monk, to whom the author is also indebted for carrying out other parts of the calculations, particularly those involving least squares methods.

[10] E. Regener, Phys. Zeits. **34**, 308 (1933); Zeits. f. Physik **74**, 433 (1932).

V. The Analysis of the Combined Data of Regener and Millikan and Cameron[10a]

When combined as just described, the data of Regener and Millikan and Cameron furnish a set of values of $I(s)$ in the range $8.25<s<240$ equivalent meters of water. From these a large graph of log I *vs.* log s was constructed; its slope was calculated numerically and the value of $f(s)$ obtained from Eq. (3). It was found preferable to use the observed rather than the interpolated values of I in Eq. (3), since the latter were subject to systematic errors of the same order as the random errors of the former; the systematic errors of the derivative could not be avoided.

If the method of solution outlined in Section II were applied to these data, the resulting function $\phi(\mu)$ would be very uncertain because of the lack of data over the range $0<s<8.25$. Fortunately, the simple device of replacing s by $s-8$ and ϕ by ϕ exp (-8μ) leaves the equation for $f(s)$ unaltered and eliminates the necessity of extrapolation. This is Heaviside's operation of *shifting;* the new function obviously represents the spectrum of the radiation at the new origin of s. The second of the two methods outlined at the end of Section II would not lend itself as readily to this modification.

It is also necessary to choose the value of s_0 (Eq. (7)). This quantity is found to determine the degree of the polynomial needed to approximate $F(x)$, but the dependence is not critical. In this case, it was found satisfactory to take $s_0=6$; the value 4 or 8 would have increased the degree considerably, but 5 or 7 would presumably have been about as satisfactory as 6. If h is the equivalent depth below the top of the atmosphere, the foregoing leads to the numerical relations

$$s=h-8,\quad \xi=s/6,\quad \alpha=12\mu,\quad x=(h-14)/(h-2). \qquad (15)$$

[10a] The new data of Kramer, Zeits. f. Physik **85**, 411 (1933) were unfortunately inaccessible at the time these calculations were made. They differ by more than the experimental error from those of Millikan and Cameron. If they had been used in the present section, some quantitative differences would have resulted, and the difficulties to be mentioned in the next section would have been accentuated. The data of Corlin, Nature **133**, 63 (1934) seem to bear out the tentative conclusion of Section IV above.

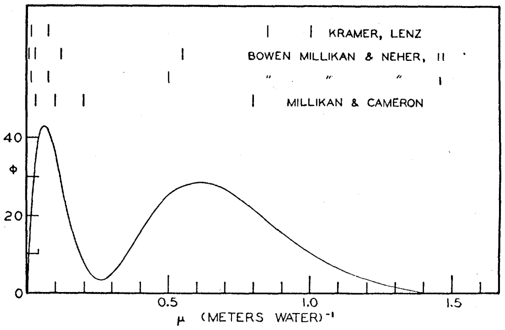

Fig. 2. Absorption coefficient spectrum of the cosmic rays at barometric pressure 8 m water = 59 cm mercury.

In Fig. 1, the points represent the values of $F(x)$ calculated as described and the curve is the graph of the polynomial

$$F(x)=7.90-4.40x+6.17x^2-6.20x^3-3.47x^4 \qquad (16)$$

(units: ions/cm³ at 1 atmos.). The coefficients of this polynomial were obtained by drawing a smooth curve through the points and interpolating the values of F for $x=0,\ \pm\frac{1}{2},\ \pm 1$. The Lagrange interpolation formula[11] was then used to find the polynomial of fourth degree defined by these five points.

From the coefficients of the polynomial, the function $\phi(\mu)$ can be calculated immediately from Eqs. (9) and (10); its graph is shown in Fig. 2. This is seen to have two well-defined but broad maxima at $\mu=0.06$ and 0.62 (meters water)$^{-1}$. For comparison, the line spectra assumed by various authors are indicated at the top of the figure.[12] To assist the reader in evaluating this comparison it should be remarked (1) that the present method has sufficient resolving power to have separated peaks at $\mu=0.5$ and 0.8 had the data required it; (2) that the numbers of disposable constants used in the present calculation and by Bowen, Millikan and Neher in their Table I synthesis are equal; (3) that the last-mentioned synthesis represents the data quite as accurately as the curve of Fig. 1, but no more so. The author believes that the comparison is unfavorable to the idea that the spectrum consists of lines.

[11] Courant-Hilbert, reference 8, 1st Ed., p. 83.

[12] In the case of Lenz's analysis only the absorption coefficients of the primary radiation are indicated. The data are taken from references 1 and 3.

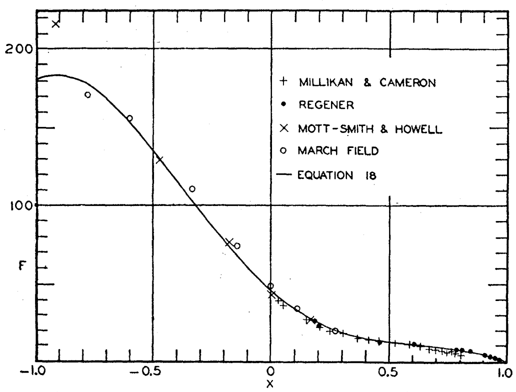

FIG. 3. Graph of $F(x)$, numerical data as in Eq. (17).

VI. The Spectrum at Higher Altitude

The spectrum of the cosmic rays at a point 4 equiv. meters below the top of the atmosphere may be obtained by combining the ionization data obtained by Mott-Smith and Howell[13] and Bowen, Millikan and Neher[1] with the underwater data discussed in the last section. The validity of this combination depends on assumptions regarding phenomena occurring at the air-water interface which have not yet been tested in sufficient detail, but which may be tentatively accepted. In this way values of $I(s)$ are obtained for the range $4<h<240$, but the data for the smaller values of h are by no means as reliable as those for larger. Choosing $s=h-4$, and $s_0=4$, so that

$$\xi=(h-4)/4, \quad \alpha=8\mu, \quad x=(h-8)/h, \quad (17)$$

it was found that

$$F(x)=47.4-131.4x+128.5x^2 +25.4x^3-85.8x^4+15.9x^5. \quad (18)$$

The coefficients of this polynomial were obtained in a slightly different way from those of Eq. (16): by graphical interpolation values of F were obtained for $x=0, \pm 0.2, \pm 0.4, \cdots \pm 1.0$. These values were then weighted proportionally to $1/(1-x)$ to take account of the preponderant amount of the data for large values of s, and the polynomial determined by least squares. No doubt it would have been better statistical practice to have omitted the interpolation and treated the original data; a glance at Fig. 3 also suggests that a polynomial of the sixth degree might have fitted the data more closely. It is not clear, however, that these technical improvements would have increased the significance of the results (see Section VII).

The solid curve of Fig. 4 represents ϕ as determined from the coefficients of Eq. (18). The two peaks have scarcely shifted their positions, but their relative and absolute heights have altered because of the smaller amount of filtering at the higher altitude. The broken curve represents ϕ as calculated from Fig. 2 by multiplying its ordinates by $\exp(+4\mu)$; the two curves should be identical, but actually the agreement is only qualitative. It is possible that the difference is due entirely to inaccuracies of the data and the calculations, but if real it must be ascribed to phenomena occurring at the air-water interface.

VII. The Influence of Secondary Radiations

The negative values of ϕ which are very pronounced in Fig. 4 require discussion. They may be considered as due to secondary radiations, which were not considered in the developments of Section II. There, ϕ was supposed to be the intensity of the primary radiation, and therefore essentially positive. If the influence of secondaries be taken into account, this restriction is removed, but the interpretation of ϕ becomes less simple.

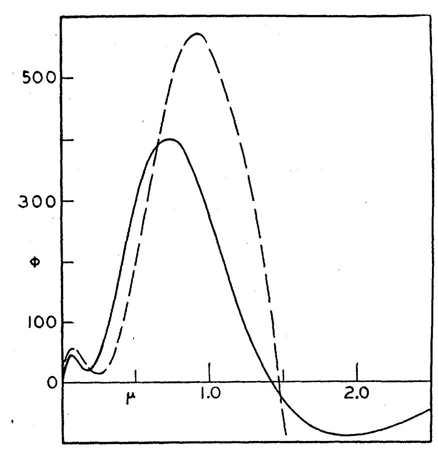

FIG. 4. Absorption coefficient spectrum of the cosmic rays at barometric pressure 4 m water = 30 cm mercury.

[13] L. Mott-Smith and L. G. Howell, Phys. Rev. **42**, 314 (1932).

The problem of the secondary radiations accompanying the primary cosmic rays is a complex one and no complete discussion of it is possible at this time. The following sketch omits many essentials, but will serve to bring out a point which has not received sufficient consideration. Let $\phi_1(\mu)$ be a function which can be given the interpretation which has previously been assigned to $\phi(\mu)$, and suppose that primary radiation of absorption coefficient μ produces secondary radiation of absorption coefficient $\mu_2>\mu$. For simplicity, tertiary, etc., radiations will be neglected. Suppose also that the secondaries travel in the same direction as the primaries; then it may be shown that[14] the ionization produced by the complex beam is given by

$$f(s)=\int\phi_1(\mu)\{a(\mu)\exp(-\mu s) - b(\mu)\exp(-\mu_2 s)\}d\mu, \quad (19)$$

where a and b are essentially positive functions. To perform the integration, μ_2 must be known as a function of μ, or preferably μ as a function of μ_2, say $\mu=g(\mu_2)$. Then the change of variable from μ to μ_2 in the second term of the integral reduces Eq. (19) to the form of Eq. (1) with

$$\phi(\mu)=a(\mu)\phi_1(\mu)-b[g(\mu)]\phi_1[g(\mu)]g'(\mu). \quad (20)$$

This new function ϕ is not restricted to positive values, and the negative ordinates of Fig. 4 are explicable.

However, it is the function ϕ_1 which is of interest for the problem of the origin and nature of the cosmic rays, and having found ϕ, the problem of determining ϕ_1 is encountered. It is by no means certain that this problem possesses a unique solution, even granting that the functions a, b and g, were known from theory, which they are not. Furthermore, Eq. (20) was derived on highly simplified assumptions; the true equation will be much more complex. It seems that no reliable conclusions concerning this point can be reached by analytical means: the problem is essentially an experimental one. Methods must be devised for measuring the primary rays alone, without admixture of secondary effects.[15] Unless the experimental data are such that the simple interpretation of Section II is valid, the function ϕ obtained by the procedure outlined there has little more significance than the original unreduced data $I(s)$.

VIII. The Latitude Effect

R. A. Millikan[16] has recently advanced an hypothesis to account for the latitude effect, according to which the cosmic rays at accessible altitudes in equatorial regions are entirely composed of photons (or neutrons) while the rays reaching the earth at higher latitudes are contaminated with electrically charged particles. Bowen, Millikan and Neher have interpreted this hypothesis to mean that only absorption data obtained in equatorial regions may be analyzed on the assumption of exponential absorption. It is therefore necessary to analyze data from the two regions in a uniform manner in order to determine whether there is an experimental basis for this distinction.

In view of the asymmetry of the directional effect in equatorial regions[17] it is not justifiable to assume that the rays are incident from all directions with uniform intensity. (This assumption is implicit in the use of Gold functions.) This point alone makes the hypothesis difficult to defend, for an explanation of the asymmetry of the directional effect apparently requires magnetic deviability of the rays. Supposing that this objection can be answered, there still remains the question, is there any experimental basis for applying the assumption of exponential absorption only to equatorial data? Such a basis is not apparent from the $I(s)$ curves;[18] the ϕ-curves cannot be given the same meaning for the two regions; there remains only the possibility that the θ-curves (cf. Eq. (4)) which do not depend on the assumption of all-sided incidence, will reveal the difference to be expected on this hypothesis.

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[14] See Kulenkampff, reference 2, and T. H. Johnson, Phys. Rev. **41**, 545 (1932). This point has also been emphasized by Millikan.

[15] For a more detailed discussion of this problem, see A. H. Compton and R. J. Stephenson, Phys. Rev. **45**, 441 (1934).

[16] R. A. Millikan, Phys. Rev. **43**, 666 (1933). Also Bowen, Millikan and Neher, reference 1.

[17] B. Rossi, Phys. Rev. **45**, 212 (1934). T. H. Johnson, Phys. Rev. **43**, 834 (1933); **44**, 856 (1933). L. Alvarez, Phys. Rev. **43**, 835 (1933).

[18] A. H. Compton and R. J. Stephenson, reference 15.

To obtain material for such a comparison, the data given in Table I, "observed" column, of the paper by Bowen, Millikan and Neher[1] were taken as representing the equatorial data. (The electrometer zero was altered in such a way as to increase each value by 0.0081 unit.) The ratio of the ionization at March Field to that in Panama and Peru was next determined from Fig. 3 of the same paper; thus two $I(s)$ curves were available, one for equatorial and one for northern latitudes. The calculations for the coefficients of θ (shifted to 4 m water below the top of the atmosphere) were then carried out uniformly (and simultaneously) with those already described in Section VI.

The resulting functions θ and their difference are plotted in Fig. 5. It is believed from various

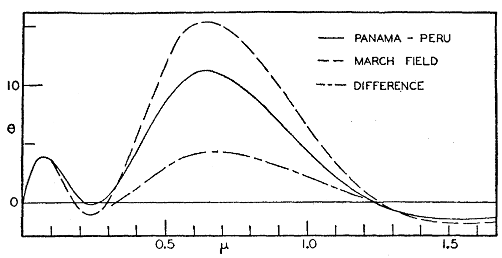

FIG. 5. Comparison of $\theta(\mu)$ at two different latitudes. From data of Bowen, Millikan and Neher, reference 1.

indications interpreted in the light of previous experience that both θ-curves are too low in the range $\mu<0.3$, and both too high in the region $0.3<\mu<1.0$. Their relative height in the latter range should not be much affected by these errors. The only experimental basis for maintaining that the Panama-Peru curve is significant, while the March Field curve is meaningless, would be a complete lack of correlation between the difference curve and that for Panama-Peru. In fact a complete and obvious correlation exists.[19] Hence, either both curves are meaningless, or both significant, so far as these data can be considered as reliable.

[19] It seems probable that an entirely similar result would have been obtained if ϕ-curves had been constructed. This is perhaps not surprising since it is also probable that the directional effect will show an asymmetry at any latitude provided the measurements are made at a sufficiently high altitude. The assumption of uniform incidence from all directions would then not be justified anywhere.

IX. Discussion

In evaluating the results obtained, it is important to understand that the successful calculation of ϕ has no theoretical significance whatever. Within limits, almost any function $f(s)$ can be represented by an integral of the type of Eq. (1), and these limits are greatly extended by the presence of secondary effects in the measurements, by experimental error, and by the impossibility of making measurements at infinite depths. Hence the justification of the assumption of exponential absorption cannot be found in the success of these calculations. Even if the cosmic rays consisted entirely of r-particles (i.e., particles having a definite range and ionizing their paths uniformly)[15] a ϕ-curve could still be deduced from the I-curve, provided only that the latter exhibited no discontinuities of slope. This condition would imply a continuous distribution of ranges. In this case, however, it would not be possible to make a unique assignment of a given value of μ to a single type of ray.[20]

The results obtained on the latitude effect cannot easily be understood on the assumption of exponential absorption. They seem to require two kinds of rays for a given value of μ, one of which is magnetically deflectable and the other not. Since it is quite easy to see that r-particles of two different ranges may contribute to the value of ϕ for a single value of μ, the artificiality of this assumption disappears when the rays are assumed to be r-particles. Neither theory affords a satisfactory explanation of the approximate proportionality of the difference between the two curves of Fig. 5 to the ordinates of either; however, this quantitative result is not to be given much weight, nor is it to be expected that either theory will be valid in the simple form discussed here.

It would seem that on any theory of the nature of the rays, the existence of the two maxima in Fig. 2 could be taken as evidence of two more or less distinct components. It should be noted, however, that there is considerable latitude in the

[20] If the number of r-particles having a range between r and $r+dr$ is given by $\mu \exp(-\mu r)dr$, then it is easily shown that $f(s)=\exp(-\mu s)$. On the r-particle basis, the ϕ-curve therefore represents the analysis of the rays into elementary distributions of this form. It is obvious that rays of every range contribute to the value of ϕ for a given value of μ.

possible descriptions of the two components. On the basis of exponential absorption, two extreme possibilities present themselves according as it is assumed that Fig. 2 is or is not appreciably affected by the presence of secondary rays not yet in equilibrium with their primaries. In the former case, the curve represents (qualitatively at least) the spectrum of the primary rays, and the two components are presumably of distinct extra-terrestrial origin. In the latter case, the primary rays may have a spectrum of only one maximum, and the minimum may be due entirely to the negative contribution of secondaries (cf. Section VII) whose origin is terrestrial. The second possibility is extreme and may perhaps be dismissed because the experiments on transition effects do not suggest the existence of secondaries of such penetrating power.

It is none the less interesting to note that these secondaries would have a mean range of about 4 m water = 30 cm mercury, which is about the range of the rays responsible for the two inflection points of the Compton-Stephenson high altitude f-curve. So far as existing data are concerned, this quantitative agreement must be ascribed to chance, for it is not possible to obtain a consistent explanation of the result on either the assumption of exponential absorption or on that of r-particles. On the former basis, the two inflection points of the f-curve would require that the ordinates of Fig. 2 be very definitely negative at $\mu = 0.25$; the data might tolerate a value $\phi = -5$ at this point, but it is doubtful if this would be sufficient. On the other hand, if the inflection points are due to r-particles of 30–40 cm range,[18] these could not possibly affect Fig. 2, which was constructed entirely from data obtained at pressures greater than 60 cm. It is desirable that further data on this point be obtained.

Perhaps the most definite conclusion to be derived from this study may be formulated as a warning against too simple an interpretation of the cosmic-ray absorption curve. It is certainly influenced by many different effects and its analysis will require further carefully planned experimental work.

It is a pleasure to acknowledge many stimulating conversations on these matters with Professor A. H. Compton, and to mention again the assistance of Mrs. G. S. Monk, without which the work would have lagged interminably.