Magnetic Refocussing of Electron Paths*

W. E. STEPHENS, *Washington University, St. Louis*
(Received January 31, 1934)

A general method of magnetic direction refocussing, i.e., the refocussing of slightly divergent electron paths in a uniform magnetic field, has been found, of which the familiar 180° refocussing is a particular case. If we use a wedge-shaped magnetic field, whose lines of force are perpendicular to the plane of motion of an electron beam, the field being produced by a solenoid or magnet pole pieces of such a shape that the effective boundaries of the field projected on the plane of motion of the electrons is V shaped; and if the electron beam enters the magnetic field perpendicular to one edge, with the strength of the field set so that the electron beam will leave perpendicular to the other edge; then the refocussing of slightly divergent electron paths will occur. The position of best refocussing is on a line through the point of divergence of the electron beam and the apex of the wedge field. A magnetic field approximating the desired field has been obtained and the theoretical results have been checked for a particular case in which the angle at the apex of the wedge field was 90°. The particular value of this general refocussing property is that, with it, velocities of electrons can be analyzed without the deflecting magnetic field straying over into the region from which the electrons originate.

INTRODUCTION

IN analyzing the velocity distribution of beams of charged particles by deviation methods, it is usually desirable to increase the intensity of the collected beam by refocussing paths of particles which diverge from a given point by a small, but finite angle. This is called direction refocussing and has been found to occur at 180° deflection in a magnetic field,[1, 2, 3] at 127° 17′ deflection in a radial electrostatic field,[4] and at 360° deflection in a magnetic field superimposed on a parallel electrostatic field.[5] The disadvantage of the regular magnetic refocussing in experiments on electron scattering is the difficulty of keeping the field in the magnetic analyzer from straying over into the region from which the electrons originate.[6] Electrostatic refocussing apparatus has the disadvantages that surface charges, contact potentials, and secondary electrons from the plates may affect the results. Therefore, it was desired to find a magnetic deflection method of analyzing electron velocities in which the deflecting field could be kept from straying over into the scattering space from which the electrons come.

It was soon realized that the 180° magnetic refocussing is a particular case of a general refocussing property of wedge-shaped magnetic fields, in which the electrons are refocussed at a deflection angle equal to the angle of the wedge bounding the field. Furthermore, the entrance slit can be some distance from the field, leaving the scattering space or collision area field free.

GENERAL CASE

The general case is illustrated in Fig. 1. If we have a homogeneous magnetic field, H, perpendicular to the paper, and bounded by planes which are perpendicular to the paper and intersect it in lines OPQ and OWV, which lines make arbitrary angles θ and γ, respectively, with the normal to the line AO, then inside the region $QPOWV$ electron paths will be arcs of circles which are tangent at the edges of the field to the direction of entrance and exit. Outside this wedge-shaped field, it is assumed, there is no field and electron paths are straight lines. A homogeneous electron beam of velocity v, which passes through an entrance slit at A making an angle θ with the base line will enter the field at

* Presented at the Cincinnati Meeting of the American Physical Society, December 1, 1933.

1 J. Danysz, Le Radium **10**, 4 (1913).

2 A. J. Dempster, Phys. Rev. **11**, 316 (1918).

3 C. D. Bock, Rev. Sci. Inst. **4**, 575 (1933).

4 A. L. Hughes, V. Rojansky and J. H. McMillen, Phys. Rev. **34**, 284 (1929).

5 H. Busch, Phys. Zeits. **23**, 438 (1922).

6 Gagge avoided this difficulty by having the field homogeneous throughout the scattering and analyzing regions. A. P. Gagge, Phys. Rev. **44**, 808 (1933).

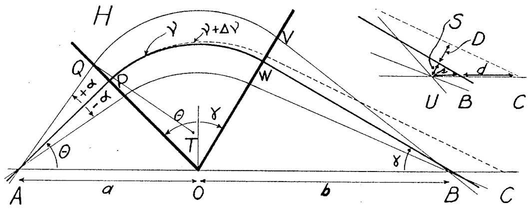

FIG. 1. Refocussing of electron paths. Offset shows region near B enlarged.

P perpendicular to OPQ. If the field H is set to turn the electron beam in an arc of radius of curvature $R=a\sin\theta=OP=OW$, where H is determined by the equation,

$$RH(e/m)=v, \tag{1}$$

then, the center of curvature of the arc PW will be O and the electron beam will leave the field at W perpendicular to the edge OVW and enter the collecting chamber at B. All other beams, such as AQV, of velocity v and passing through A, but making a small but finite angle $\pm\alpha$ with the original beam will be refocussed so that they will cross very close to B. Hence, for a given point of divergence A of an electron beam entering a wedge-shaped field perpendicular to one edge and leaving perpendicular to the other edge, best refocussing will occur at a point B at a distance b from the apex O of the field and on a line through A and O, where $b=a\sin\theta/\sin\gamma$.

The departure from perfect refocussing will be called the spread and will be defined as the distance S in the offset of Fig. 1. This is the separation, at the position of best refocussing B, between the original or normal electron path $APWB$ and paths such as AQV making a small angle $\pm\alpha$ with the normal path at A. UB or s is the "spread along the base line" and when multiplied by $\sin\gamma$ gives S, the spread.

If now a beam of slightly greater velocity $v+\Delta v$ starts out along AP, its radius of curvature in the magnetic field will be larger, it will follow the dotted line APC, and intersect the base line at C. The dispersion of the apparatus is defined as the distance D, on the offset of Fig. 1. This dispersion is the ability of the apparatus to separate beams of slightly different velocities. The dispersion along the base line is d or BC and when multiplied by $\sin\gamma$ gives D. The spread and dispersion are defined as perpendicular to the electron path, since in practice, the plane of the collector slit is usually placed perpendicular to the path. The function of practical importance is, of course, the ratio of D to S which gives a measure of the theoretical resolving power of the apparatus.

To find the spread, S, in terms of α, θ, γ and a, we proceed as follows:

In Fig. 1, let AOB be the X axis and let the Y axis be perpendicular at A. Considering the path $AQVU$ of the beam of velocity v making an angle $+\alpha$ with the normal beam, we proceed by standard methods of analytic geometry to obtain the coordinates (x_1, y_1), (x_2, y_2), (x_3, y_3), and (x_4, y_4) of the points Q, T, V and U, knowing that $QT=R=a\sin\theta$, and QT is perpendicular to AQ.

$$\begin{aligned} x_1&=a\cos\theta\cos(\theta+\alpha)/\cos\alpha,\\ y_1&=a\cos\theta\sin(\theta+\alpha)/\cos\alpha, \end{aligned} \tag{2}$$

$$\begin{aligned} x_2&=a[(1/\cos\alpha)\cos\theta\cos(\theta+\alpha)+\sin\theta\sin(\theta+\alpha)],\\ y_2&=a[(1/\cos\alpha)\cos\theta\sin(\theta+\alpha)-\sin\theta\cos(\theta+\alpha)], \end{aligned} \tag{3}$$

$$\begin{aligned} x_3&=[-n+(n^2-4mc)^{\frac{1}{2}}]/2m,\\ y_3&=(x_3-a)\cot\gamma, \end{aligned} \tag{4}$$

where $m=\csc^2\gamma$

$n=-2(x_2+y_2\cot\gamma+a\cot^2\gamma)$,

$c=(x_2^2+y_2^2+2ay_2\cot\gamma+a^2\cot^2\gamma-a^2\sin^2\theta)$,

$$\begin{aligned} y_4&=0,\\ x_4&=x_3+y_3(y_3-y_2)/(x_3-x_2). \end{aligned} \tag{5}$$

Then

$$s=(a+b)-x_4 \tag{6}$$

or

$$S=a(\sin\gamma+\sin\theta)-\{\sin\gamma[x_3(x_3-x_2)+y_3(y_3-y_2)]/(x_3-x_2)\}. \tag{7}$$

Now by expanding functions of α and approximating by dropping powers of α greater than two, we get

$$S=(a\alpha^2/2)[(\sin^2\theta/\sin\gamma)+(\sin^2\gamma/\sin\theta)]. \tag{8}$$

When $\theta=\gamma$, this reduces further to $S=a\alpha^2\sin\theta$, and for $\theta=\gamma=90°$, the familiar 180° case, we get $S=a\alpha^2$, a result already known.

To find D, the dispersion, we know that the increase in velocity Δv causes an increase in the radius of curvature ΔR such that $\Delta R/R = \Delta v/v$, if the field H is kept constant. If we draw in the new radius as $R+\Delta R$ and determine the new position and the new angle with which the beam leaves the field, we can find its intercept on AOB. We get this distance in terms of a, θ, γ, and ΔR, dropping all powers of ΔR greater than one, and substituting for ΔR, we find,

$$D = a(\sin\theta/\sin\gamma)(\Delta v/v)(\sin\theta + \sin\gamma). \quad (9)$$

When $\theta=\gamma$, this reduces to $D=2a\sin\theta(\Delta v/v)$, and for the 180° case, $D=2a(\Delta v/v)$, which is the known value, where a is then the radius of curvature. To get an idea of the values of θ and γ which will give the best combination of D and S, we plot the ratio of D to S in Fig. 2 in units of $(2/\alpha^2)(\Delta v/v)$, as a function of θ and γ. The greater the ratio of D to S, the better the theoretical resolving power. The graph indicates that the maximum value of D/S occurs for values of θ and γ determined by $2\sin\gamma = \sin\theta$, giving a value of D/S one and a third times that for either the 180° case or the ($\theta=45°$, $\gamma=45°$) case. The value given by the graph for $\gamma=0$, or $\theta=0$, is meaningless, since refocussing does not occur under those conditions. To satisfy the requirements of our original problem, that of keeping the deflecting field from straying past the entrance slit, we choose $\theta=45°$, so that the entrance slit will be distant from the edge of the field by an amount equal to the radius of curvature in the field. Then we choose $\gamma=45°$, so that the edges of the field are 90° apart, so that we may secure the desired field with a square solenoid. For this case then, $S=a\alpha^2/2$, $D=2a(\Delta v/v)$, and $D/S=(2/\alpha^2)(\Delta v/v)$, (from Eqs. (8) and (9)). Now, since $H\propto I$, where I is the current through the solenoid producing the field H, and since $RH\propto v$ (Eq. (1)), then we get $\Delta H/H=\Delta I/I$, and $\Delta R/R=\Delta v/v=-\Delta H/H$. Now, if we let $S=D$, we get,

$$\Delta I/I = -\alpha^2/2, \quad (10)$$

where ΔI is the change in current through the solenoid necessary to compensate for a change α in original direction of the beam. Also, since $V\propto v^2$, we find,

$$\Delta V/V = 2(\Delta v/v) = 2(\Delta I/I), \quad (11)$$

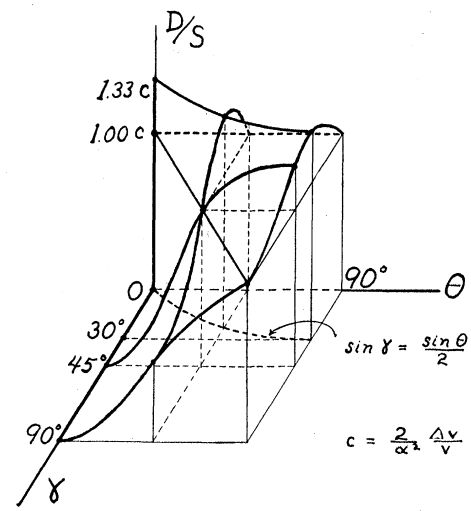

FIG. 2. Theoretical values of D/S (resolving power) as a function of θ and γ.

where ΔI is the change in current necessary to compensate for a change ΔV in accelerating voltage. These Eqs. (9) and (10), can be checked experimentally.

Apparatus and Results

A diagrammatic sketch of the apparatus is shown in Fig. 3a. The cross-hatched square marked H represents the projection on the plane of cross section of the apparatus of the square solenoids which produced the 90° wedge-shaped field. These two solenoids (8×8×10 cm, 10 turns per cm) were placed end to end and separated about two centimeters to permit insertion of the apparatus as shown in Fig. 3b. The intensity of the field produced in this manner as measured along the perpendicular bisector of the edge of the solenoids in the plane of cross section of the apparatus is given in Fig. 5b by the dashed curve. Although this field does not accurately correspond to the theoretical field, since it does not cut off sharply at the edge of the solenoids, nevertheless, it was considered satisfactory for the present experiment since it did give good refocussing.

An oxide-coated filament f, Fig. 3a, supplies an electron current of one milliamp. which is accelerated through a 150 mesh grid g by a potential of 100.0 volts. This produces a fan-shaped beam of electrons which sprays the walls of the cylinder G_0. Those electrons which pass

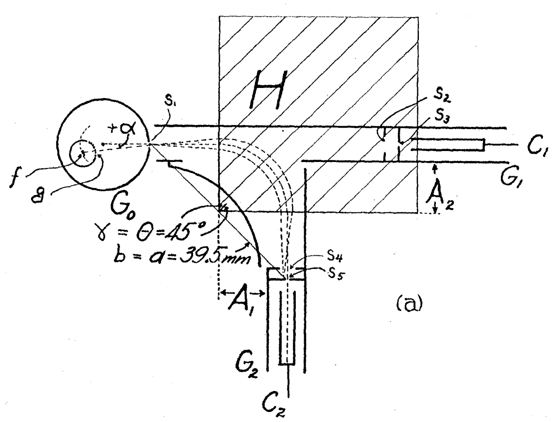

FIG. 3a. Diagrammatic cross section of apparatus for securing refocussing at $\theta = 45°$, $\gamma = 45°$.

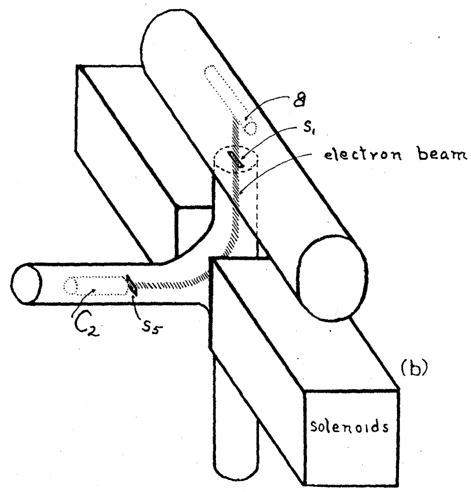

FIG. 3b. Perspective view of apparatus, showing arrangement of the solenoids.

through the slit S_1 (1×5 mm) form a narrow beam with an entrance angle α, which can be varied by rotating a ground glass joint to which the gun is attached. With no magnetic field on, this beam travelled straight down the tube through a guard slit S_2 (3×7 mm), and a collecting slit S_3 (1×5 mm) to the collector C_1 giving a current called C_1. Zero angle was assumed for α for the position which gave a maximum current C_1. When the field H was applied, the electron beam was bent as shown by the dotted lines on Fig. 3. It then passed through a similar set of slits S_4 and S_5 to collector C_2. Its path length from S_1 was 100 mm in both cases. The radius of curvature of the beam was calculated to be 28 mm. The currents C_1 and C_2 were always expressed as C_1/G_0 and C_2/G_0 where G_0 is the gun emission. The metal parts, shield, slits and collectors, indicated by heavy lines in Fig. 3a, were made of copper or brass and those parts subjected to electron bombardment were sooted to reduce secondaries. This was all enclosed in a glass tube and evacuated to a pressure below 10^{-5} mm Hg.

For different angle settings α of the gun, the magnetizing current I necessary to give a maximum current C_2 was determined. I in amperes was plotted against α in degrees in Fig. 4 for different positions of the solenoids relative to the apparatus as indicated by the distances A_1 and A_2 in mm (see Fig. 3a). The dotted curve in Fig. 4 is the theoretical curve $\Delta I/I = -\alpha^2/2$, (Eq. (10)), plotted arbitrarily from $\alpha = 0$, $I = 1.6$ amp. The shape of the curve is important rather than the relative positions. The flatter the curve the better the refocussing. The position of the coils is not critical as it can be seen that changes of a few millimeters in A_1 or A_2 does not affect the refocussing appreciably.

To find the dispersion of the apparatus, I was found for maximum current to C_2 for different accelerating voltages V of the electrons. If I in amperes is plotted against V in volts, the experimentally determined points in Fig. 5a are obtained. The straight line is the theoretical curve, Eq. (11), drawn arbitrarily from the point $V = 100$ and $I = 1.47$.

Discussion

The graphs in Fig. 4 indicate the excellent refocussing obtained. Comparing the shape of the curves for positions of A_1 and A_2 near the calculated position, curves Fig. 4b, c, and d, with the shape of the theoretical dotted curve e, we see that refocussing is better than calculated. This is probably due to the finite size of the collector slit. Furthermore, due to the fact that the solenoids are small and square the field bulges out more at the center of the edge than at the corners, so the effective edge of the field in the plane of the electron beam is not parallel to the edge of the solenoids, but slightly rounded.

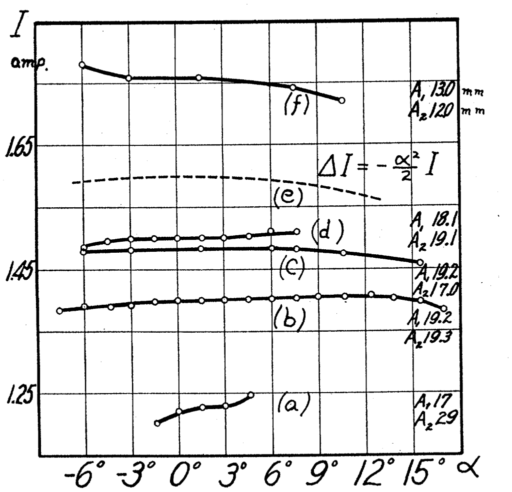

FIG. 4. Experimental test of spread. *Dotted line*, theoretical; *solid lines*, experimental (I, α) curves for different positions of solenoids.

This would have the effect of improving the refocussing since if the two diverging rays leave the field sooner than they would with a straight edge to the field, they will not be deflected so much and will come nearer perfect refocussing. Another contributing factor to the good refocussing may be the fact that the field is not perfectly homogeneous, but falls off towards the apex of the solenoids, fulfilling to some degree the conditions for the third order refocussing described by Bock.[3] The "$-\alpha$"-ray will then be deflected in a circle of greater radius and will, therefore, come closer to perfect refocussing. The interpretation of the curves in Fig. 4 can be carried further. It will be noticed (see Fig. 1) that at the position of best refocussing, the $+\alpha$-divergent ray ($AQVU$) and $-\alpha$-divergent ray both fall on the near side of the normal ray. Thus the field has to be decreased to bring both the $+$ and $-\alpha$-rays back to the collector, giving a maximum in the (I, α) curves as shown in Fig. 4b, c, d, and e. If, however, we move the collector towards the solenoid along the electron path, or equivalently, move the solenoid closer to the collector, i.e., increase A_1 and A_2, the $+\alpha$-ray will now fall on the opposite side of the normal ray from the $-\alpha$-ray. For this position, the field has to be increased for $+\alpha$ and decreased for $-\alpha$ to give the maximum C_2. This is what the curve Fig. 4a indicates, and we see from the large A_1 and A_2 (see Fig. 3) that the solenoids are closer to the collector than best position.

If the collector is moved away from the edge of the solenoids along the electron path or equivalently, the solenoids moved away, or A_1 and A_2 decreased, then the rays start to diverge again and the $+$ and $-\alpha$-rays are on different sides of the normal ray in such a manner that for $+\alpha$ the field has to be decreased, while for $-\alpha$ it has to be increased as indicated by curve Fig. 4f. Thus these (I, α) curves give criteria for determining the best position of the solenoids for best refocussing on a given apparatus: the curve must decrease for $+$ and $-\alpha$ and the maximum should be as flat as possible for best

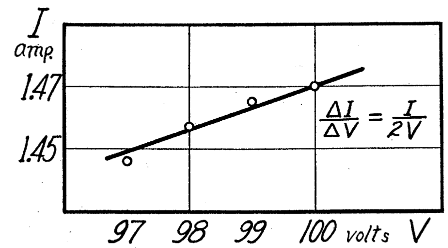

FIG. 5a. Experimental test of dispersion. *Solid line*, theoretical; *points*, experimental.

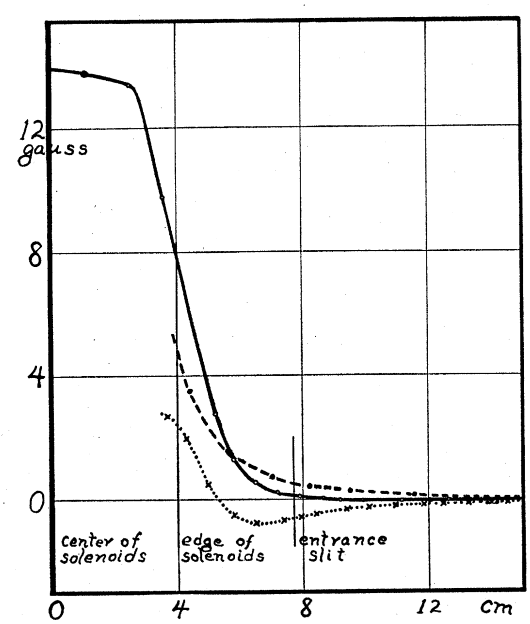

FIG. 5b. Straying of magnetic field due to separation of solenoids. *Dashed line*, field without compensating coils; *dotted line*, field due to compensating coils; *solid line*, field due to solenoids and compensating coils together.

refocussing. This best position is approximately that calculated. The apex of the solenoids should be placed on a line connecting the entrance and collecting slits, with the distances a and b determined from $b=a \sin\theta/\sin\gamma$. In the experimental case for $\theta=\gamma=45°$, and $a=b$, the calculated values were (A_1, 18.2, A_2, 18.2 mm).

It has been found possible to get a magnetic field which more exactly satisfies the theoretical conditions desired, i.e., homogeneity and a sharp cut-off. The most practical method is to add two small narrow square coils which fit over the ends of the large solenoids at each side of the gap.[7] These produce a field shown in Fig. 5b by the dotted line. The number of turns and the position and dimensions of these coils can be so fixed that the stray field outside the entrance slit will be approximately compensated and the field will be of the form of the solid line. This also makes the field inside the solenoids more homogeneous. In an analyzer, designed and built by Dr. Hergenrother of this laboratory for analyzing scattered electrons, two small coils ($10.2\times10.2\times0.7$ cm) carrying the same number of ampere turns per cm as the solenoids are added on each side of the gap of the solenoids which are similar to the ones used in this experiment. This combination gives a field whose value is shown in the solid line of Fig. 5b. A more direct, but less practical method of reducing the stray field is by reducing the effective gap of the solenoids. This effective decrease in gap can be secured either by reducing the thickness of the apparatus and putting the solenoids closer together or by increasing the cross-sectional area of the solenoids. If the cross-sectional area is increased, care must be taken that the solenoids are long enough to avoid stray fields from the outer ends.

A further application of this general refocussing property would result theoretically in perfect refocussing. Suppose for a convenient θ, say 50°, we let $\gamma=180°+\theta$. Then $\sin\theta=-\sin\gamma$, $\cos\theta=-\cos\gamma$, $S=0$, and $D=0$. With this arrangement the boundaries of the magnetic field would have an angle of $180°+2\theta$ and the beam would refocus at A, the point of entrance. There would be perfect direction and velocity refocussing to the accuracy of our approximations. For $\alpha=\pm3°$, the direction refocussing would be accurate to 1 part in 10,000. For $\Delta V=1$ volt for 100 volt electrons, the velocity refocussing would also be accurate to 1 part in 10,000. To use this arrangement for e/m or mass analysis the plane of the initial beam would have to be tilted slightly so that the refocussing point would be separated from the entrance point. For velocity analysis, two charged deflecting plates would have to be inserted where the beam leaves the magnetic field to give a vertical separation between beams of different velocities.

Another way of securing more perfect refocussing is to so shape the right-hand boundary of the magnetic field, see Fig. 1, that the diverging rays will refocus perfectly at B. The shape of the edge for this third order refocussing can be most easily obtained by graphical methods. Lines from B are drawn tangent to the circles of curvature in the magnetic field for different values of α. The loci of the points of tangency of such lines will be the desired edge of the field. This can be secured approximately by so shaping one side of the solenoids. This third order refocussing method is an alternative to the variation in intensity of the magnetic field method described by Bock.[3]

The refocussing described here occurs only in the plane of the electron beam. If the set-up shown in Fig. 1 were duplicated for every plane through AOB in the figure of revolution obtained by turning the figure about AOB as axis, then all rays leaving A making an angle $\theta\pm\alpha$ with AOB would be refocussed at B. The desired field could be obtained by a ring solenoid of proper shape in which the wires with which the solenoid is wound are far enough apart to permit the electrons to pass through without unduly distorting the field. This arrangement would constitute an electron "lens" if the rays that had angles of entrance other than $\theta\pm\alpha$ were cut off by proper diaphragms. The magnifying properties of the lens would depend on the ratio of θ to γ and focussing would be accomplished by variation of the magnetizing current.

In conclusion, I wish to thank Professor A. L. Hughes, under whose direction this work was carried out.

[7] Similar coils on the end to increase homogeneity inside solenoids were used by A. Buhl and F. Coeterier, Phys. Zeits. **33**, 773 (1932).