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An Investigation of the Klein-Nishina Formula for X-Ray Scattering, in the Wave-Length Region 50 to 20 X-Units

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An x-ray tube and a spectrograph were used to produce monochromatic beams of wave-length from 50 to 20 x-units. The absorption coefficient per electron for each beam was measured in carbon and aluminum. Results indicate it is unlikely that the Klein-Nishina formula for the scattering coefficient per electron is in error by more than one percent in this region of wave-lengths.

INTRODUCTION

A BEAM of x-ray photons, in passing through matter, loses intensity partly as a result of Compton scattering by the extranuclear electrons. We shall call the Compton scattering per electron ${}_e\sigma$ and the total absorption per electron ${}_e\mu$.

In 1928 Klein and Nishina[1] derived an expression for the dependence of ${}_e\sigma$ on the frequency of the radiation. This was based on Dirac's relativistic interpretation of the quantum mechanics, and supplanted a formula derived by Dirac[2] from the older quantum mechanics.

The most advantageous region of wave-lengths in which to test the Klein-Nishina formula is from 12 to 50 x-units, where its predictions differ appreciably from those of the older theories, and where there is good reason to believe other modes of absorption are relatively small. For longer wave-lengths the photoelectric absorption becomes increasingly important. For shorter wave-lengths there is the possibility of additional absorption due to the production of positive and negative electron pairs, to nuclear absorption, and to nuclear scattering.

The total absorption coefficient for the filtered gamma-rays from thorium C'' (wave-length 4.7 x.u.) has been measured by several experimenters. The values for ${}_e\mu$ so obtained for elements of atomic number less than 14 are, on the average, about 1.28×10^{-25}, whereas the value of ${}_e\sigma$ calculated from the Klein-Nishina formula for this wave-length is 1.23×10^{-25}. Also Chao[3] has scattered the 4.7 x.u. radiation from aluminum at suitable angles to produce monochromatic beams of wave-length 7.0, 9.6, 15.5, 29.4, and 47 x.u., and measured the absorption coefficient per electron in aluminum. The values he obtained are plotted in Fig. 3. In general the values for ${}_e\mu$ are about four percent greater than the values for ${}_e\sigma$ to be expected from the Klein-Nishina formula, and the excess appears to exist for wave-lengths from 4.7 to 29.4 x.u. The excess at wave-lengths between 12 and 29.4 x.u., if real, can scarcely be due to nuclear absorption or scattering, while it is equally unlikely to be due to photoelectric absorption.

[1] Klein and Nishina, Zeits. f. Physik **52**, 853 (1928).
[2] Dirac, Proc. Roy. Soc. **A111**, 405 (1927).
[3] Chao, Phys. Rev. **36**, 1519 (1930).

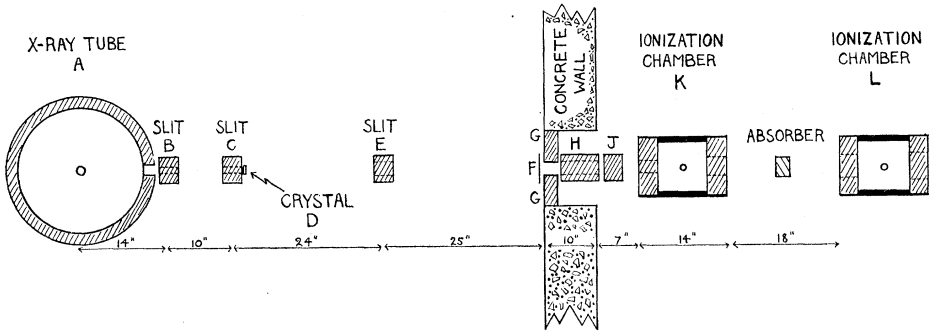

FIG. 1. Arrangement of the apparatus.

It is the purpose of this investigation to determine the absorption per electron in aluminum and carbon for monochromatic beams in the wave-length region 20 to 50 x-units, and to compare the results with the values of ${}_e\sigma$ calculated from the Klein-Nishina formula.

APPARATUS

The arrangement of the apparatus is given in Fig. 1. The x-ray tube A could be excited by voltages up to 1000 kv. Slits in the lead blocks B and C defined a plane horizontal x-ray beam, which was incident on the internal atom planes of the rocksalt crystal D at such an angle that radiation of the desired wave-length was reflected. A slit in the lead block E allowed the monochromatic beam to pass on into the ionization chambers, while the block itself absorbed the unreflected radiation. The lead filter F and the slit H reduced the intensity of stray radiation which might reach the chambers. The sliding lead block J acted as a shutter.

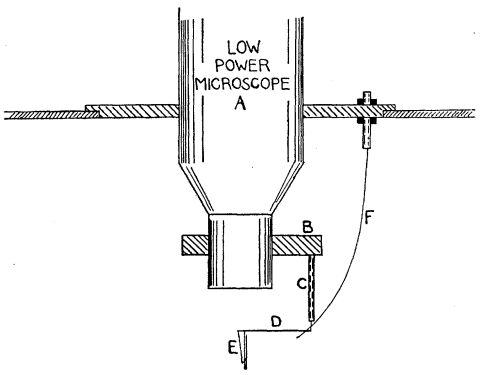

FIG. 2. Construction of the ionization chamber.

The construction of the aluminum ionization chambers is shown in Fig. 2. The low power microscope A had an eyepiece scale across which the foot of the L-shaped gold coated quartz fiber E appeared to move. Charge could be put upon the fiber system by rotating the wire F so as to touch the wire frame D, which was insulated by an amber pillar C.

EXPERIMENTAL PROCEDURE

A radioactive source was used to produce a constant rate of ionization in the chambers, and a discharge curve, scale reading against time, was graphed for each chamber. Since the rate of ionization was constant, a unit on the time axis could be regarded as an arbitrary unit of ionization, so that the ratios of ionizations producing discharges over different portions of the eyepiece scale could be determined. It was verified that the graphs were independent of the magnitude of the ionization currents, and that the sum of the ionizations produced by two radioactive sources acting separately equalled the ionization produced by the sources acting together.

The monochromatic x-ray beam was passed through the chambers without any absorber between them. Shutter J was opened for a time sufficient to permit the more rapidly moving fiber to cover almost the whole eyepiece scale. The ionization in chamber L was then expressed in terms of that in chamber K. The discharge was repeated with an absorbing screen midway between the chambers. From the ionization in chamber K, and the known response of L in terms of K with no absorber present, the ionization which would have occurred in L had

no absorber been present, was calculated. The actual ionization in L was known from the observed discharge, so that the absorption coefficient could be found. The ionizations were always corrected for stray radiation by observing the discharge of the chambers under the same conditions, except that the angle of the crystal was altered so that it no longer reflected the monochromatic beam down the slit system.

The absorption coefficient was measured for several thicknesses of the absorber, and it was found that there was no hardening of the beam with increasing thickness. The x-ray tube was operated at a peak voltage less than twice that corresponding to the wave-length reflected by the crystal. Therefore radiation of half the desired wave-length, reflected in the second order, was not present.

The wave-length of the radiation was determined by photographing the monochromatic ray and the unreflected ray at a distance of 169 cm from the crystal, where the separation of the rays was from 1 to 2 cm. A microphotometric trace of the photographic film made it possible to determine the wave-length with an error of less than one percent.

Results

The results of the measurements are given in Table I, and are plotted in Fig. 3 together with the Klein-Nishina curve for $_e\sigma$ the Dirac curve for $_e\sigma$, and the values obtained by Chao.

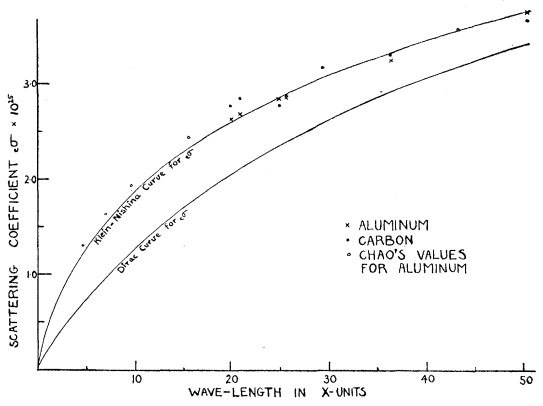

FIG. 3.

Sources of Error

Calculation shows that the x-rays scattered by the absorber, yet still able to enter chamber L, would produce an error of less than 1/10 percent in the observed absorption. The wave-length width $\Delta\lambda$ at half maximum of the monochromatic ray was from 10 to 20 percent of the wave-length λ corresponding to the peak. Since $_e\sigma$ does not vary rapidly with λ this spread would not cause the effective wave-length to differ appreciably from the peak wave-length. The error in the wave-length should be less than two percent.

Nine observations were necessary to obtain one absorption coefficient. If the maximum likely errors in these observations were cumulative,

TABLE I. *Measured values of x-ray scattering coefficient and comparison with Klein-Nishina values.*

Wave-length λ (x.u.)	$\Delta\lambda$ (x.u.)	Element	Experimental value ($\times 10^{25}$)	Probable error ($\times 10^{25}$)	No. of obs.	Weighted mean ($\times 10^{25}$)	Deviation from Klein-N. value	Klein-Nishina value ($\times 10^{25}$)
50.4	4.1	Aluminum	3.75	0.02	4	3.73	−0.5%	3.755
		Carbon	3.64	0.04	3			
36.3		Aluminum	3.24*	0.07	5	3.31	−1.2	3.350
		Carbon	3.31	0.01	3			
25.6	5.0	Aluminum	2.86	0.01	2	2.87	−1.0	2.902
		Carbon	2.88	0.01	2			
24.8	3.7	Aluminum	2.85	0.02	4	2.84	−0.7	2.864
		Carbon	2.77*	0.05	3			
20.8	3.3	Aluminum	2.68	0.03	3	2.69	+1.1	2.657
		Carbon	2.85*	0.10	3			
19.9	3.7	Aluminum	2.63	0.01	4	2.64	+1.1	2.606
		Carbon	2.76*	0.03	4			

they would lead to an error of three percent in the coefficient.

The target of the x-ray tube was carried by a steel column about 15 feet long, and was cooled by water circulated by thermo-syphon action. A small change of temperature would cause the target to move with respect to the slit system, which would alter the ratio of response of the chambers and the amount of background radiation. This was the most troublesome source of error as it could be detected only by a diminution of x-ray intensity, and by the lack of agreement between values of the absorption coefficient obtained under otherwise identical conditions. It is probable that the values in Table I marked by asterisks were affected by such an occurrence.

Discussion of the Results

The probable errors listed in Table I have not much significance, since the number of observations is small. They are added to give an idea of the agreement between the different observations.

Mention has been made of a possible cause of the larger probable errors of the four values in Table I marked by asterisks. If a smooth curve were drawn among all the points these four would deviate most from it. In each case the value obtained with the same wave-length, but with the other element, has a much smaller probable error, and would fall much closer to the curve. It would therefore seem that little weight should be attached to the four values marked by asterisks, as compared with the corresponding values for the other element.

It is to be expected that ${}_e\tau$, the photoelectric coefficient of absorption, should be less than the experimental error in this region. Otherwise the total absorption coefficient should be greater for aluminum than for carbon, and the difference should diminish regularly with increasing frequency. Furthermore, if we assume Gray's empirical law for the photoelectric absorption of lead, and also that the law ${}_e\tau \propto Z^3$ which holds for longer wave-lengths holds in this region also, then for aluminum for a wave-length of 50 x.u. ${}_e\tau$ is 0.008×10^{-25}. For all other wave-lengths used, and for carbon, ${}_e\tau$ calculated in this manner is much less than the experimental error.

The weighted means given in Table I are obtained by compounding the values for aluminum with those for carbon, giving each a weight inversely proportional to the square of its probable error. If these weighted means were plotted, and a smooth curve were drawn among them, it would not deviate from the Klein-Nishina curve by more than one percent at any point between 50 and 20 x.u.

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