In the *Annals of Mathematics* for April, 1934, I derive the conditions for the Stäckel result in such a form that I have been able to determine all the real type forms so that the space with the fundamental form

$$ds^2 = H_1^2dx_1^2 + H_2^2dx_2^2 + H_3^2dx_3^2 \tag{4}$$

is euclidean, and I have shown that they satisfy the condition (3). These forms and the relation between the coordinates x_i and cartesian coordinates are as follows:

$$H_i = 1, \text{ cartesian}; \tag{I}$$

$$H_1 = H_2 = 1,\ H_3 = x_1, \text{ cylindrical polar coordinates}; \tag{II}$$

$$H_1 = 1,\ H_2 = x_1,\ H_3 = x_1 \sin x_2, \text{ polar coordinates}; \tag{III}$$

$$H_1^2 = 1,\ H_2^2 = H_3^2 = a^2 (\cosh 2x_2 - \cos 2x_3),$$
elliptic cylinder coordinates;
$$x = x_1;\ y = a \cosh x_2 \cos x_3,\ z = a \sinh x_1 \sin x_3; \tag{IV}$$

$$H_1^2 = 1,\ H_2^2 = H_3^2 = x_1^2 x_3^2,$$
parabolic cylinder coordinates;
$$x = x_1,\ y = x_2^2 - x_3^2,\ z = x_2 x_3; \tag{V}$$

$$H_1^2 = 1,\ H_2^2 = H_3^2 = x_1^2[k^2 cn^2(x_2, k) + k'^2 cn^2(x_3, k')],$$
$$k^2 + k'^2 = 1,\ x = x_1 dn(x_2, k) sn(x_3, k'),$$
$$y = x_1 sn(x_2, k) dn(x_3, k'),\ z = x_1 cn(x_2, k) cn(x_3, k'); \tag{VI}$$

$$H_1^2 = H_3^2 = x_1^2 + x_3^2,\ H_2^2 = x_1^2 x_3^2,$$
parabolic coordinates,
$$x = x_1 x_3 \cos x_2,\ y = x_1 x_3 \sin x_2,\ z = \tfrac{1}{2}(x_1^2 - x_3^2); \tag{VII}$$

$$H_1^2 = H_3^2 = a^2 (\sinh^2 x_1 + \sin^2 x_3),$$
$$H_2^2 = a^2 \sinh^2 x_1 \sin^2 x_3,$$
prolate spheroidal coordinates,
$$x = \sinh x_1 \sin x_3 \cos x_2,\ y = a \sinh x_1 \sin x_3 \sin x_2,$$
$$z = a \cosh x_1 \cos x_2; \tag{VIII}$$

$$H_1^2 = H_3^2 = a^2 (\cosh^2 x_1 - \sin^2 x_3),$$
$$H_2^2 = a^2 \cosh^2 x_1 \sin^2 x_3,$$
oblate spheroidal coordinates,
$$x = a \cosh x_1 \sin x_3 \cos x_2,\ y = a \cosh x_1 \sin x_3 \sin x_2,$$
$$2 = a \sinh x_1 \cos x_3; \tag{IX}$$

$$H_i^2 = (x_i - x_j)(x_i - x_k)/f(x_i),$$
$$f(x_i) = 4(\alpha - x_i)(\beta - x_i)(\gamma - x_i),\ (i, j, k \neq),$$
confocal ellipsoidal coordinates; (X)

$$H_i^2 = (x_i - x_j)(x_i - x_k)/f(x_i),$$
$$f(x_i) = 4(a - x_i)(b - x_i),\ (i, j, k \neq),$$
confocal parabolic coordinates,
$$x = (x_1 + x_2 + x_3 - a - b)/2,$$
$$y^2 = (a - x_1)(a - x_2)(a - x_3)/(b - a)$$
$$z^2 = (b - x_1)(b - x_2)(b - x_3)/(a - b),\ x_1 > b > x_2 > a > x_3. \tag{XI}$$

In each case the coordinate surfaces consist of confocal quadrics including the cases when one or more families consist of planes. All such systems yield solutions and only these. Consequently the only orthogonal systems of coordinates in which the three-dimensional Schrödinger equation can be solved by separation of the variables are the above types.

LUTHER PFAHLER EISENHART

Fine Hall,
Princeton, New Jersey,
February 24, 1934.

On the Inversion of Doublets in Alkali-Like Spectra

In a recent paper[1] we considered the effect of the polarization of the core on the doublet separations in alkali-like spectra. We are indebted to Professor Van Vleck for pointing out to us that such an effect may be formulated as a third order perturbation in a systematic application of perturbation theory. If H is the Hamiltonian the third order correction to the energy of the state i is

$$\delta E_i = -\Sigma_{jj'} \frac{H_{ij}H_{jj'}H_{j'i}}{(H_{ii} - H_{jj})(H_{ii} - H_{j'j'})}. \tag{1}$$

A careful examination shows that, for our case, and for both the coupling schemes which we considered for the excited core states, the terms in (1) reduce to those calculated by us. But whereas (1) is clearly independent of the representation chosen for the excited core states, our results, based on two different assumptions for this representation, differed by a factor of two. We have found that in the table of doublet separations on p. 646 the values given for terms arising from triplet parents are incorrect: the coefficients of ξ_1 for these three doublets should all be reduced by a factor of three. When this is done, and the corresponding correction is made in Table III, the result, on the assumption of LS coupling, reduces to $\delta E = -(F_0G/E_0^2)\xi_1 - 2(fG + gF_0)/3E_0$, in complete agreement with that given on p. 648 for the other coupling scheme considered.

MELBA PHILLIPS

Department of Physics,
University of California,
February 22, 1934.

[1] Melba Phillips, Phys. Rev. **44**, 644 (1933).

Artificial Radioactivity Produced by Deuton Bombardment

Following the discovery by I. Curie and F. Joliot[1] that radionitrogen is formed when boron is bombarded with alpha-particles, it seemed probable, as they in fact suggest, that this new radioactive element might be formed by bombarding carbon with high speed protons and deutons. Indeed, in the light of our recent experiments in which

[1] Curie and Joliot, Nature **133**, 201 (1934).

neutrons and protons were found to be emitted from many elements when bombarded with deutons, the possibility presented itself that in these nuclear reactions new radioactive isotopes of many of the elements might be formed.

Using both deutons and protons we have bombarded all the elements, except neon and sulphur, from lithium through chlorine and also calcium. Except possibly in the case of carbon, we observed no radioactivity induced by bombardment with one and one-half million volt protons. But every substance was observed to emit both gamma-radiation and ionizing particles of approximately electronic mass for some time after bombardment with three million volt deutons. In the light of the Curie-Joliot experiments these particles are presumably positrons.

Preliminary measurements of the half-lives of the radioelements produced in the several targets are as follows: Calcium fluoride, forty seconds; calcium chloride, thirteen minutes; boric oxide, sodium phosphate, lithium carbonate and ammonium nitrate about two minutes; aluminum three minutes; magnesium nine minutes; beryllium nine minutes with some evidence of a shorter lived group of about three minutes; carbon twelve minutes. The possibility that some of the observed effects are ascribable to impurities remains to be investigated.

We are indebted to Professor J. R. Oppenheimer for many helpful discussions, to Professor Gilbert N. Lewis for furnishing the necessary deuterium, to the Research Corporation and The Chemical Foundation for their financial support and to Commander Telesio Lucci and Alexander Hildebrand for their assistance.

MALCOLM C. HENDERSON
M. STANLEY LIVINGSTON
ERNEST O. LAWRENCE

Radiation Laboratory,
Department of Physics,
University of California,
Berkeley, California,
February 27, 1934.

Invariants of Quadrics and Electrical Circuit Theory

It is the purpose of this letter to call attention to a paper upon a *Classification of Quadrics in Affine n-space by Means of Arithmetic Invariants* which I wrote in 1931 and subsequently published;[1] and which, when translated into electrical language, indicates the answers to a number of questions raised at the end of the recent paper by Nathan Howitt.[2]

Howitt showed, in his paper on *Group Theory and the Electric Circuit*, that two-terminal linear passive networks of a finite number of meshes consisting of inductances, capacitances and resistances, form a group under the real linear affine non-singular transformations of matrix B, on the charges and currents, with the driving-point impedance function as an absolute invariant; i.e., to a given impedance function there corresponds an infinite number of networks, any one of which can be obtained from the other by a special affine transformation of the instantaneous mesh currents and charges. This was done by applying the matrix B to the matrices of the three fundamental quadratic energy forms of the network, resulting in the formation of new quadratic forms congruent (equivalent) under the transformation used.

Consider the two-terminal linear network (with positive or negative circuit parameters) of total instantaneous magnetic energy T, power loss R, and electrostatic energy V. Let A denote the matrix of a real fundamental energy form F, (T, R or V), which is quadratic in the instantaneous currents (or charges). Suppose the currents (and charges) are subject to a transformation of matrix B, which leaves the driving-point current (and charge) in nth mesh invariant. Let R, S; r, s, denote the ranks and signatures, respectively, of A and a, where a is matrix A with last row and column deleted. Then, according to my paper, we have the answer to certain questions raised by Howitt.

If $(R-r)\neq 2$, A can be reduced by a transformation B to a diagonal form for which F is the sum of the squares of the (instantaneous) mesh currents, p being positive and q being negative, where $2p=R+S$ and $2q=R-S$; and, theoretically, all mutual parameters of type F can be eliminated, R being the minimal number of type F circuit parameters. If $(R-r)=2$, F may be reduced to the sum of the squares of the mesh currents, p positive and q negative, plus twice the product of the currents in $(n-1)$th and nth meshes, where $2p=r+s$ and $2q=r-s$; and, theoretically, all mutual parameters of type F can be eliminated, save one, $(R-1)$ being the minimal number of type F circuit parameters. If the circuit is passive, $R=S$, $r=s$ and we can dispense with the signatures altogether.

The reduction of F to normal form indicated in the proofs of the theorems of my paper should simplify computations considerably. It is hoped that these methods may prove of practical value. Though we may not need to know the value of the impedance function many times, and we may characterize it by other means, I hardly think that the concept of impedance can be scrapped, because any absolute invariant is of fundamental significance from the mathematical point of view.

If instead of imposing the invariance of the current in nth mesh, we impose the invariance of the current in jth mesh, the above results go through with n and j interchanged, the transfer impedance being an absolute invariant then.

The extension of these methods to pairs and triples of energy forms and the resulting network interpretation I hope to indicate later in an extended paper.

RICHARD S. BURINGTON

Case School of Applied Science,
Cleveland, Ohio,
February 26, 1934.

[1] R. S. Burington, Am. Math. Monthly **39**, No. 9, 528–532 (1932).

[2] N. Howitt, Phys. Rev. **37**, 1583–1595 (1931).