The Fine Structure of the Balmer Lines

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(Received September 16, 1933)

A new method of treating interferometer patterns of doublets has been applied to the study of the Balmer series. The method involves the measurement of the intensities of the minima between the members of the doublet and between successive orders of interference. This gives for the separations of the centers of gravity for $H\beta$, $\Delta\nu=0.3298\pm0.0004$; for $H\gamma$, $\Delta\nu=0.3388\pm0.0004$; for $H\delta$, $\Delta\nu=0.3451\pm0.0006$; and for $H\epsilon$, $\Delta\nu=0.3506\pm0.0006$. These values are far too small to agree with the ordinary theory of these lines and it must be concluded that the structure is not quite that expected. The discrepancy may lie in the neglect of the radiation reaction in the calculation of the energy levels.

Because the present quantum theory permits all of the features of the hydrogen spectrum to be calculated without any approximations of a mathematical nature, it is of interest to study this spectrum with especial care. Previous work on the fine structure of the Balmer series has led to the conclusion that the positions of the individual lines are those given by the Sommerfeld formula, when the selection rules derived on the basis of the electron spin idea are used.[1] Since the Dirac theory of this spectrum leads to the Sommerfeld formula for the positions, and to selection rules which are analogous to those of the alkalis, the observations have been taken as lending strong support to the validity of Dirac's relativistic equation. However, in spite of the apparently satisfactory way in which the observed positions have agreed with those to be expected, there has usually been some discrepancy between the observed and calculated relative intensities of the two members of each doublet. This has usually been ignored since it has been felt that the knowledge of the conditions of excitation is insufficient to give an accurate prediction of the intensities.

Because of the satisfactory way in which the apparent separations of the hydrogen doublets, i.e., the separations of the centers of gravity of the two component groups, seemed to fit the theory, we hoped to use this separation as a means of experimentally determining the fine structure constant, which has been the subject of so much recent discussion. This determination would require the measurement of this apparent separation for the higher members of the series, since for the higher members the separation of the centers of gravity approaches the separation of the lower states. To obtain the desired accuracy in the fine structure constant it would be necessary to measure this apparent separation with a precision of the order of 0.1 percent. The first two members of the series are not very suitable for this purpose, because the two component groups, which are observed, are composed of several lines whose separation is a very considerable fraction of the apparent doublet separation; but since the theory must be assumed to connect the fine structure pattern with the fine structure constant, it can be assumed also to make the suitable corrections for the first two lines.

It has been found possible to approach the required precision on the first five members of the series, but the results are so far from those to be expected that it is clear we are not measuring the fine structure constant, but have attained to that degree of precision in which the theory is no longer satisfactory. One possible explanation of this is that the effect of the interaction between the radiation field and the atom has been neglected in computing the frequencies.

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[1] G. Hansen, Ann. d. Physik **78**, 558 (1925). W. V. Houston, Astrophys. J. **64**, 81 (1926). Sommerfeld and Unsöld, Zeits. f. Physik **36**, 259 (1926); **38**, 237 (1926). Kent, Taylor and Pearson, Phys. Rev. **30**, 266 (1927).

The results presented here give an indication of the amount by which the theory is in error, but because of the complexity of the patterns, they do not give a great deal of information as to the exact nature of the discrepancy.

Description of the Apparatus

The source of light is a simple discharge tube, 2 cm in diameter and about 50 cm long. It is bent into the form of a U so that the central part can be immersed in a thermos bottle full of liquid air. The window is so arranged that only the part of the tube which is under the liquid air can be seen from the slit of the spectrograph. The excitation is by means of a direct-current generator.

The hydrogen is prepared by electrolysis, and is admitted in small quantities to the discharge tube through which helium at a pressure of about 0.5 mm is kept circulating. The circulating system contains two traps immersed in liquid air, one of which is filled with charcoal. After the system has been sealed up for several weeks, most of the impurities are absorbed on the charcoal, and the spectrum in the center of the discharge tube consists almost entirely of the Balmer series. The stronger helium lines appear in the long exposures. The structures seem to come out most clearly when the pressure is not over 0.5 mm and the current is not over 0.1 amp. These conditions are not very sharply defined, however, and we hope later to investigate more thoroughly the effect of the conditions of excitation.

The spectrographic apparatus is the usual combination of a Fabry-Perot interferometer with a prism spectrograph. The camera has a focal length of 80 cm. This is about twice that of the collimator. The magnification of the slit image, which is obtained in this way, helps to eliminate the effect of the photographic plate grain.

The interferometer was loaned us by Dr. Babcock of the Mt. Wilson observatory, and we wish to express here our appreciation of his kindness in permitting us to use this magnificent instrument. The plates are about 11 cm in diameter, and since the diameter of the collimator lens is only 3 cm, the best part of the plate surfaces can be selected for use. The plates are silvered frequently, by evaporation,[2] with enough silver to give about 92 percent reflection. The transmission is then about 4 percent, and the resolving power seems to be the maximum consistent with sufficient light intensity. The exposure times range between 2 and 5 minutes, so that no especial precautions are taken to control the temperature of the spectrograph.

The method used for measuring the apparent doublet separation requires the determination of relative intensities. For this purpose intensity marks are put on each plate by the arrangement shown in Fig. 1.[3] A 500 watt lamp S is enclosed

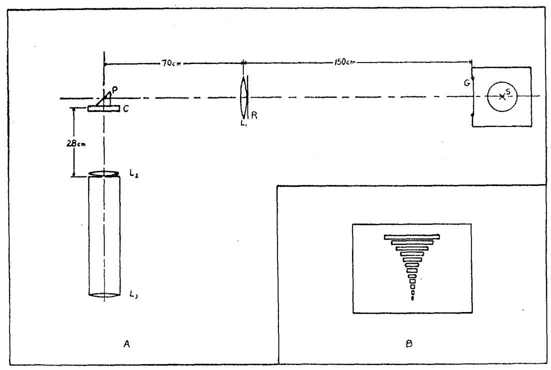

FIG. 1. (A) The arrangement of the apparatus for putting density marks on each plate. (B) The form of the screen for producing the density marks.

in a box with an opal glass window G. This provides a fairly uniform source of light about 9.5 cm square. By means of a lens L_1, this source is focussed upon the cylindrical lens C through the 90° prism P. At R is a grating of the form shown in 1B. The longest slot is 48 mm and the shortest is 0.5 mm long. There are twelve slots in all and they are each 2 mm wide. This grating is focussed on the slit by the cylindrical lens C whose axis is horizontal. The lens L_2 focusses C upon the collimator lens L_3.

[2] The silver is applied with the apparatus developed by Dr. J. D. Strong and described in Rev. Sci. Inst. **2**, 189 (1931). We are much indebted to him for his assistance in this work.

[3] This method has been described by G. Hansen, Zeits. f. Physik **29**, 356 (1926).

When properly adjusted, this arrangement gives a series of illuminated strips whose intensity in the center is proportional to the length of the corresponding slot in R. Although no elaborate tests have been made to check the intensity ratios, they appear to have the expected values.

Method of Measurement

It is probably impossible to attain the required precision by the ordinary methods of measurement. The two component groups in each series member are so wide that the uncertainty in the location of the maximum, or the center of gravity, introduces more error than can be permitted. Hence it is necessary to make the measurements by analyzing the intensity distribution.

There are three principal reasons for the breadth of the groups in the Balmer series. In the first place each of the individual lines has a certain inherent width due to the radiation damping. In the second place, the observations are made on the two component groups of lines which constitute the doublet. Each of these groups consists of either two or three individual lines, and so the whole group has a width which is at least as great as the separation of these individual lines. The third and predominant cause, however, is the Doppler effect, due to the temperature motion of the atoms. It has been assumed throughout that all causes of broadening, except the Doppler effect and the complex nature of the groups, can be neglected.

The shape of a single line which is broadened by the Doppler effect is[4]

$$I=I_0e^{-\alpha(\delta\lambda)^2},\quad \text{with}\quad \alpha=mc^2/2kT\lambda^2. \tag{1}$$

I is the intensity of the line at the wave-length $\lambda\pm\delta\lambda$, I_0 is the intensity of the line at the center, c is the velocity of light, k is the Boltzmann constant, T is the absolute temperature, and m is the mass of the atom or molecule which emits the line. If $\Delta\lambda$ and $\Delta\nu$ represent the half width of such a line at half maximum intensity, measured in wave-length and wave number, respectively, then

$$\Delta\lambda/\lambda=\Delta\nu/\nu=3.57\cdot 10^{-7}(T/M)^{\frac{1}{2}}. \tag{2}$$

In this equation, M is the molecular weight of the atom or molecule which emits the line. It is to reduce as far as possible the broadening of this kind that the discharge tube is operated under liquid air, but even under these conditions, the individual lines have a considerable width. If a temperature of 100°K is assumed, the calculated half widths range from 0.05 cm^{-1} for the individual lines of Hα to 0.09 cm^{-1} for those of Hϵ. The apparent doublet separation which is to be measured is only two to three times as large as the minimum separation that could be observed with individual components as wide as this.

Because of these facts it is impossible, at least at the available temperature, to resolve the component groups, of a doublet, into the individual lines of which they are composed. It is only possible to compute the theoretical shape and to determine experimentally the distribution of intensity. However, instead of trying to determine a large number of points on the curve of intensity as a function of wave-length, we have for the present concentrated on the determination of the positions of the centers of gravity of each of the component groups. Other characteristic points would serve as well, but the subsequent analysis shows that the center of gravity is a point which is easily located. If it is assumed that the relative intensitives and the positions of the individual lines are those required by the theory of Dirac, the scale of the pattern can be determined by determining the separation of the centers of gravity of the component groups. This can be done with considerable precision by the methods indicated below. If, however, this assumption is not made, the measurements we have made give a somewhat less accurate value for some sort of an apparent doublet separation.

The ordinary method of measuring an interferometer pattern requires the estimation of the location of some characteristic point on each fringe, usually the center of gravity, either on the original plate or on a microphotometer curve. From the location of these points the diameters of the corresponding circles can be determined, and these give the value of the fractional order of interference at the center of the pattern for each line. In terms of these orders of inter-

[4] Rayleigh, Phil. Mag. **29**, 274 (1915).

ference, the wave-number difference between the two lines is

$$\Delta\nu = \Delta p/2d, \tag{3}$$

where Δp is the difference between the orders of interference of the two lines and d is the separation between the surfaces of the interferometer plates.[5] The idea of our procedure is to find the plate separation for which $\Delta p = 0.500$, without actually measuring the fractional orders of interference. If the pattern is due to a pair of single lines, each of which is symmetrical about its center, the pattern will then consist of a series of fringes separated by minima which are all of the same intensity. When the difference between the orders of interference is greater or less than 0.500, the minima between two fringes with the same integral order of interference will be less intense or more intense, respectively, than the minima between fringes of successive orders of interference. This behavior of the minima provides a sensitive test for the satisfaction of this condition. The method, then, consists in photographing the interference pattern with different values of d and selecting that value which satisfies this test.

Since it is not practical to construct a series of interferometer separators in which the thicknesses differ by much less than 0.05 mm, it is necessary to find a reliable method of interpolation between the separations used. This requires the definition of a measure of the lack of equality between the intensities of the two kinds of minima. This measure should be somewhat independent of the widths of the fringes, so as not to be too much affected by changes in the exposure time and in the accuracy of the adjustment of the interferometer. These conditions are satisfactorily met by the following analysis.

If the spectrum consists of two single lines, both symmetrically broadened by the Doppler effect, the intensity distribution in the interference pattern is given by

$$I(x) = \sum_{n=-\infty}^{\infty} e^{-\beta(x-n)^2} + A \sum_{n=-\infty}^{\infty} e^{-\beta(x-h-n)^2}. \tag{4}$$

[5] W. V. Houston, Astrophys. J. **64**, 81 (1926).

In this expression x is the distance, measured in orders of interference, from one of the maxima which is taken as the origin. h is the wave-number difference between the two spectral lines divided by the spectral range of the interferometer pattern. β is the constant which gives the width of the individual lines. Because of the units in which x is measured, β is equal to the α of Eq. (2) multiplied by the square of the spectral range of the pattern. In making the approximations it will be assumed that $\beta > 16$. This condition is satisfied for the hydrogen lines with the interferometer separations used. A is the ratio of the intensities of the two lines which compose the pattern, and for convenience the origin is taken so that A is not greater than one.

From Eq. (4) it can be shown that the positions of the first two minima on the positive side of the origin are given by

$$x_1 = (1/8 + \epsilon/4)\{1 + (8A - \beta)/(8 - \beta)\}$$

and (5)

$$x_2 = 1 - (1/8 - \epsilon/4)\{1 + (8A - \beta)/(8 - \beta)\},$$

where $\epsilon = h - 0.500$. ϵ is a measure of the amount by which the fringes due to the line of intensity A are distant from the position halfway between the adjacent fringes due to the other line. Because of the assumed lower limit for β it is unnecessary to use more than one term in each of the sums in Eq. (4) to determine the intensities at these minima. Let M_1 be the intensity at x_1, let M_2 be the intensity at x_2, and let $(8A - \beta)/(8 - \beta) = 1 + S$. Then, with sufficient accuracy

$$\begin{aligned} M_1 &= e^{-\beta(\frac{1}{4}+\epsilon/2)^2}\{e^{-\beta(\frac{1}{4}+\epsilon/2)^2 S} + A e^{\beta(\frac{1}{4}+\epsilon/2)^2 S}\}, \\ M_2 &= e^{-\beta(\frac{1}{4}-\epsilon/2)^2}\{e^{-\beta(\frac{1}{4}-\epsilon/2)^2 S} + A e^{\beta(\frac{1}{4}-\epsilon/2)^2 S}\}. \end{aligned} \tag{6}$$

Since S and ϵ are both small compared with unity, this gives

$$\log M_1/M_2 = -\beta\epsilon/2. \tag{7}$$

If the quantity β were strictly constant, and did not vary at all from one exposure to the next, Eq. (7) would be sufficient to determine the variation of ϵ with the interferometer separation. It would only be necessary to plot log (M_1/M_2) against the separation, d, when the slope would give the value of β, while the intercept would be the value of d for which $\Delta p = 0.500$. However,

β does vary a little from one exposure to the next and so it is desirable to evaluate it for each plate. This can be done from the relative intensities of the maxima and the minima. For small ϵ and small S, Eq. (6) gives

$$M_1+M_2=2(1+A)e^{-\beta/16}. \tag{8}$$

Then, if L_1 and L_2 are the intensities of the two maxima, Eq. (4) gives

$$\begin{aligned} L_1+L_2&=(1+A)\{1+2e^{-\beta(\frac{1}{2}+\epsilon)^2}\} \\ &=(1+A)(1+2e^{-\beta/4}). \end{aligned} \tag{9}$$

From these it follows that

$$\frac{L_1+L_2}{M_1+M_2}=\frac{1+2e^{-\beta/4}}{2e^{-\beta/16}}. \tag{10}$$

Thus, from a knowledge of the ratio in (10), the quantity β can be determined. With this, ϵ can be determined from (7).

Symmetry of the Component Groups in Each Doublet

Since the method which has just been described is based on the assumption that the two spectral lines, whose wave-number separation it is desired to measure, have the shape given by Eq. (2), it is necessary to investigate the extent to which this is true of the component groups in the Balmer series. If two individual lines of intensities 1 and A are separated by the distance t, the distribution of intensity due to the two of them is

$$\begin{aligned} I&=\exp\left[-\alpha z^2\right]+A\exp\left[-\alpha(z-t)^2\right] \\ &=\exp\left[-\alpha(z-g)^2\right]\left\{\exp\left[-\frac{\alpha t^2A}{1+A}\left(\frac{2z}{t}-\frac{A}{1+A}\right)\right]+A\exp\left[\frac{\alpha t^2}{1+A}\left(\frac{2z}{t}-\frac{1+2A}{1+A}\right)\right]\right\}, \end{aligned} \tag{11}$$

where $g=At/(1+A)$. If the separation of these lines is much less than their half width, i.e., if $\alpha t^2\ll 1$, this intensity distribution is very similar to that of a single line at the point $z=g$, and with the intensity $1+A$. For large enough values of z, of course, this will be a poor representation, but for these values of z the whole intensity will be so low that the discrepancy will not be important. For the component groups in the Balmer series, the theoretical positions of the individual lines are such that this is a good representation for all except $H\alpha$ and $H\beta$. These must be separately considered.

Although Eq. (11) shows that the general shape of the composite group of two lines is well given by an exponential curve, it is not well adapted to showing the extent to which the group is symmetrical about its center of gravity. If it is to be perfectly symmetrical, it is necessary that all of the odd powers of z, in a power series expansion of the bracket, shall vanish. This they obviously do not do. It is sufficient, however, for the application of the method described above, that these odd powers should be negligible in the regions in which the intensity of the minima are measured. The determination of the conditions under which this amount of symmetry is present is more difficult from Eq. (11) than from an expansion in terms of the orthogonal functions of the Hermite polynomials. If a group consists of a set of individual lines of intensity A_i, whose centers are located at the points h_i, the intensity due to the whole group can be represented as follows. Let

$$\Phi_s=(2^s s!\sqrt{\pi})^{-\frac{1}{2}}\mathrm{H}_s(z)e^{-z^2/2}; \tag{12}$$

then

$$\begin{aligned} \sum A_i e^{-\alpha(z-t_i)^2}&=\pi^{\frac{1}{4}}(2\alpha)^{-\frac{1}{4}}\sum_{s=0}^{\infty}(2^{s-2}s!)^{-\frac{1}{2}}(2\alpha)^{s/2}\sum_i A_i e^{-\alpha t_i^2/4}t_i^s\Phi_s(\xi) \\ &=(2\alpha)^{-\frac{1}{2}}e^{-\alpha z^2}\sum_{s=0}^{\infty}(2^{s-1}s!)^{-1}(2\alpha)^{s/2}\mathrm{H}_s(\xi)\sum_i A_i e^{-\alpha t_i^2/4}t_i^s, \end{aligned} \tag{12a}$$

where $\xi=(2\alpha)^{\frac{1}{2}}z$.

In order that the expression (12a) shall have the required symmetry, it is necessary that the coefficients of the odd numbered polynomials be zero. The coefficient of H_1 can be made zero by adjusting the origin from which z and t_i are measured. The position so determined is just the center of gravity except that each individual line is given a weight equal to its intensity multiplied by $e^{-\alpha t_i^2/4}$ instead of simply equal to its intensity. This factor, however, makes a negligible difference since in the cases in which the required symmetry exists the factor is very close to unity.

The sufficiency of the symmetry then depends upon the magnitude of the coefficient of H_3 multiplied by the value of H_3 at the point where the minimum in the interference pattern occurs. The leading term in H_s is $(2^{\frac{3}{2}}\alpha^{\frac{1}{2}}z)^s$, and the coefficient is $(2\alpha)^{s/2}\sum_i A_i t_i^s$. Hence if the quantity $(4\alpha t_i z)$ is sufficiently small at the value of z for which the minimum occurs, the convergence of the series is rapid enough to neglect all of the odd polynomials of index greater than one. As an example, this criterion may be applied to the violet component group of $H\beta$. Table II gives the theoretical structure of this group. The individual lines are located so that $\Delta\nu/R\alpha^2$ has the values 0.06543 and 0.07324, measured from the position given by the simple Balmer formula. It is important to notice that the α in this table is the fine structure constant and has nothing to do with the α used above to give the width of the lines. The two uses can be easily distinguished by the context. The point at which the minimum will be located is roughly halfway between the two component groups or at 0.04240. The two intensities A_1 and A_2 are 7.81 and 1.535, while the values of t are $t_1 = 0.00128$ and $t_2 = -0.00653$. The value of z which is of interest is $z = -0.02956$. The value of α, when these units are used for position, is about 2500. With these values, the term in the summation over s, in Eq. (12a), for which $s=0$ is 18.69. The term for which $s=1$ is zero because of the use of the center of gravity as the origin. The term for which $s=2$ is 1.51, while the term for which $s=3$ is 0.3. Thus the term in H_3 contributes about 1.6 percent of the intensity at this minimum when the quantity $(4\alpha t_i z)$ has the value 1.95. For $H\gamma$, where $(4\alpha t_i z) = 0.98$, the contribution of the H_3 term is small enough to be neglected. A similar analysis can be applied to the other component group.

The result is, then, that for $H\gamma$ and the higher members of the series, the two component groups of which the doublet is composed are sufficiently symmetrical for the application of the method described above. It also appears that the groups in $H\beta$ are not symmetrical enough to give good results. To make sure of this point a curve was plotted of the intensity distribution to be expected from the theoretical structure of $H\beta$. By actually drawing the interference pattern to be expected it was shown that the method which we use will give the wave-length difference between the centers of gravity with an error less than the experimental uncertainty. A similar graphical treatment of $H\alpha$ showed that the results would require a correction of the order of one percent.

Measurement of the Plates

To apply the method described we have made a microphotometer curve of each interference pattern and of the intensity marks put on each plate. From this curve it is possible to read the intensity of each maximum and each minimum in terms of the intensity of the strongest intensity mark. Since the intensity of illumination is not quite uniform over the image, it is necessary to compare each minimum with the average of the two adjacent minima. This ratio, taken for each minimum and averaged over the whole plate, gives the quantity M_1/M_2. In a similar way the ratio $(L_1+L_2)/(M_1+M_2)$ is averaged over the whole plate. From the plate average of this quantity, the quantity β is determined by Eq. (10). Eq. (7) then gives the value of ϵ which is characteristic of the plate.

The quantity ϵ is then plotted as a function of the separation of the interferometer surfaces, d. The point at which this curve crosses the axis is then the value of d which is inserted in Eq. (3) to find the wave-number separation of the centers of gravity of the component groups. If each component group is strictly given by the exponential curve, these graphs should be straight lines. If, however, the groups have any form whatever which is sufficiently symmetrical,

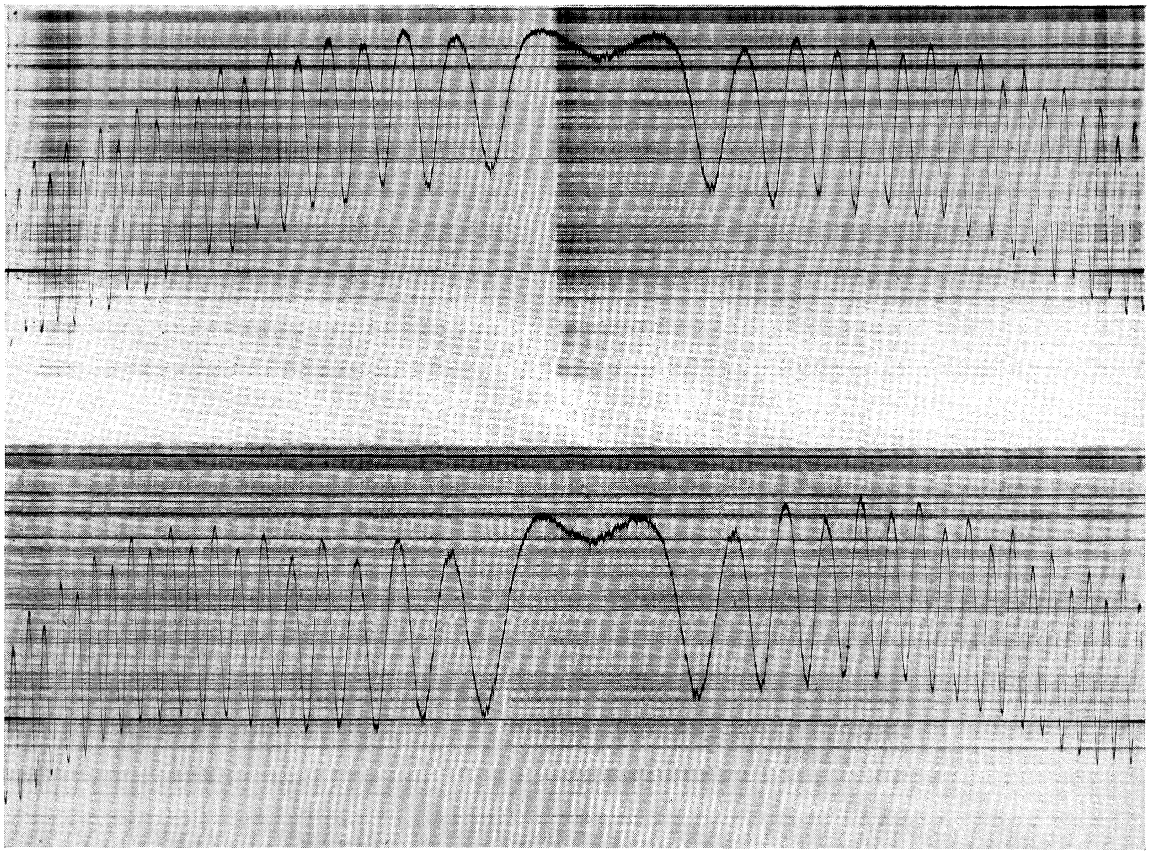

FIG. 2. (a) Microphotometer curve for the line Hβ at 7.499 mm separation. It can be seen from this curve that the more intense minimum has the stronger component inside of it and the weaker component outside of it. Hence $\Delta p<0.50$. (b) Microphotometer curve for the separation 7.611 mm. The two minima are almost the same intensity, but a careful examination shows that $\Delta p>0.50$.

the intersection of the curve with the axis will still give a value of d which will give the wave-number separation of the centers of symmetry of the groups. The detailed analysis of this fact has not been presented, but it can be easily seen from the general method. If the component groups are not symmetrical, the point at which the graph crosses the axis will still give the value of d for which the two minima are of the same intensity, and if the shape of the groups were known experimentally or theoretically, this datum would give their wave-number separation.

RESULTS

Fig. 2 shows a pair of microphotometer curves for Hβ. These show clearly the change in the interferometer pattern as the plate separation, d, is changed. It is possible to estimate from an inspection of these curves, the value of d at which $\Delta p=0.500$. Fig. 3 shows the curves in which $(1/\beta)\log(M_1/M_2)$, in arbitrary units, is plotted against d. The vertical lines extending above and below the plotted points represent the mean deviation of the values from different plates. These are of the same order of magnitude, but are in general a little smaller, than the deviations within the individual plates. Hence the uncertainty is possibly greater than is indicated by these lines. Table I gives the measured values of the separations of the centers of gravity. The uncertainties in this table are merely estimated from the spread of the points in the curves.

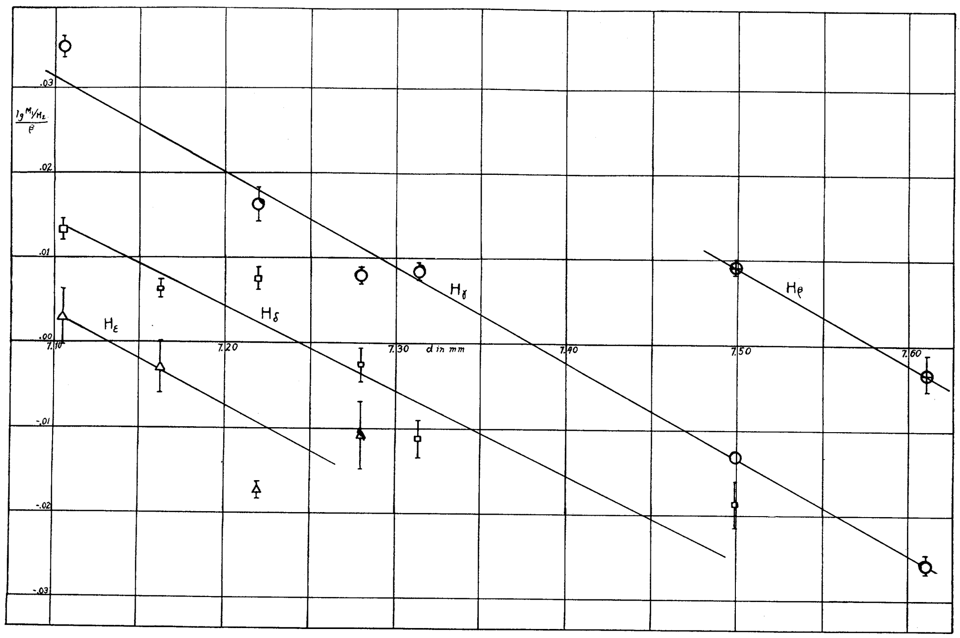

FIG. 3. Curves in which $(\log M_1/M_2)/\alpha$ is plotted against the interferometer separation in mm.

TABLE I.

Line	d_0 (mm)	$\Delta\nu$ (cm^{-1})
Hβ	7.58 $\pm$0.01	0.3298$\pm$0.0004
Hγ	7.38 $\pm$ .01	.3388$\pm$ .0004
Hδ	7.245$\pm$ .015	.3451$\pm$ .0006
Hϵ	7.13 $\pm$ .015	.3506$\pm$ .0006

The method of measurement we have used is not directly applicable to Hα, and so we have not given much attention to it. We have, however, made five measurements of three plates in the ordinary fashion, and have obtained $\Delta\nu = 0.3171 \pm 0.0020$. On the other hand, with an interferometer separation of 7.91 mm the minima were far from being equal. A rough estimate indicates that the minima would be equal at about 8.1 or 8.2 mm. If this could be applied directly to determine the separation of the centers of gravity of the lines, the separation would be 0.3086 or 0.3049. Of course this direct application is not justified in the case of Hα, but the graphical analysis based on the theoretical form of this line, indicated that these results need to be increased by only about one percent to get the correct separation. Hence we must conclude that the method of direct visual measurement gives results which are too large, at least for this line. This is not difficult to understand, since the peak of the line stands out much more on a plate than does the weaker companion, and so the measurement tends to be of the strong lines, rather than of the center of gravity. We hope to make a further analysis of Hα immediately, but this observation explains the difference between our present results and previous work on these doublets.

DISCUSSION OF THE RESULTS

Table II shows the position and the intensity of each of the components of the first five members of the Balmer series, as computed from the present theory, without the inclusion of the effect of a nuclear magnetic moment. The positions are those given by Sommerfeld's formula, which gives the energy levels permitted by Dirac's theory. The intensities are calculated by the methods of Sommerfeld and Unsöld.[1, 6] For small atomic numbers, this method gives the same results as a more rigorous calculation with relativistic functions.

[6] Saha and Banerji, Zeits. f. Physik **68**, 704 (1931), get the same results from Dirac's relativistic theory.

TABLE II. *Theoretically predicted structure of the Balmer lines.*

Component	Hα I	Hα $\Delta\nu/R\alpha^2$	Hβ I	Hβ $\Delta\nu/R\alpha^2$	Hγ I	Hγ $\Delta\nu/R\alpha^2$	Hδ I	Hδ $\Delta\nu/R\alpha^2$	Hε I	Hε $\Delta\nu/R\alpha^2$
$2s_{1/2}-np_{1/2}$	1.04	0.05035	1.41	0.06543	1.58	0.07132	1.667	0.07407	1.808	0.07552
$2p_{1/2}-ns_{1/2}$	0.097		0.125		0.137		0.143		0.146	
$2s_{1/2}-np_{3/2}$	2.08	.06887	2.81	.07324	3.16	.07532	3.333	.07639	3.616	.07698
$2p_{1/2}-nd_{3/2}$	5.00		5.00		5.00		5.00		5.00	
CG viol.	8.217	.06631	9.345	.07196	9.877	.07462	10.143	.07598	10.570	.07671
$2p_{3/2}-ns_{1/2}$	0.195	.01215	0.250	.00293	0.274	.00862	0.286	.01157	0.293	.01302
$2p_{3/2}-nd_{3/2}$	1.00	.00637	1.00	.01074	1.00	.01282	1.00	.01389	1.00	.01448
$2p_{3/2}-nd_{5/2}$	9.00	.01254	9.00	.01335	9.00	.01416	9.00	.01466	9.00	.01497
CG red	10.195	.01146	10.250	.01284	10.274	.01388	10.286	.01440	10.293	.01487
$\Delta CG/R\alpha^2$		.05485		.05912		.06074		.06158		.06184
$\Delta\nu$	1.243	.3191	1.098	.3440	1.040	.3534	1.014	.3583	0.974	.3598
Observed			1.089	.3298	1.041	.3388	1.009	.3451	1.006	.3506
Obs./Calc.				.959		.959		.963		.974

The first column gives the designation of the components in the notation for alkali spectra. The succeeding columns are devoted to the separate series members. The first column under each member gives the intensity, while the second column gives the displacement of the component from the position given by the Balmer formula, in terms of $\Delta\nu/R\alpha^2$. The corresponding quantities are given for the centers of gravity of the red and violet components. The fourth from the last row gives the separation of the centers of gravity in terms of $\Delta\nu/R\alpha^2$, while the third from the last row gives this separation in cm^{-1} when $R\alpha^2$ is taken as 5.818. This is the value given by Birge[7] as the best. In the same row, in the intensity columns, are given the ratios of the intensity of the red component to the violet component. The second from the last row gives the corresponding observed quantities, while the last row gives the ratio of the observed to the calculated separations. Hα is not included in these observed values as we have not yet made a careful measurement on it.

The predicted separations are in every case greater than those observed. The two sets could be brought together fairly well by using $(1/\alpha)=139.9$. This change is so great, however, that it seems impossible that there should be any such error in the constants composing α, and we are forced to the conclusion that the theory, as we have used it, is inadequate to explain the observations.

One possible explanation of this discrepancy would be that the conditions of excitation are such that the relative intensities are not those assumed in Table II. It is impossible to absolutely exclude this possibility without some further investigation, but the fact that the observed intensity ratios of the component groups of the doublets are so near to the calculated values, gives some support to the belief that the intensities are all about as calculated.

Another possibility is that the effect of the nuclear magnetic moment is sufficient to so distort the structure that the centers of gravity have not the calculated separation. If it is assumed that the spin moment of the nucleus is $\frac{1}{2}$, and that the magnetic moment is that to be normally expected in view of the mass of the nucleus, the separation of the levels into which the $2s$ term is split is 0.002 cm^{-1},[8] while the separations of all other levels is much smaller. The relation of these two levels to the original is such that for transitions to this particular state, the center of gravity of the line is not displaced at all. For some of the other components there is a displacement of the center of gravity, but it is so small as to be entirely within the uncertainties of our measurement. Of course, if the g-factor of the nucleus were very much

[7] R. T. Birge, Phys. Rev. **40**, 207 (1932).

[8] G. Breit, Phys. Rev. **35**, 1447 (1931). H. Bethe, Handb. d. Physik **24**, 388.

larger than is to be expected there might be some effect.[9] On this account we have not included the effect of this hyperfine structure in Table II and believe we are justified in ignoring it.

Another explanation of these results has been pointed out to us by Professor Bohr and Professor Oppenheimer. The calculation of the energy levels has been made without the inclusion of the interaction between the electron and the radiation field. The customary procedure is to compute the levels in this way and then to introduce the interaction with the field as a perturbation. It is not known at the present time, how to combine the atom and the field into one system, but it is clear that when such a combination is made, a relative displacement of the levels may be expected, and it should be of the order of α times the fine structure separation. This is the order of magnitude of the effect we have observed.

The order of magnitude of the effect is also indicated by calculations of the natural width of the lines. the $2p$ level has the shortest lifetime of any of those involved. Its life is 0.16×10^{-8} sec.[10] By the uncertainty principle this gives a width of 0.02 cm^{-1}, which is larger than any of the observed discrepancies. Hence it seems to us very probable that this is the cause of the discrepancy we have observed.

[9] The value of the magnetic moment observed by Estermann, Frisch and Stern, Nature **132**, 169 (1933), is still much too small to produce any observable effect.

[10] H. Bethe, Handb. d. Physik **24**, 444.

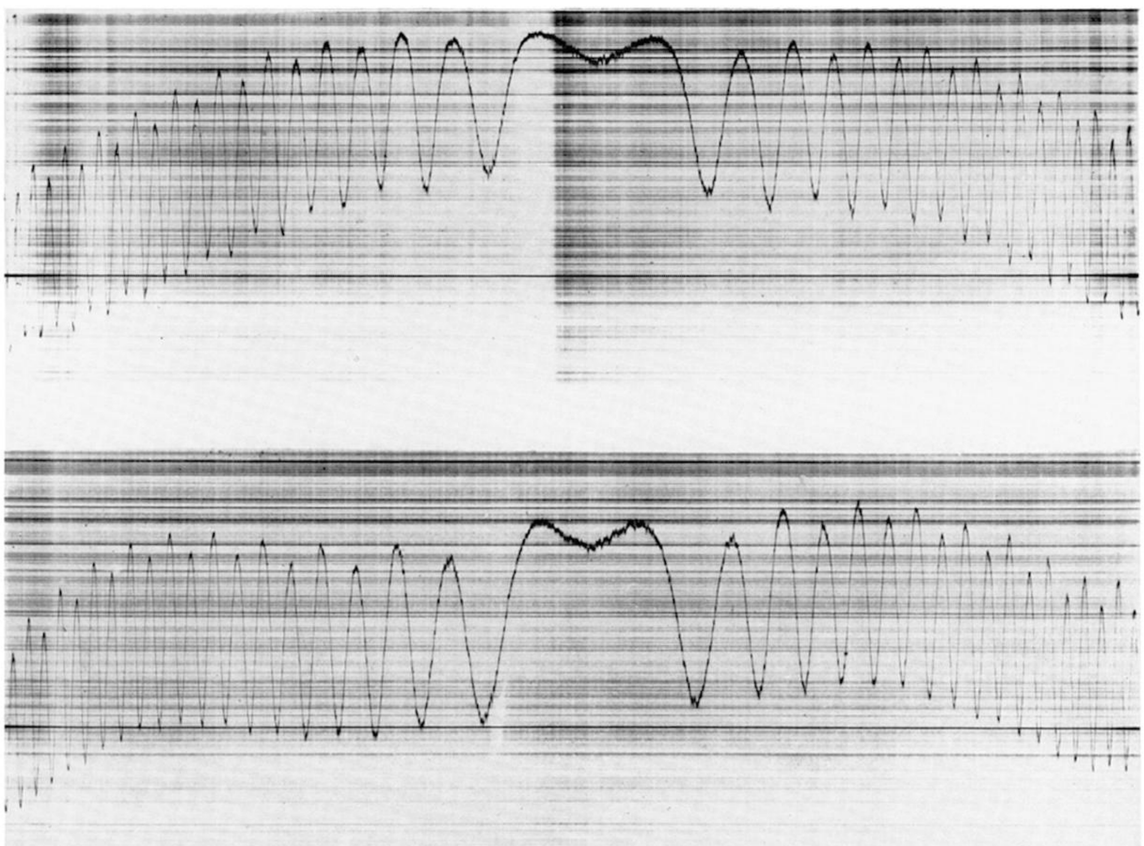

FIG. 2. (a) Microphotometer curve for the line Hβ at 7.499 mm separation. It can be seen from this curve that the more intense minimum has the stronger component inside of it and the weaker component outside of it. Hence $\Delta p < 0.50$. (b) Microphotometer curve for the separation 7.611 mm. The two minima are almost the same intensity, but a careful examination shows that $\Delta p > 0.50$.