

the Dirac delta-function. The error in the previous paper amounted to omitting the factor,  $1/N!$ , on the right-hand side of the above equation. The summation over the index,  $n$ , may be carried out with the aid of the completeness relation, Eq. (5), and the subsequent integrations over the  $p'_1 \cdots p'_{3N}$ , performed in the usual way.

$$\int_{-\infty}^{+\infty} F(p') \delta(p' - p) dp' = F(p). \quad (6)$$

Finally, one obtains,

$$\sigma = [1/N!(2\pi h)^N] \int_{-\infty}^{+\infty} \cdots \int \Phi^*(p, q) e^{-\beta H} \Phi(p, q) dp_1 \cdots dp_{3N} dq_1 \cdots dq_{3N}. \quad (7)$$

This equation becomes identical with Eq. (10) of the previous article, except for the extra factor  $1/N!$ , when the functions  $\Phi(p, q)$ , given by Eq. (2) are explicitly introduced. Since Eqs. (20) and (21) are merely modifications of Eq. (10), they must also contain this factor. The same result is obtained when the completeness relation for the configuration wave functions,  $\psi_n(q)$ , is employed, instead of that of the momentum wave functions,  $S_n(p)$ .

The additional factor,  $1/N!$ , while relatively trivial from a mathematical point of view, is of considerable physical importance. When the right side of Eq. (7) is expanded in powers of Planck's constant, and only the first term is

retained, one obtains the following semiclassical expression for  $\sigma$ .

$$\sigma = [1/N!(2\pi h)^N] \int_{-\infty}^{+\infty} \cdots \int e^{-\beta H(p, q)} dp_1 \cdots dp_{3N} dq_1 \cdots dq_{3N}, \quad (8)$$

where  $H(p, q)$  is the classical Hamiltonian function. The entropy of the system is given by the usual formula,

$$S = k[\log \sigma - \beta(\partial \log \sigma / \partial \beta)]. \quad (9)$$

Thus, the factor,  $1/N!$ , in  $\sigma$  provides the necessary term,  $k(N - N \log N)$  in the entropy, after  $\log N!$  has been approximated by Stirling's formula. In classical statistics, this factor,  $1/N!$ , has to be introduced somewhat arbitrarily. Gibbs accomplishes this by counting the group of  $N!$  permutations or specific phases, included in each generic phase only once instead of  $N!$  times. However, when classical statistics is treated as a limiting case of quantum statistics, the whole question is automatically taken care of by the symmetry restrictions imposed upon the wave functions by the Pauli exclusion principle.

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#### The Relative Abundance of the Lithium Isotopes

Although many attempts have been made to measure directly the relative abundance of the lithium isotopes the results so far reported are quite discordant. Bainbridge, in a paper describing his experiments on this problem,<sup>1</sup> has given a critical survey of the other work in this field. In this connection we thought it of interest to report the results of a set of experiments, extending over a period of several weeks, performed in connection with one of our laboratory courses.

A mass-spectrometer was constructed in the form of a glass tube bent in the shape of a semicircle having a filament and accelerating slit at one end and a slit and Faraday collector at the other. The walls of the tube were heavily silvered. The tube was surrounded by a solenoid capable of furnishing a magnetic field of 400 gauss. Lithium ions emitted by a spodumene coating on the filament described a semicircular path of 5.5 cm radius and those entering the collector were detected by means of an electrometer. The resolving power was sufficient to separate isotopes 6 and 7 quite well. The total emission from the filament was measured with a galvanometer. The conditions specified by Bainbridge,<sup>1</sup> that (1) the emission be not limited by space

charge and (2) that ions reach the collector by identical paths, were fulfilled.

The results of a number of determinations for the current ratio  $\text{Li}^7/\text{Li}^6$  all fell between 8 and 9 with an average value of 8.4. The results seemed to be independent of the current density within the errors of measurement and no change in the ratio with age of the filament was perceptible. Our results are in disagreement with those cited in reference (1) but agree with those of Wijk and Koeveringe<sup>2</sup> and Nakamura.<sup>3</sup> These authors found 7.2 and 8, respectively, from band spectra. All measured values are in bad agreement, however, with the abundance calculated from the masses of the isotopes and the atomic weight.

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<sup>1</sup> K. T. Bainbridge, J. Frank. Inst. **212**, 317 (1931).

<sup>2</sup> v. Wijk and v. Koeveringe, Proc. Roy. Soc. **A132**, 98 (1931).

<sup>3</sup> G. Nakamura, Nature **128**, 759 (1931).

#### Multiple Laue Spots

Double Laue spots have been frequently observed when thick crystals composed of atoms of low atomic number were used. Cork<sup>1</sup> has described triple spots observed with crystals of quartz, calcite and Rochelle salt. A Laue

photograph of a slab of crystalline quartz was recently made for the purpose of determining the directions of the

<sup>1</sup> Cork, Phys. Rev. **42**, 749 (1932).