the Dirac delta-function. The error in the previous paper amounted to omitting the factor, $1/N!$, on the right-hand side of the above equation. The summation over the index, n, may be carried out with the aid of the completeness relation, Eq. (5), and the subsequent integrations over the $p'_1 \cdots p'_{3N}$, performed in the usual way.

$$\int_{-\infty}^{+\infty} F(p')\delta(p'-p)dp' = F(p). \qquad (6)$$

Finally, one obtains,

$$\sigma = [1/N!(2\pi\hbar)^N]\int_{-\infty}^{+\infty} \cdots \int \Phi^*(p, q)e^{-\beta H}\Phi(p, q)dp_1 \\ \cdots dp_{3N}dq_1 \cdots dq_{3N}. \qquad (7)$$

This equation becomes identical with Eq. (10) of the previous article, except for the extra factor $1/N!$, when the functions $\Phi(p, q)$, given by Eq. (2) are explicitly introduced. Since Eqs. (20) and (21) are merely modifications of Eq. (10), they must also contain this factor. The same result is obtained when the completeness relation for the configuration wave functions, $\psi_n(q)$, is employed, instead of that of the momentum wave functions, $S_n(p)$.

The additional factor, $1/N!$, while relatively trivial from a mathematical point of view, is of considerable physical importance. When the right side of Eq. (7) is expanded in powers of Planck's constant, and only the first term is

retained, one obtains the following semiclassical expression for σ.

$$\sigma = [1/N!(2\pi\hbar)^N]\int_{-\infty}^{+\infty} \cdots \int e^{-\beta H(p, q)}dp_1 \\ \cdots dp_{3N}dq_1 \cdots dq_{3N}, \qquad (8)$$

where $H(p, q)$ is the classical Hamiltonian function. The entropy of the system is given by the usual formula,

$$S = k[\log \sigma - \beta(\partial \log \sigma/\partial\beta)]. \qquad (9)$$

Thus, the factor, $1/N!$, in σ provides the necessary term, $k(N - N \log N)$ in the entropy, after $\log N!$ has been approximated by Stirling's formula. In classical statistics, this factor, $1/N!$, has to be introduced somewhat arbitrarily. Gibbs accomplishes this by counting the group of $N!$ permutations or specific phases, included in each generic phase only once instead of $N!$ times. However, when classical statistics is treated as a limiting case of quantum statistics, the whole question is automatically taken care of by the symmetry restrictions imposed upon the wave functions by the Pauli exclusion principle.

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The Relative Abundance of the Lithium Isotopes

Although many attempts have been made to measure directly the relative abundance of the lithium isotopes the results so far reported are quite discordant. Bainbridge, in a paper describing his experiments on this problem,[1] has given a critical survey of the other work in this field. In this connection we thought it of interest to report the results of a set of experiments, extending over a period of several weeks, performed in connection with one of our laboratory courses.

A mass-spectrometer was constructed in the form of a glass tube bent in the shape of a semicircle having a filament and accelerating slit at one end and a slit and Faraday collector at the other. The walls of the tube were heavily silvered. The tube was surrounded by a solenoid capable of furnishing a magnetic field of 400 gauss. Lithium ions emitted by a spodumene coating on the filament described a semicircular path of 5.5 cm radius and those entering the collector were detected by means of an electrometer. The resolving power was sufficient to separate isotopes 6 and 7 quite well. The total emission from the filament was measured with a galvanometer. The conditions specified by Bainbridge,[1] that (1) the emission be not limited by space

charge and (2) that ions reach the collector by identical paths, were fulfilled.

The results of a number of determinations for the current ratio Li^7/Li^6 all fell between 8 and 9 with an average value of 8.4. The results seemed to be independent of the current density within the errors of measurement and no change in the ratio with age of the filament was perceptible. Our results are in disagreement with those cited in reference (1) but agree with those of Wijk and Koeveringe[2] and Nakamura.[3] These authors found 7.2 and 8, respectively, from band spectra. All measured values are in bad agreement, however, with the abundance calculated from the masses of the isotopes and the atomic weight.

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[1] K. T. Bainbridge, J. Frank. Inst. 212, 317 (1931).
[2] v. Wijk and v. Koeveringe, Proc. Roy. Soc. A132, 98 (1931).
[3] G. Nakamura, Nature 128, 759 (1931).

Multiple Laue Spots

Double Laue spots have been frequently observed when thick crystals composed of atoms of low atomic number were used. Cork[1] has described triple spots observed with crystals of quartz, calcite and Rochelle salt. A Laue

photograph of a slab of crystalline quartz was recently made for the purpose of determining the directions of the

[1] Cork, Phys. Rev. 42, 749 (1932).

crystallographic axes. The slab, 0.52 cm thick, was mounted so that the face upon which x-rays from a tungsten target tube operated at 55 kv were incident, was 5.0 cm from the photographic film. The spots observed were all either double or triple.

The occurrence of triple spots may be explained by assuming that the crystal is ideally perfect except near the two surfaces where it is more nearly ideally imperfect. Let λ be the wave-length of the x-rays reflected by the Bragg planes parallel to the direction OC. Because of the imperfection of the crystal at the point of incidence, O, the diffracted ray OB will have considerable angular width and will be correspondingly intense. However, because of the
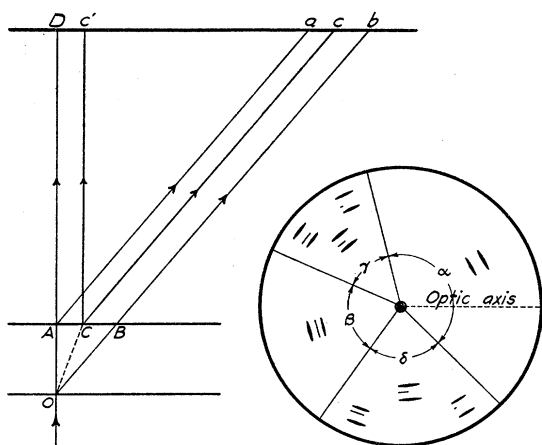

FIG. 1.

perfection of the interior of the crystal there will be a narrow angular range having approximately the direction, OB, within which rays of wave-length λ cannot proceed far without suffering total reflection after which their direction of propagation is approximately parallel to OA.[2] Similarly rays of wave-length λ having directions close to that of OA will suffer total reflection and assume a direction of propagation nearly parallel to OB. It follows that the x-rays lying in this narrow angular range will suffer successive total reflection as they proceed through the crystal. The direction of propagation will always be nearly parallel to either OA or OB but the path can never be far from the

line OC and the energy will emerge from the crystal near the point C in two rays Cc and Cc'. The rays OA and OB will have relatively large angular width but will have no energy within a narrow range of angular width equal to that of the ray Cc. The imperfection of the crystal at A will give rise to a relatively strong diffracted ray Aa. Thus there should appear upon the photographic film three spots on a radial line, the outside spots, a and b, relatively intense with a less intense spot, c, between them. The triple spots observed are of this character.

If the body of the crystal is not ideally perfect, but is made up of small mosaic blocks within each of which primary extinction is effective, no spot will appear at c but instead a slight fogging of the film in a band extending from a to b. This is, the character of the double spots observed.

The absolute position of the outside spot of each set was studied by making a gnomonic projection and the relative positions of the other spots were measured. The inside and outside spots of each set lie in positions a and b within the accuracy of observation. This confirms the conclusions of Colby and Harris[3] who studied the variation of the relative intensity of doublet spots when the crystal surfaces were treated. The intermediate spot was measured for 31 triplets; 21 of these spots lie accurately in position c; 8, slightly inside; and 2, slightly outside this position.

The circular insert shows typical drawings of the multiple spots as they appear on various parts of the photographic film. Note that in regions γ and δ the intermediate spot is cut off on the side toward the α-region, this being more pronounced as the boundary of the α-region is approached. This is the type of distribution to be expected if the incident ray OA traversed the slab close to the edge of a block of nearly perfect crystal.

It seems feasible by this method to locate large perfect blocks within a quartz crystal and to survey their boundaries. Laue photographs of several other specimens have been made in some of which triple spots occur. As yet no other specimen has been found in which such a block extends completely across the crystal.

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[2] Darwin, Phil. Mag. 27, 675 (1914).

[3] Colby and Harris, Phys. Rev. 43, 562 (1933).

On the Magnetic and Gyromagnetic Properties of Pyrrhotite

The g-value and other magnetic properties of pyrrhotite (Fe_7S_8) may be obtained theoretically from the following simple model: We consider one d-electron (or several independent d-electrons) per atom, with fairly large spin-orbit interaction (but not as large as the interatomic exchange energies), in a hexagonal crystal which has the iron atoms in successive principal planes not directly opposite one another, but staggered, or else far apart. In the absence of x-ray data on the positions of the iron atoms in Fe_7S_8, this seems to be a reasonable assumption.

One effect of the large spin-orbit interaction is that, assuming ferromagnetic saturation and its large crystal spin, the lowest states will be those in which the orbital momenta are orientated antiparallel to the spins. Taking projections in the direction of the magnetization, each m_s is $-1/2$ and each m_l is 2. The g-value is then $(2m_s+m_l)/(m_s+m_l) = 1/(3/2) = 2/3$. Incomplete orientation of the orbit would make a deviation toward smaller g-values. For pyrrhotite, Coeterier[1] observes $g = 0.63$.

[1] Coeterier, Helv. Phys. Acta 6, 483 (1933).