The Value of e/m from the Zeeman Effect

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The measurement of e/m from the Zeeman effect of the red singlet lines of Cd and Zn has been repeated with careful attention to all of the sources of experimental uncertainty. The magnetic field has been measured, under the actual conditions of operation, with an uncertainty of about one part in three thousand. The final result is

$$e/m = 1.7570 \pm 0.0010.$$

IN the report published about two years ago on the determination of e/m from the Zeeman effect,[1] the value was given as $e/m = 1.7579 \pm 0.0025$ e.m.u. This work confirmed the "spectroscopic value" in being much lower than the older "deflection values" but was considerably lower than the spectroscopic values previously reported.[2] The precision was not great enough, however, to make sure whether this new discrepancy was real or not. The present paper is a report of the continuance of this work in the effort to reduce the uncertainty of the result. The apparatus and the methods have not been essentially changed so the reader is referred to the previous paper for details of the experimental arrangement.

The major cause of uncertainty in the published value was the uncertainty in the magnetic field. The two methods used in its determination gave slightly different results and so a large part of the present work has been directed toward the removal of this source of uncertainty. We have given especial attention to the following points: (a) the duplication and interchange of all standards and instruments, resistances, standard solenoids, potentiometers, etc., to locate constant errors in the individual pieces; (b) the use of standard resistances and cells recently measured at the Bureau of Standards; (c) careful measurement and control of temperature, both because of its effect on resistance and its effect in causing expansion of solenoids; (d) the calibration of the solenoid both before and after its use in making spectroscopic measurements.

The equation for determining e/m by this method is

$$e/m = (4\pi c/K)(\Delta\nu/I)(1/\bar{a}).$$

We shall take up successively the determination of K, $\Delta\nu/I$, and $\bar{a}$.

MEASUREMENT OF THE MAGNETIC FIELD

The intensity of the magnetic field in absolute gauss was determined from the equation $H = KI$, where K is a constant to be determined experimentally and I is the current. The current was measured by a potentiometer and a standard resistance. The determination of K for weak fields and its use for strong fields is not satisfactory, because the rise of temperature and the mechanical stresses in the solenoid may affect this constant. It has therefore been necessary to determine K under the actual conditions of operation. This determination was made by comparison with long single-layer solenoids, whose constants could be computed from their dimensions.

a. The standard solenoids

Four different solenoids were used in the calibration. Two of these consisted of a single layer of No. 12 bare copper wire wound on linen Bakelite tubes which had been threaded to take ten turns per inch. The other two were wound with No. 20 enameled wire on tubes threaded for 28 turns per inch. One of these

[1] J. S. Campbell and W. V. Houston, Phys. Rev. 39, 601 (1932).

[2] R. T. Birge, Phys. Rev. Supplement 1, 1 (1929).

was wound on linen Bakelite and the other on a brass tube. All of these solenoids were slipped inside the large solenoid and were insulated from it, as was described in the previous paper.

The numbers of turns per centimeter were determined by measurement with a glass cathetometer scale used in such a way that coincidences were observed between the edges of turns and marks on the scale. This scale was calibrated against a Gaertner type M 1001 standard meter at Pomona College and also against a glass decimeter scale which belongs to the Mt. Wilson Observatory and which had been calibrated at the Bureau of Standards. Both calibrations led to the result that our glass scale is uniformly 0.032 percent too long at 21°C.

The Bakelite solenoids have the advantage that it is easy to avoid leaks between the winding and the tube but they also change in dimensions with the season of the year. This is presumably due to the change in humidity and the corresponding change in the moisture content of the Bakelite. This explanation is supported by the fact that all three Bakelite solenoids seem to change together at about the same rate. The turn density of the brass solenoid showed no measurable change at all and after careful winding it was not possible to detect any leak between the winding and the core.

These standard solenoids were usually used at a temperature slightly different from that at which they were measured. Consequently, it was necessary actually to measure the temperature and to hold it fairly constant during a series of readings. A temperature coefficient of 2×10^{-5} was applied to all of the solenoids. This coefficient is near to those of brass, copper and Bakelite. The validity of this correction is apparent from the greater coherence of measurements, made over a temperature range of 30°, when it is applied. Table I gives the data on the solenoids. The constants K_s of these solenoids are determined from the relation

$$K_s = 0.4\pi n \cos\alpha \cos\phi,$$

where n is the number of turns per cm, α is the angle subtended at the center by the radius of the end of the winding and ϕ is the pitch of the winding.

TABLE I. a. *Data on the standard solenoids.*

Solenoid	Effective diameter in cm	Length	$0.4\pi \cos\alpha \cos\phi$
Bakelite No. 1	6.00	89.7	1.25371 ± 3
Bakelite No. 2	5.97	89.5	1.25374 ± 1.5
Bakelite No. 3	5.92	90.4	1.25395 ± 2
Brass	5.86	90.0	1.25397 ± 2

b. *Measurement of Turn Density*

Solenoid	Date	Readings	Turns per cm	K_s
Bakelite No. 1	12/21/31	40	3.9326 ± 4	4.9303 ± 5
	1/29/32	30	3.9340 ± 3	4.9321 ± 4
Bakelite No. 2	12/21/31	50	3.9389 ± 2	4.9384 ± 3
	1/10/32	44	3.9403 ± 7	4.9401 ± 9
	6/20/32	60	3.9353 ± 2	4.9338 ± 3
	3/3/33	18	3.9406 ± 3	4.9405 ± 4
Bakelite No. 3	12/23/31	30	11.0295 ± 15	13.8304 ± 19
	2/4/32	34	11.0322 ± 8	13.8338 ± 11
Brass	12/23/31	30	11.0220 ± 10	
	2/4/32	30	11.0232 ± 5	13.8224 ± 10
	2/1/33	30	11.0235 ± 10	

b. Other measuring apparatus

All currents were measured with standard resistances and potentiometers. Four different potentiometers were used and these were checked against each other. In some cases deflection potentiometers were used but usually the ordinary type was preferred because of their greater sensitivity.

The one and ten ohm shunts were new ones, with certificates from the Bureau of Standards and negligible temperature coefficients. The other shunts were calibrated by the Bureau for several different currents. The same Weston standard cell was used for all measurements. It was compared at various times with new cells from the Bureau and found to be sufficiently constant. A final determination was made in comparison with two new cells kept in a thermostat. Over a three weeks' period the maximum deviation from the mean value was 0.008 percent and the mean value was 1.01857 ± 0.00004 volts.

c. The null method calibration

The methods of making the calibration were described in detail in the previous paper. The null method consists in balancing the field of the large solenoid against that of a standard solenoid which is placed inside of it. The balance is determined from the deflection of a ballistic galvanometer when a flip coil is turned over in the center of the solenoids. If I is the current in the large solenoid when the balance is obtained, and I_s is the current in the standard, then

$$K = K_s I_s / I.$$

Three flip coils were used. One contained 50,000 turns while the others contained 4000 and 7000 turns, respectively. The chief purpose of the null method was to test the reliability of the apparatus, the coherence of the standard solenoids and the constancy of the large solenoid with time. The results are summarized in Table II.

TABLE II. *Null method calibration.*

Solenoid	Date	No. values	Mean I_s/I	K_s	K
Bakelite No. 1	12/23/31	10	7.4768±10	4.9303	36.863±5
Bakelite No. 2	12/23/31	10	7.4665±11	4.9384	36.873±6
	6/28/32	8	7.4728±5	4.9338	36.869±3
Bakelite No. 3	12/24/31	10	2.6659±3	13.8304	36.870±6
Brass	12/25/31	8	2.6679±4	13.8224	36.877±6
	12/5/32	8	2.6677±2	13.8224	36.874±4
				Mean	36.871±5

d. The mutual inductance calibration

The essential point of this method is to compare the throw of a ballistic galvanometer when the flip coil is turned over with its throw when the current is reversed in the primary of a mutual inductance. In this way it is possible to obtain a relation between the strength of the magnetic field and the magnitude of the mutual inductance. By doing this both with the large solenoid and with the standards, K can be determined from the relation $K = K_s(I_s'/I_m')(I_m/I)$. I_s' is the current in the standard solenoid for which the throw is equal to that with I_m' in the primary of the mutual, while I is the current in the large solenoid which gives the same throw as the current I_m in the mutual. The accuracy and reliability of this method depend upon the following factors:

(1) The constant of the galvanometer must not change during the calibration of any one solenoid against the mutual inductance. The effects of all the solenoids on the galvanometer were eliminated by removing it to a room some 40 feet distant. The stray magnetic field at this distance was negligible. Deflections of about constant magnitude and the same direction were always used. With these precautions the galvanometer readings showed the required consistency.

(2) The method also requires that the mutual inductance shall have the same value for a wide range of currents in its primary. It was necessary to change the current by a factor of 200. Two different mutual inductances were used. The first was described in the previous paper. The second was larger and was used in a different location. Both were made without any trace of ferromagnetic material and were placed as far as possible from the floor and ceiling which might have contained steel reinforcement. Both coils led to the same result, as can be seen from Table III. As a further check upon this constancy, the deflection due to the reversal of different currents in the primary was compared with the steady deflection due to constant currents through the galvanometer. This showed that the mutual inductance was constant with an uncertainty of less than 0.03 percent.

TABLE III. a. *Calibration of the mutual inductances with standard solenoids.*

Solenoid	I_s/I_m	K_s	K_sI_s/I_m	Mean
Mutual inductance No. 1 in January, 1932				
Bakelite No. 1	95516±14	4.9321	47109±8	47098±11
Bakelite No. 2	95341±15	4.9401	47099±11	
Bakelite No. 3	34048±7	13.8338	47101±11	
Brass	34063±9	13.8224	47083±13	
Mutual inductance No. 2 in March, 1933				
Bakelite No. 2	27089±3	4.9405	133833±18	133843±21
Brass	96840±17	13.8224	133856±25	

b. *Calibration of the large solenoid in terms of the mutual inductances*

Current amp.	Mutual No. 1 January, 1932 I_m/I	Mutual No. 1 January, 1932 K	Mutual No. 2 March, 1933 I_m/I	Mutual No. 2 March, 1933 K
1	78266±16	36.862±9	27546±8	36.868±11
5			27549±4	36.872±7
15	78266±32	36.862±15		
150	78251±17	36.855±9		
190	78236±33	36.848±15	27529±6	36.846±9
Adopted value of K under operating conditions 36.835±12				

(3) To be sure of the constancy of the flip coils two different ones were used. These gave internally consistent readings and agreed with each other. The flip coils were insulated from the surrounding solenoid by a glass tube.

(4) A fourth and very important factor in this method is the resistance of the galvanometer circuit. If this should change in any way, the deflection would change correspondingly. The two points at which heating could take place are the flip coil and the secondary of the mutual inductance. The flip coil might be expected to absorb heat from the surrounding solenoid, especially when a heavy current is being used. This was avoided by blowing a current of air around the glass tube which enclosed the flip coil. With this arrangement, temperature equilibrium was attained in about ten minutes and was maintained indefinitely. Some heating was found to occur in the secondary of the mutual inductance due to the current passing through it. This was reduced to a negligible amount by immersing the whole mutual inductance in a bath of transformer oil. These precautions kept the change of resistance to less than 0.02 percent during the time of a series of measurements.

Table III gives the results of the calibration with the mutual inductances. The values at the low currents are in good agreement with those obtained by the null method. At the higher currents the constant decreases somewhat. This may be attributed to two causes. In the first place, the mechanical stresses caused by the large currents may distort the winding enough to affect the constant. This effect would tend to lower the constant. In addition to this, the correction for the increase in temperature may be inadequate. All of the values in the table have been reduced to 22°C. The temperature of the solenoid was measured by measuring the temperature of the outside of the brass case and this was probably somewhat lower than the effective temperature in the winding. The measured temperature averaged around 40° for the heavy currents. Since, however, the Zeeman effect measurements and the calibration measurements were made under identical conditions, an error in this correction will have no effect upon the final result. The adopted value of K given in Table III differs from that tabulated for 190 amp. by the amount of this temperature correction and by 0.005 percent to change from international to absolute gauss. This adopted value, then, represents the constant of the solenoid as determined under the actual conditions of operation.

e. The measurement of the current

The other essential factor in the measurement of the magnetic field is the measurement of the current. This was measured by a potentiometer connected across the terminals of a 0.001 ohm resistance. This resistance was calibrated by the Bureau of Standards for two different currents. It showed a change of 9 parts in 10,000 between 60 amp. and 300 amp. This was evidently due to heating and so care was taken always to permit this shunt to attain approximate temperature equilibrium before use. By measuring the temperature changes, it was found that this procedure assured a constancy of this shunt to within 0.01 percent. The actual value of this shunt is of no importance in the final result, since it was used in the same way both for the calibration and for the spectroscopic work.

The constancy of the current was maintained within 0.015 percent by manual control of the current in the field coils of the motor-generator set.

Spectroscopic Measurements

The same interferometer and method was used as was described previously. The source of light was changed in that a stop was placed at the back of the constriction, so that no light could reach the spectrograph except that from the center of the magnetic field. It was also operated at a considerably lower temperature which made the lines somewhat sharper.

Photographs were taken for a range of currents at each separation of the interferometer plates. These currents were so selected as to be more or less evenly divided between those which gave too wide a separation of the components for even spacing in the interferometer pattern and those which gave too narrow a separation. In this way errors due to overlapping of the components were eliminated from the average. The currents used ranged from 175 amp. to 195 amp., while the interferometer separations

TABLE IV. *Spectroscopic measurements.*

Order difference	Cadmium λ6438 Number of plates	$2\Delta\nu/I$	Zinc λ6362 Number of plates	$2\Delta\nu/I$
1.5	2	34356 ± 5		
3.5	12	34350 ± 14	9	34352 ± 22
4.5	18	34346 ± 16	12	34355 ± 19
Means		34348 ± 15		34354 ± 20
$\bar{a}$		0.99978		0.99996
$2\Delta\nu/Ia$		34356 ± 15		34355 ± 20
$2\pi c/K$		51141 ± 17		51141 ± 17
e/m				1.7570 ± 0.0010

were such as to separate the Zeeman effect components from 1.5 to 4.5 orders of interference. Table IV shows the results of these measurements. These results appear to be lower than those previously given by about 0.1 percent. This is due to an error discovered in one of the potentiometers. This error also gives K close to the value previously reported, but, in fact, the difference in the final result should really be attributed to the magnetic measurements. The indicated uncertainties in $\Delta\nu/I$ represent the mean deviations of the values of this quantity from different plates. The uncertainty is essentially that in $\Delta\nu$, although there may be a little due to the uncertainty in the reading of the potentiometer and the control of the current.

The quantity $\bar{a}$ was determined theoretically in exactly the manner previously described. It involves the g factors of the levels involved. The uncertainty ascribed to the value of e/m is only that indicated by the spread of the experimental values and takes no account of the possible uncertainty in the theory of the Zeeman effect. The uncertainty indicated throughout is the mean deviation. Had probable errors been used their values would have been approximately one-half of the given deviations. In all of the tabulated data the agreement between various mean values is very close to that to be expected from the mean deviations.

The value we have obtained is in good agreement with the recent value of Dunnington[3] by a deflection method, and is certainly lower than the previously accepted spectroscopic value. The uncertainty has been reduced as far as it seems practical to go with the present apparatus.

[3] F. G. Dunnington, Phys. Rev. **43**, 404 (1933).