LETTERS TO THE EDITOR

Prompt publication of brief reports of important discoveries in physics may be secured by addressing them to this department. Closing dates for this department are, for the first issue of the month, the twentieth of the preceding month; for the second issue, the fifth of the month. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

An Attempt to Discover Radiation at Low Critical Potentials in Mercury Vapor

Low critical potentials have been observed in mercury below 4.7 volts by Jarvis,[1] by Nielsen,[2] by Pavlov and Sueva[3] and others. Kondratjew[4] has proposed a metastable state at about 2 volts based upon thermochemical considerations. R. G. Loyarte[5] attempts an explanation (not well received) on the basis of the rotation of the mercury atom as a free axial rotator, whose energy values change by multiples of 1.39 volts. This figure fits in pretty well with the differences of critical potentials observed by Jarvis and Nielsen.

The authors undertook a spectroscopic study to determine whether any radiation could be observed at these low potentials. The tubes used were of the same general type as those used by Foote and Mohler (Bureau of Standards) in the stage excitation of the magnesium spectrum. High capacity emission filaments were secured from Dr. E. F. Lowry of the Westinghouse Laboratories. A fast glass spectrograph of low dispersion was used for wave-lengths above 6500A and the Hilger E-3 for wave-lengths below 6500A. The Eastman infrared plates, types *N*, *P*, and *Q* were used. The exposures were from 2 to 10 hours. In all cases except one the results were negative and in this one there is a slight doubt as to whether the effect was identical with that obtained from the filament alone, no mercury present. Work is to be continued from a somewhat different angle.

The authors wish to express their appreciation to the National Research Council and to Ohio Wesleyan University for grants-in-aid and funds to carry on this investigation.

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E. N. Shawhan

Ohio Wesleyan University,
Delaware, Ohio,
June, 1933.

[1] C. W. Jarvis, Phys. Rev. **27**, 808A (1926).
[2] Nielsen, Phys. Rev. **31**, 1123A, 1134A (1928).
[3] Pavlov and Sueva, Zeits. f. Physik **54**, 236 (1929).
[4] Kondratjew, Phys. Zeits. **32**, 288 (1931).
[5] R. G. Loyarte, Phys. Zeits. **30**, 923 (1929).

The Representation of Radiation Reaction in Wave Mechanics[1]

It is well known, from a theorem of E. H. Kennard, that the wave-mechanical ψ equation leads to the conclusion that the centroid of the wave-mechanical electron moves according to the classical electrodynamic equation of motion in which, however, the terms representing what is commonly called radiation reaction are absent. Kennard's demonstration applies only to the nonrelativistic case, but any relativistic ψ equation which is to represent the facts must lead to a similar conclusion with, however, the mass acceleration replaced by $md/dt\{\mathbf{v}/(1-v^2/c^2)^{\frac{1}{2}}\}$, where $\mathbf{v}$ is the velocity of the centroid of the electron. We shall short circuit the difficult question of establishing this theorem, and suppose only that there exists a ψ equation in terms of which it can be established so that in fact the ψ equation leads to

$$\frac{m}{e}\frac{d}{dt}\frac{\dot{\bar{\mathbf{s}}}}{(1-\dot{\bar{\mathbf{s}}}^2/c^2)^{\frac{1}{2}}}=\int\int\int\left(\mathbf{E}+\frac{[\mathbf{uH}]}{c}\right)\rho d\tau, \tag{1}$$

where $\dot{\bar{\mathbf{s}}}$ is the velocity of the centroid, $\mathbf{E}$ and $\mathbf{H}$ are the external electric and magnetic fields, ρ and $\rho\mathbf{u}$ are the charge density (normalized to unity) and the current density, which the wave-mechanical equation itself provides as the proper quantities for this purpose satisfying the equation of continuity.

The equation of motion including radiation reaction terms may be regarded as the equation which is obtained by operating on the left-hand side of (1) with the operator $\{1-\alpha_1 kd/dt+\alpha_2 kd/dt(kd/dt)\cdots\}$ which we shall call P^{-1}. Here α_1, α_2, etc., are constants, and $k=(1-v^2/c^2)^{-\frac{1}{2}}$.

In the ψ equation will occur the vector and scalar potentials $\mathbf{U}$ and φ, in terms of which the electric and magnetic fields are given by $\mathbf{E}=-(1/c)\partial\mathbf{U}/\partial t-\text{grad }\varphi$ and $\mathbf{H}=\text{curl }\mathbf{U}$. In general, the sole property which need be invoked of $\mathbf{U}$ and φ in order that they shall preserve the invariance of the equation is that U_x, U_y, U_z, $ic\varphi$ shall form a 4-vector. Such other properties as restrict them to

[1] A more extended exposition of the content of this paper will appear in an early issue of the Journal of The Franklin Institute.

satisfy the relations div $\mathbf{U} = -(1/c)\partial\varphi/\partial t$; $\nabla^2\mathbf{U} - (1/c^2)\partial^2\mathbf{U}/\partial t^2 = 0$; $\nabla^2\varphi - (1/c^2)\partial^2\varphi/\partial t^2 = 0$, while important for endowing them with their usual significance in classical electromagnetic theory, play no part in the story of the invariance, even though they may play a part in the derivation of the ψ equation itself from certain assigned postulates. Hence, if in the ψ equation we replace $\mathbf{U}$ and φ by quantities obtained from them by operating with an invariant operator, the quantities so obtained, being still 4-vectors, will provide for the invariance of the new ψ equation. The operator kd/dt is an invariant operator when operating on a 4-vector, and P which is a function of kd/dt is an invariant operator. Hence, if in the ψ equation $\mathbf{U}$ and φ are replaced by $\mathbf{U}_1 = P(\mathbf{U})$ and $\varphi_1 = P(\varphi)$, the invariance will be preserved. The fact that the unmodified equation leads to (1) now results in the modified equation leading to (1) with $P(\mathbf{E})$ and $P(\mathbf{H})$ replacing $\mathbf{E}$ and $\mathbf{H}$. For a case where $\mathbf{H}$ is sensibly constant throughout the electron, it can then easily be shown that the modified right-hand side of (1) assumes the form $P(\mathbf{E}) + [P(\mathbf{H})\mathbf{v}]/c$.

If we now take a system of axes in which the centroid is at rest, and if zero subscript refers to this system of axes, the equation assumes the form

$$(m/e)(d\dot{\mathbf{s}}_0/dt_0) = P_0(\mathbf{E}_0),$$

which after operation on both sides by P_0^{-1} yields

$$(m/e)P_0^{-1}(d\dot{\mathbf{s}}_0/dt_0) = \mathbf{E}_0.$$

On transforming back to the system of axes in respect to which the centroid moves with velocity $\mathbf{v}$, we obtain the desired equation with the radiation reaction terms, *viz.*,

$$\frac{m}{e}P^{-1}\frac{d}{dt}\frac{\mathbf{v}}{(1-v^2/c^2)^{\frac{1}{2}}} = \mathbf{E} + \frac{[\mathbf{vH}]}{c}.$$

The use of $\mathbf{v}$, the velocity of the centroid, in the operator P is admittedly artificial, although it provides a logically consistent story. However, it is possible to carry through the argument in a form in which $\mathbf{v}$ in this operator is replaced by the velocity $\mathbf{u}$ which occurs in $\rho\mathbf{u}$, the expression for the current density. The velocity $\mathbf{u}$ varies of course throughout space. A few restrictions have to be placed upon the argument in this form; but, it is believed that they are of a trivial nature.

The purpose of the whole investigation is not to establish the equation of motion with the radiation reaction terms, but to use the power to yield that equation as evidence in favor of the ψ equation which secures that end. It is admitted that such a method of deciding upon the ψ equation is not unique, but it is one which assumes a very natural relation to the mathematical structure of the theory. It is also realized that the spirit of the foregoing demonstration is that of the older forms of wave-mechanical theory rather than that of the more modern forms as typified by Dirac's theory. However, it is believed that the differences in the matter under discussion may be more apparent than real. If one admits the foregoing method of realizing the modified ψ equation, the ultimate result is simple in statement, and consists in replacing the perturbing field specified by $\mathbf{E}$ and $\mathbf{H}$ by a field specified by $P(\mathbf{E})$ and $P(\mathbf{H})$. Thus, for example, in the simplest case, for small velocities, $\mathbf{E}$ becomes replaced by $\mathbf{E} + \alpha\partial\mathbf{E}/\partial t$ as a first approximation.

W. F. G. SWANN

Bartol Research Foundation
of The Franklin Institute,
October 22, 1933.

Thermoelectric Force of Thin Films

We have recently had occasion to sputter very thin thermocouples of antimony and bismuth and have found a significant change of thermoelectric e.m.f. with varying thickness of bismuth. On the other hand, no change of e.m.f. with varying thickness of antimony has been observed.

Each metal was sputtered in purified argon on mica through suitable templates. Heavy layers of gold were sputtered over the ends of each metal to provide good electrical contact. The sputtering chamber was cleaned between each sputtering to assure freedom from contamination.

The e.m.f. of the thermocouples was measured within a day of manufacture with a Leeds and Northrup Type K potentiometer. The resistance of the couples was not sufficient to impair the sensitivity of the measurements. The couples were mounted in an apparatus composed of three sets of heavy copper blocks bored to permit circulation of water. The hot junction was in contact with the middle set, the two cold junctions with the outer sets. Temperatures were determined by thermometers immersed in the blocks. The reliability of the temperature measurements was tested with a copper-constantan couple.

All values given below for thermal e.m.f. are average values of several sets of measurements at different temperature differences between the hot and cold junctions.

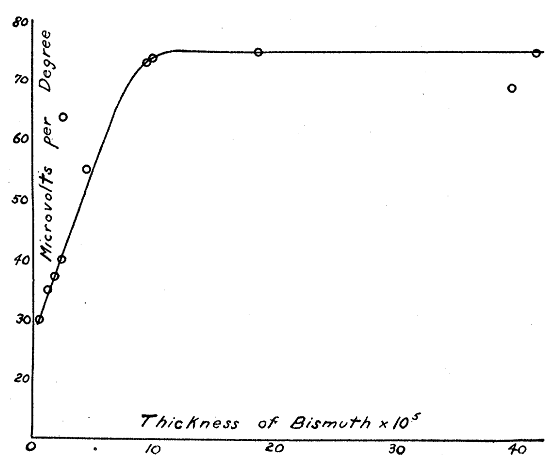

FIG. 1.