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Electro-Optical Kerr Effect in Gases

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The Kerr constant B of CO_2 has been measured as a function of density from 0.06 to 0.32 g/cm^3 and as a function of temperature from 7.2 to 34.6°C. The results are in agreement with the Langevin-Born theory. The absolute value of B for CO_2 has been measured and is 0.34×10^{-10} per atmosphere for wave-length 4550A and temperature 34.6°C. This is also the theoretical value of B for CO_2. The probable error of the measured value is 2 percent. The absolute value of B for N_2 measured by comparison with CO_2 is 0.057×10^{-10} for the above wave-length and temperature, with a probable error of 3 percent. The corresponding theoretical value for N_2 is 0.050×10^{-10}. The Kerr constant of H_2 is shown to be less than 0.01×10^{-10} which is not in disagreement with the theoretical value of 0.005×10^{-10}.

I. INTRODUCTION

IN 1875 Kerr[1] discovered that if an optically isotropic substance be placed in an electric field it acquires the optical properties of a uniaxial crystal with the optic axis in the direction of the electric field. The law governing this phenomenon, proposed by Kerr, states that if n_1 and n_2 are the refractive indices for the components of the light vector vibrating parallel and perpendicular to the lines of force in the substance, respectively, their phase difference D in radians after passing through the field E, in absolute electrostatic units, is

$$D=2\pi l(n_1-n_2)/\lambda=2\pi BlE^2, \qquad (1)$$

where l is the length of the light path through the field, λ is the wave-length of the light in a vacuum and B is the so-called Kerr constant of the substance.

The theory of the Kerr effect as worked out by Langevin,[2] Born[3] and others connects the Kerr constant with the index of refraction, dielectric constant and the light scattering coefficient (depolarization constant).[4, 5]

Langevin assumes that the molecules are electrically and optically anisotropic and, as a result of the orienting action of the electric field upon the induced doublets in the molecules, the substance as a whole becomes doubly refracting. Born extended the theory to include the permanent moment of the molecule. Raman and Krishnan[6] have further modified the Langevin-Born theory so that it becomes applicable to liquids in which the contribution of the molecules to the field becomes large. The quantum mechanics of the Kerr effect has yielded little more than these classical theories.[7]

From the Langevin-Born theory, for a nonpolar gas

$$B\equiv B_mN=\pi N(n^2+2)^2(\epsilon+2)^2\theta/27n\lambda, \qquad (2)$$

$$\theta=(1/45kT)[(a_1-a_2)(b_1-b_2)+(a_2-a_3)(b_2-b_3)+(a_3-a_1)(b_3-b_1)], \qquad (2a)$$

where B_m is the Kerr constant per molecule; N, the number of molecules per unit volume; n, the ordinary index of refraction; ϵ, electrostatic value of dielectric constant; k, Boltzmann's constant; T, absolute temperature; $b_1b_2b_3$, moments induced in the molecule along its three principal axes of optical anisotropy by unit electrical force in the light wave acting along the three axes respectively; and $a_1a_2a_3$, moments induced along the same axes by unit electrostatic force. From (2) and (2a)

$$B\equiv B_mN\propto N(n^2+2)^2(\epsilon+2)^2/n \qquad (3)$$

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[1] Kerr, Phil. Mag. (4) **50**, 337 (1875).

[2] Langevin, Le Radium **7**, 249 (1910).

[3] Born, Ann. d. Physik **55**, 177 (1918).

[4] See Beams, Rev. Mod. Phys. **4**, 133 (1932).

[5] See Szivessy, Handbuch d. Physik Vol. XXI, Julius Springer (1929).

[6] Raman and Krishnan, Proc. Roy. Soc. **A117**, 1, 589 (1927).

[7] See Van Vleck, *Electric and Magnetic Susceptibilities*, p. 366, Oxford University Press (1932).

if λ and T are held constant while the density is varied. It is also seen from (2) and (2a) that if N and λ are held constant and T is varied $BT = \text{constant}$, since, for a nonpolar substance, ϵ and n are practically independent of the temperature.

From a result of the theory of light scattering by molecules[8] and an assumption due to Gans[9] it may be shown that for a nonpolar gas at low densities,

$$B=\frac{(n^2-1)(\epsilon-1)}{8\pi Nn\lambda kT}\,\frac{3\Delta}{6-7\Delta}, \tag{4}$$

where Δ is the light-scattering coefficient (depolarization constant) of the molecule. Formula (4) is sufficiently accurate for this experiment.

The theory for the transmission of plane polarized light through the Kerr cell, which is the same as through a uniaxial crystal, shows that the intensity of the Kerr beam is

$$I=I_0 \sin^2 (D/2), \tag{5}$$

where I_0 is the intensity of light through the Kerr cell and D is the phase difference in radians between the ordinary and the extraordinary ray. Using the value of D from (5) in (1) gives

$$B=(1/\pi lE^2) \arcsin (I/I_0)^{\frac{1}{2}}, \tag{6}$$

which for most work in gases may be written

$$B=(1/\pi lE^2)(I/I_0)^{\frac{1}{2}}. \tag{7}$$

From (7)

$$B \propto I^{\frac{1}{2}} \tag{8}$$

if I_0E, and l are constant. Eq. (7) is the one by which B is determined from experiment. The error in using (7) instead of (6) in reducing the measures in this experiment is never greater than 0.1 percent.

Three tests may be applied to test the validity of the Langevin-Born theory. These are: (1) a comparison of the experimental value of the Kerr constant as a function of density with the theoretical value as given by Eq. (3), (2) the examination of the constancy of BT when the density and the wave-length of the light are held constant while the temperature is changed and (3) a comparison of the absolute values of B given by Eq. (4) with the values given by experiment using Eq. (7). Needless to say experimental values of n and ϵ are used in Eqs. (3) and (4). These three tests may be taken as the object of this paper.

II. Experimental Arrangement

A modification of the method of Stevenson and Beams[10] is used. The novelty of their method consists in placing the polarizer and analyzer inside the Kerr cell containing the gas under high pressure. This method eliminates the effect of strains in the windows of the Kerr cell and allows higher field strength to be used. On account of difficulty with the nicol prisms under high pressure they were replaced by a double image polarizer and analyzer. Fig. 1 shows the optical system of the final arrangement. C_1 is a photronic cell used to correct for small variations in the light source F. The light from F is collimated by L_1. The stop S_1 limits the size of the beam of light entering the polarizer P which is a calcite crystal, set so that the plane of polarization of the light makes an angle of 45° with the field between the plates K of the Kerr cell. S_2 cuts out the extraordinary ray. The analyzer is a Cotton[11] prism made of crystal quartz. It is set parallel to the polarizer so that with no field on the Kerr cell there is a single beam of light coming from the analyzer, called hereafter the light beam. With the field applied light through the Kerr cell

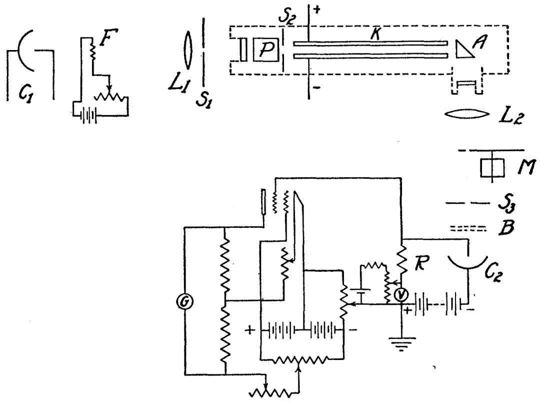

FIG. 1. Optical system and circuit of d.c. amplifier.

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[8] See Marx, *Handbuch der Radiologie*, p. 781.

[9] Gans, Ann. d. Physik **65**, 97 (1921).

[10] Stevenson and Beams, Phys. Rev. **38**, 133 (1931).

[11] Cotton, Comptes Rendus **193**, 268 (1931).

becomes elliptically polarized so that two beams of light leave the analyzer, the ordinary ray comparable in intensity with the light beam and the extraordinary ray of small intensity, hereafter called the Kerr beam. The lens L_2 brings the light to a focus at S_3, which may be set to cut out the ordinary ray but to allow the Kerr beam to pass through the filter B to the evacuated potassium hydride photoelectric cell C_2, used in conjunction with a d.c. amplifier to measure the light intensity. A rotating sector M driven at a high speed by a motor may be placed in the path of the light beam to cut down its intensity to approximately that of the Kerr beam so that by a slight shift of S_3 it could also be measured to obtain absolute values of the Kerr constant. The plates K are held in position by Bakelite rings and held apart by Bakelite separators 4 mm thick. The plates are 90 cm long and 2.5 cm wide. Electrical connections with the plates are made through ordinary spark plugs, the bases of which are soldered into holes bored in the tube.

Fig. 1 also gives the electrical circuit of the d.c. amplifier. By a separate experiment the deflection of the galvanometer was found to be proportional to the light falling on C_2 in the range used. The resistance R is 10^{10} or 10^{11} ohms depending on the sensitivity required. This entire circuit was placed inside a metal box for shielding. The inside of the box was kept dry by P_2O_5. This circuit is a modification of the one given by DuBridge.[12]

Fig. 2 shows the construction of the analyzer end of the Kerr cell. Copper washers at C and lead foil at L form gas-tight seals. The window W is glass 3 cm thick and 3 cm in diameter. By the worm gear G the analyzer may be set accurately parallel to the polarizer. The barrel of the Kerr cell is a cold-drawn steel tube 3.8 cm inside in diameter with walls 1.25 cm thick.

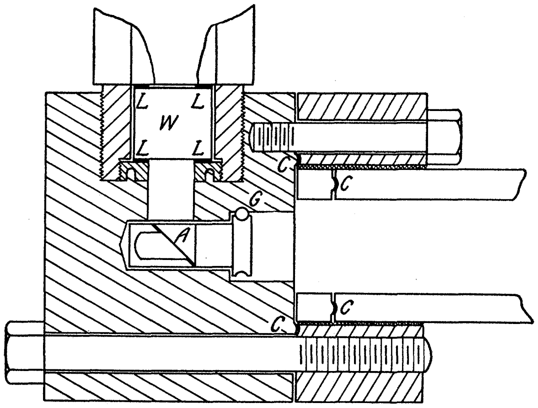

FIG. 2. Construction of end of Kerr cell.

The high voltage applied to the Kerr cell was obtained by rectifying the current from a five kilowatt x-ray transformer. The high voltage circuit is essentially the same as that of Stevenson and Beams.[13] The voltage applied to the plates of the Kerr cell was measured by a Wulf electrostatic voltmeter capable of measuring up to 50,000 volts with an accuracy of 0.5 percent. The period of the quartz fiber of the voltmeter is such that it will detect a 60-cycle ripple in the voltage. Under the conditions of this experiment the ripple was too small to be detected.

The gas pressures were measured to 0.1 atmosphere by a carefully calibrated bourdon spring gauge used in conjunction with a dead weight piston gauge. Amagat's[14] data on CO_2 were used to turn pressures into densities.

The gases used were from commercial cylinders but tests on CO_2 showed it to be better than 99.5 percent pure. The purity of the hydrogen and nitrogen is not known. The gases were dried by allowing them to pass through P_2O_5 before being used.

III. Experimental Procedure and Results

The variation of the Kerr effect with density was determined without the rotating sector and was standardized by setting the analyzer slightly off the parallel position. In this way about 0.2 percent of the light through the Kerr cell goes along with the Kerr beam to the photoelectric cell. This light measured with the field off furnishes a complete check on the line-up of the optical system as well as a check on the sensitivity of the amplifier.

The variation of B with density and the absolute values were measured at a temperature of 34.6°C held constant by placing the cell in an electrically heated water bath, the temperature of which was regulated by a mercury-in-glass

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[12] DuBridge, Phys. Rev. **37**, 392 (1931).

[13] Stevenson and Beams, Phys. Rev. **38**, 133 (1931).

[14] Amagat, Comptes Rendus, 663 (1885).

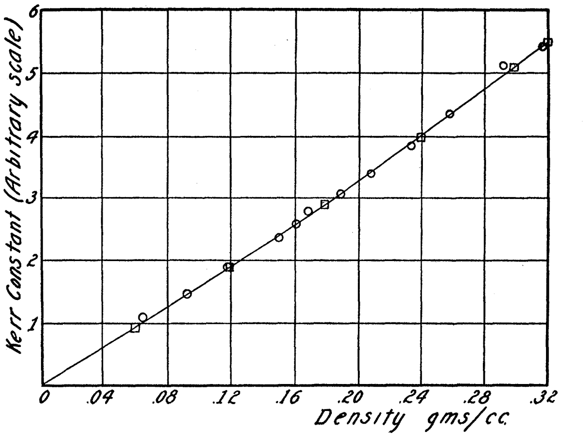

FIG. 3. Variation of Kerr constant with density.

thermostat. It is estimated that the temperature varied less than 0.01°C.

Fig. 3 shows the variation of the Kerr effect with density in CO_2 compared with the variation as computed from Eq. (3). The constant of proportionality in (3) was so chosen as to make the theoretical curve agree with the experimental at one point other than at the origin. The squares represent points computed by Eq. (3). Values for ϵ were taken from the data of Uhlig, Keyes and Kirkwood[15] and n from Phillips[16] measures on the index of refraction for 4358A. The average deviation of the experimental points from the curve drawn from theory is less than 1 percent if one omits the first measure for which the Kerr effect is small.

Fig. 4 shows the variation of the Kerr constant per molecule with density in CO_2. Again the curve drawn is a theoretical one computed from Eq. (3). The Kerr constant per molecule increases with density as predicted by theory.

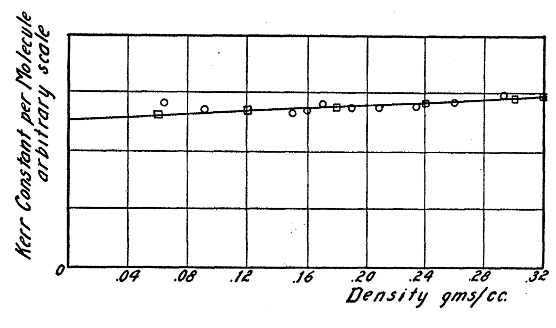

FIG. 4. Variation of Kerr constant per molecule with density.

Table I gives measurements on the variation of the Kerr constant with temperature in CO_2. BT is constant within the limits of error *of the measurements*, estimated to be approximately 0.5 percent.

TABLE I. *Variation of Kerr constant with temperature.*

Temperature (absolute)	307.6	280.2
Density, g/cm^3	0.1035	0.1035
(Intensity of Kerr beam)$^{\frac{1}{2}}$	1.36	1.50
T (Intensity of Kerr beam)$^{\frac{1}{2}}$ or BT	418.	420.

The absolute value of B for CO_2 was measured at 34.6°C for a density of 0.1775 g/cm^3 and wave-length 4550A. This value of B reduced to a density corresponding to a pressure of one atmosphere is 0.34×10^{-10} and is probably not more than 2 percent in error. The most uncertain quantity is the wave-length of light used. A Zeiss C filter gave a narrow band of wave-lengths, the effective wave-length of which depends on the character of the source and the variation of the sensitivity of the photo-cell with wave-length. With a monochromator an intensity wave-length curve was obtained from which the effective wave-length was determined to be 4550A.

For comparison with the previously measured value of B for CO_2 it is necessary to reduce the above value to wave-length 5890A and 17.5°C. From the data of Phillips[16] on n and from Eq. (4) the reduction gives 0.28×10^{-10}. The theoretical value of B computed by (4) is 0.28×10^{-10}. The experimental value obtained by Szivessy[5] is 0.24×10^{-10}. The agreement here between experiment and theory shows the essential correctness of the theory. From the variation in the values of the light scattering coefficient Δ as determined by different observers[17] it would appear that for non-polar gases the light-scattering coefficient might be determined more accurately from measures of the Kerr constant than from direct measures.

The Kerr constant B was measured for N_2 in terms of that for CO_2 since the value of B for N_2

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[15] Uhlig, Keyes and Kirkwood, J. Chem. Phys. **1**, 155 (1933).

[16] Phillips, Proc. Roy. Soc. **A97**, 226 (1920).

[17] Wien-Harms, Handbuch der Exp. Physik, Vol. XIX.

is so small as to make the use of the rotating sector impractical. The Kerr constant for N_2 is one-sixth that of CO_2, or $B(N_2)$ is 0.057×10^{-10} for 4550A and 34.6°C. The probable error of this value is 3 percent. Formula (4) gives $B(N_2) = 0.050\times10^{-10}$. This measure was made at a density of 0.0915 g/cm^3.

An attempt was made to measure the Kerr constant of H_2. With a pressure of 82 atmospheres and a field of 70,000 volts per cm the effect was too small to be detected. This means that the Kerr constant of H_2 is less than 0.01×10^{-10}, which is what one would expect, the theoretical value given by (4) being 0.005×10^{-10}.

IV. ACKNOWLEDGMENTS

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