where

$$\tfrac{1}{2}mu'^2=\chi_0-h\nu,\qquad a=m/2kT,\qquad \log A(\text{Fermi})=\epsilon^*/kT.$$

If u^* is defined by $Ae^{-au^2}=1$, two cases arise as usual, namely $u'>u^*$ and $u'<u^*$, wherein the logarithm may be expanded in powers of Ae^{-au^2} and its reciprocal, respectively, with the result that probability integrals occur for which series approximations may be written at once. The results are:

$u'>u^*$ $(h\nu<h\nu_0)$:

$$N_B=[(2m)^{\frac{3}{2}}\pi k^2T^2/h^3\epsilon^{*\frac{1}{2}}]\{s_2(e^x-e^{2x}/4+e^{3x}/9-\cdots)$$
$$-\tfrac{1}{2}\gamma s_3(e^x-e^{2x}/8+e^{3x}/27-\cdots)+\cdots\},\quad (1)$$

$u'<u^*$ $(h\nu>h\nu_0)$:

$$N_B=[(2m)^{\frac{3}{2}}\pi k^2T^2/h^3\epsilon^{*\frac{1}{2}}]\{s_1\chi^2/2+\pi^2/6$$
$$-s_2(e^{-x}-e^{-2x}/4+\cdots)$$
$$-\tfrac{1}{2}\gamma s_3(e^{-x}-e^{-2x}/8+\cdots)+\cdots\},\quad (2)$$

where

$$x=(\epsilon^*-\epsilon')kT=h(\nu-\nu_0)kT,$$
$$\gamma=kT/\epsilon^*=kT/(\chi_0-h\nu_0),$$
$$s_1=1+\delta/3+3\delta^2/16+\delta^3/8+\cdots,$$
$$s_2=1+\delta/2+3\delta^2/8+5\delta^3/16+\cdots,$$
$$s_3=1+3\delta/2+15\delta^2/8+35\delta^3/16+\cdots,^1$$
$$\delta=(\epsilon^*-\epsilon')/\epsilon^*.$$

The two expressions are given to the first approximation in $\gamma(=kT/\epsilon^*)$ which is always small (of the order of 0.04 at most). Eqs. (1) and (2) differ from Fowler's (a) in the additional γ-term, and (b) in the introduction of the series in δ, which when multiplied by $1/(\chi_0-h\nu_0)^{\frac{1}{2}}$ replace the factor $1/(\chi_0-h\nu)^{\frac{1}{2}}$ and provide the desired correction for frequencies away from the threshold.

Using Eqs. (1) and (2) in further development after Fowler's method, on the assumption that the photoelectric current $I\propto N_B$:

$$I=A'T^2(\chi_0-h\nu_0)^{-\frac{1}{2}}f'(x),$$

where

$$f'(x)=s_2(e^x-e^{2x}/4+e^{3x}/9+\cdots)$$
$$-\tfrac{1}{2}\gamma s_3(e^x-e^{2x}/8+e^{3x}/27-\cdots)+\cdots,\qquad x\leqq 0$$
$$=s_1x^2/2+\pi^2/6-s_2(e^{-x}-e^{-2x}/4+\cdots)$$
$$-\tfrac{1}{2}\gamma s_3(e^{-x}-e^{-2x}/8+\cdots)+\cdots,\qquad x\geqq 0.$$
$$\log I/T^2=B'+F'(x),$$

where

$$F'(x)=\log f'(x),$$

and $B'=\log A'-\frac{1}{2}\log(\chi_0-h\nu_0)$, whose last term is strictly constant.

The function $F'(x)$ is seen to be dependent upon δ and γ as well as upon their ratio $x(=\delta/\gamma)$, and thus its universal character is gone to the extent of this higher approximation. When $\delta=0$ (at the threshold frequency, hence $x=0$) $F'(x)$ reduces to Fowler's $F(x)$ except for the added term in γ. In extreme cases (T large and ϵ^* small) this might amount to 2 percent of $f(x)$ or a difference, $F'-F=-0.01$. The deviations of $F'(x)$ from $F(x)$ depend upon the value of δ. Practical limitations on current measurement prevent δ from becoming more negative than -0.1. Such would be the case for Cs where $\lambda_0=6600$A, $\epsilon^*=1.53$ e.v. (assuming one free electron per atom) and incident light of $\lambda=7170$A. Then $F'-F$ would vary from -0.01 at $\delta=0$ to $\log(s_2+\frac{1}{2}\gamma s_3)=-0.016$ at $\delta=-0.1$.

On the short wave-length side of the threshold δ may become 0.2 for Ni illuminated with 2050A light, or 0.4 for Cs illuminated with 5000A light. In the case of Ni, $F'-F$ varies from -0.01 at $\delta=0$ to 0.03 at $\delta=0.2$, while for Cs, $F'-F$ varies from -0.01 to 0.07 at $\delta=0.4$, the upper limit being almost independent of temperature in each case. By examining Eqs. (1) and (2) it is seen that for values of x greater than 5, the γ term is negligible and $F'-F=\log s_1$, a function of δ only. So there is a slight change of shape in Fowler's theoretical curve to which experimental curves are shifted, depending upon the distance from the threshold.

If DuBridge's method[2] of plotting isochromatic instead of isothermal curves is used, for each curve there will be a constant δ and an almost constant difference $F'-F$ which produces a negligible change of shape and hence is absorbed in the arbitrary vertical shift. This method then introduces less error for frequencies far from the threshold.

If at any time the exact value of the proportionality constant between photoelectric current and the number of available electrons becomes of value, it will be given more accurately by this extension of Fowler's analysis.

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June 16, 1933.

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[1] These series converge for $\delta<1$.

[2] DuBridge, Phys. Rev. **39**, 108 (1932).

Nuclear Spin and Magnetic Moment of Sodium from Hyperfine Structure

Until recently attempts to determine the nuclear spin of sodium had led to inconsistent results. Recently Rabi and Cohen[1] from a Stern-Gerlach field deflection method and Joffe and Urey[2] from alternating intensities in band spectra have reported the nuclear spin of sodium to be 3/2. The writers have made measurements of the relative intensities of the hfs components of the sodium D lines which also yield 3/2 for the nuclear spin. In this research a liquid air cooled Schüler tube with argon as a foreign gas was used as a source. The hfs was resolved by means of a glass Fabry-Perot interferometer with spacers ranging

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[1] Rabi and Cohen, Phys. Rev. **43**, 582 (1933).

[2] Joffe and Urey, Phys. Rev. **43**, 761 (1933).

from 14 mm to 25 mm. As has been reported by Schüler[3] each D line was found to consist of two components the separation between which is approximately the splitting in the $3\,^2S_{\frac{1}{2}}$ level since the splitting in the upper levels ($3\,^2P_{\frac{3}{2}}$ and $3\,^2P_{\frac{1}{2}}$) are too small to be resolved. A print of the Fabry-Perot pattern of the D lines is shown in Fig. 1. The j value for the lowest state is $\frac{1}{2}$, so that the number of resolvable components in both D_2(λ5890) and D_1(λ5896) is $(2j+1)=2$ independent of the value of the nuclear spin. In order to determine the spin it was thus necessary to resort to intensity measurements of the hfs components.

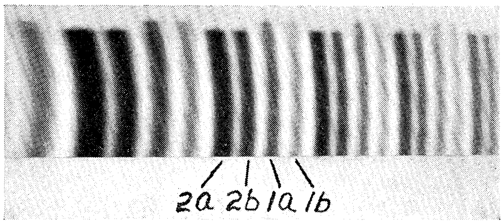

FIG. 1.

The relative intensity of the components as given out by the source was determined photographically by comparing them with intensity marks put on the plate by means of a tungsten filament lamp. In doing this considerable care was taken to determine and correct for the influence of the diffuse background which is always present in the pattern formed by the Fabry-Perot interferometer.

In order to interpret the results of these measurements it was necessary to determine the effect of absorption in the source. This was done by measuring the relative intensity of D_2 and D_1 and by making use of formulas and tables given by Ladenburg and Levy.[4] For any value of the ratio D_2/D_1 could be calculated the relative intensities of the hfs components given out by the source, for any assumed value of the nuclear spin. By comparing the measured ratios with those calculated for the same value of D_2/D_1 but for various values of the spin, the nuclear spin could be determined. The data from one of the plates are given in Table I. D_{2a} and D_{2b} are the components of D_2 and D_{1a} and D_{1b} those of D_1. These and the other data obtained are consistent only with a nuclear spin of 3/2.

TABLE I.

	$\frac{D_{2a}}{D_{2b}}$	$\frac{D_{1a}}{D_{1b}}$	$\frac{D_{2b}}{D_{1a}}$	$\frac{D_{2a}+D_{2b}}{D_{1a}+D_{1b}}$
Average measured ratios	1.62	1.59	1.19	1.92
Calculated ratios	1.90	1.95	1.00	for $i=1$
	1.59	1.63	1.19	$i=\frac{3}{2}$
	1.45	1.48	1.31	$i=2$

By careful measurements on the separations between the components of D_2 and D_1 a determination of the total hfs splittings in the $3\,^2P_{\frac{1}{2},\frac{3}{2}}$ levels, which are too small to be resolved directly, has been made. Because of the hfs of the $3\,^2P_{\frac{3}{2}}$ and $3\,^2P_{\frac{1}{2}}$ levels the separation between the components of D_2 and D_1 is not quite the same. After having determined the nuclear spin and by using the theoretical relation between the interval factors of the $3\,^2P_{\frac{3}{2}}$ and $3\,^2P_{\frac{1}{2}}$ levels, it is possible to compute the ratios of the splittings of these levels to the resultant difference in the separations between the resolvable components of the D_2 and D_1 lines. The results of the measurements are

$$\Delta\nu(D_2)=0.0555\pm0.001\text{ cm}^{-1},$$

$$\Delta\nu(D_1)=0.0612\pm0.001\text{ cm}^{-1},$$

$$\Delta\nu(D_1)-\Delta\nu(D_2)=0.0057\text{ cm}^{-1}.$$

The writers feel that the accuracy in this difference is better than that in either interval because in the method employed it is not likely that one interval is overestimated and the other underestimated. The difference obtained above is consistent with a total splitting of 8.3×10^{-3} cm^{-1} for the $3\,^2P_{\frac{1}{2}}$ level and $3/5\times8.3\times10^{-3}=5.0\times10^{-3}$ cm^{-1} for the $3\,^2P_{\frac{3}{2}}$ level. This splitting obtained for the $3\,^2P_{\frac{3}{2}}$ level is in fair agreement with results obtained by Heydenburg and Ellett[5] for this same level from polarization of resonance radiation. From this information it may be concluded that the magnetic moment of the sodium nucleus is approximately 2.6/1840 Bohr magnetons.

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June 16, 1933.

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[3] Schüler, Naturwiss. **16**, 512 (1928).

[4] Ladenburg and Levy, Zeits. f. Physik **65**, 189 (1930).

[5] Heydenburg and Ellett, Bull. Am. Phys. Soc. **8**, 23 (1933). (Paper presented at the Chicago meeting of the Physical Society, June, 1933.)

The Isotopes of Calcium by the Magneto-Optic Method

We have checked a large number of the minima reported by Allison[1] to an accuracy of 0.02 units. In addition to the two isotopes which Allison reports for calcium, we always find a faint third minimum. Thus in a dilute HCl solution which shows no minima in the calcium region, we find, upon the addition of a trace of calcium chloride, minima located at 18.45, 18.59, and 18.66, expressed in terms of Allison's units. The values reported by Allison are 18.44 and 18.66. From the position of the third minimum we believe it is due to the presence of Ca^{41}.

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June 17, 1933.

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[1] F. Allison, J. Am. Chem. Soc. **52**, 3798 (1930).

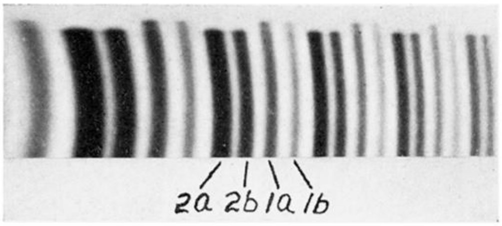

Fig. 1.