On the Existence of Time Lags in the Faraday Effect

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The nature of the initial or surge currents, and of the oscillatory currents, found in a magneto-optic apparatus similar to that used by F. Allison, has been investigated. Assuming the existence of a time lag in the Faraday effect, the nature of minima which could be produced by these currents is examined. The possible minima are found to be extremely broad, a matter of several meters rather than a few centimeters, and have such a small intensity change for a unit movement of the trolley as to be incapable of detection. Large and easily visible minima characteristic of the organic liquids used have been observed, whose explanation is found in terms of Verdet constant and a change in magnitude of current as a result of trolley movement. The effect of concentration on the appearance of minima arising from a Faraday effect is found not to agree with the experimental results of Allison. No sharp minima were found either with visual or photoelectric observation, and the conclusion is reached that the cause of the results of Allison is not a time lag in the Faraday effect, but is some as yet unknown cause, possibly a peculiar sort of Kerr phenomenon.

J. W. BEAMS and F. Allison,[1] in an attempt to explain certain results obtained on organic liquids with a magneto-optic apparatus, advanced the idea that a small but measurable time existed between the application of a magnetic field to the liquid and the resultant rotation of the plane of polarization of light passing through the liquid parallel to the magnetic field. Allison has carried on these original experiments, and has devised a unique method of chemical analysis which he reports capable of detecting compounds when in concentrations as low as one part in 10^{11} of the solvent.[2] Allison's explanation of his experimental results is that there exists a measurable time lag in the Faraday effect, the time lag being characteristic of the chemical compound, and he has reported numerous proofs to support this idea and to show that other hypotheses are inadequate.[2]

The apparatus used in experiments performed at Cornell University and at the University of Minnesota is similar to that described by Allison and Murphy[3] and is shown in Figs. 1 and 2. The variable condenser C (which was usually adjusted to a value near 0.0005 mf) is charged from the source of high potential (25 kv Thordarsson transformer with kenotron as rectifier) until the voltage across the spark gap has risen to a value sufficient to produce a breakdown. (The gap distance was usually fixed at 4 mm, between ends of narrow magnesium rods.) The number of breakdowns per second may be varied by a rheostat in series with the transformer primary; about 60 per second is a good number. Too great a number renders the spark very unsteady. L_1 and L_2 represent coils consisting of 54 turns of No. 18 annunciator wire (1.02 mm diameter) wound on a glass cylinder of 2.5 cm diameter. The high frequency resistance of each coil measured at 400 kc, the normal frequency of the oscillating circuit, is 0.5 ohm. The inductance is 30 microhenries. Within these coils are placed glass cylinders, to hold liquids, with end faces cemented on with water glass in the case of organic liquids, and picein or other wax when aqueous solutions are used. The coils are connected to the spark gap and condenser through a set of wires, upon which are movable connections or trolleys T_1 and T_2. T_1 is variable in steps only, but T_2 slides along the wires and may be placed at any desired position. The purpose of

[1] Beams and Allison, Phys. Rev. **29**, 161 (1927).

[2] Allison, J. Chem. Ed. **10**, 70 (1933); and Phys. Rev. **43**, 38–50 (1933).

[3] Allison and Murphy, J. A. C. S. **52**, 3796 (1930).

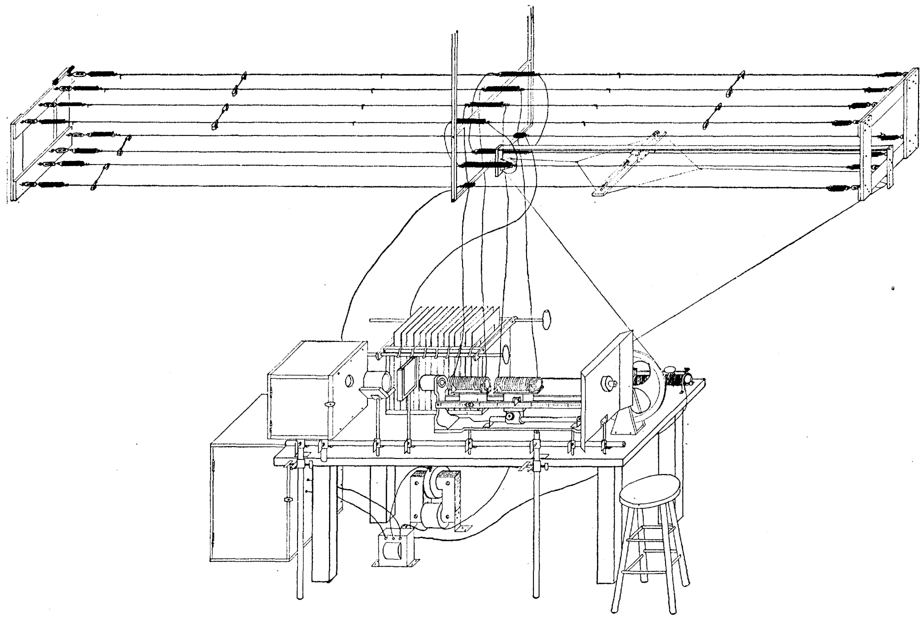

FIG. 1.

the wire system and trolleys is to vary the time of arrival of the surge (which travels out from the spark gap when a breakdown occurs) at the coil L_2 with respect to its time of arrival at L_1. I_1 and I_2 represent radiofrequency ammeters which were placed in the circuit for purposes of experiment, but which were not present when the circuit was used in the usual way. Light from the spark gap passed through a lens, a color filter F which transmits a narrow band centering on the Mg line 4481, a nicol N_1, then through the liquids in L_1 and L_2 and finally through a nicol N_2, crossed with respect to N_1, to the eye.

The results obtained by Allison, and their interpretation in terms of a time lag in the Faraday effect may be summarized as follows: With the same liquid (say CS_2) in L_1 and L_2, the light passing through the two cells and nicols is seen to be a minimum when the trolley slide T_2 is placed at such a point that the wire path from the spark gap to L_1 and L_2 is the same length. When such is the case, the surge arrives at L_1 and L_2 at the same time, and since the coils are so connected that the magnetic fields are opposed there is no resultant rotation and the light is stopped by N_2. (Because of the symmetry of the apparatus, the currents in L_1 and L_2 are approximately equal for this case.) When the slide T_2 is at any other position than the one for equal wire path, the surge arrives at one coil before the other, and for a short interval the plane of polarization is rotated in one coil and not in the other. Hence a small amount of light can pass N_2; a minimum of light passing

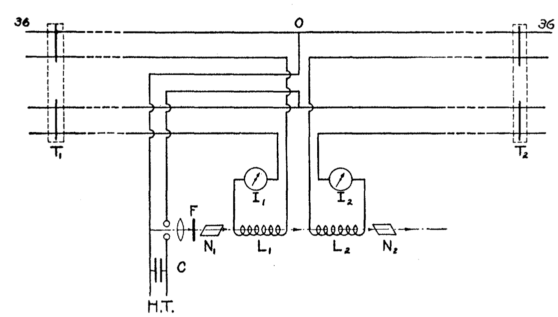

FIG. 2.

when the wire paths to the coils are equal. With different liquids (say CS_2 and a water solution of NaCl) in L_1 and L_2, a minimum of light was found when the slider T_2 was at a position for which the wire paths were unequal; in fact it was found that with CS_2 in one cell, and a series of salt solutions in the other, minima existed whose positions on the wire system appeared characteristic of the compounds. The explanation is that for two different compounds, different times must elapse between the application of the magnetic field and the optical rotation. Then in order to have the rotations occur at the same time and produce a minimum of light by annulling one another, the wire paths must be adjusted to some unequal value so that the magnetic fields (produced when a surge arrives at the coil) are applied at different times. The rotations may also be made to annul one another by allowing the surges to arrive at the same time but displacing one cell from the other, thus varying the time of arrival of the light. The adjustment of wire path is the common procedure, however.

When different liquids are used, the Verdet constants are usually unequal, and the rotations can never completely balance one another, for, as mentioned above, the currents are approximately equal. This difference of Verdet constant has, however, no effect on the position or production of these minima observed by Allison.

A graphic representation of the conditions existing when the trolley T_2 is at some random position which allows light to pass N_2 is shown in Fig. 3. Fig. 4 is a representation of the quantities when they are so related as to produce a minimum.

The change in light intensity, for which the foregoing explanation has been advanced, has the remarkable property of being localized to a very small displacement of cell or trolley slide for Allison states,[4] "They are extremely sharp, as evidenced by the fact that different observers can agree in settings in the dark within three

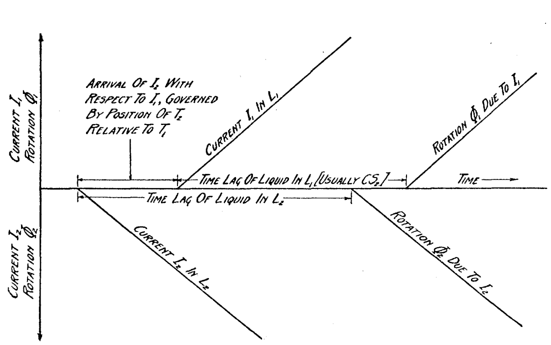

FIG. 3.

[4] Allison, Chem. Ed. **10**, 73 (1933).

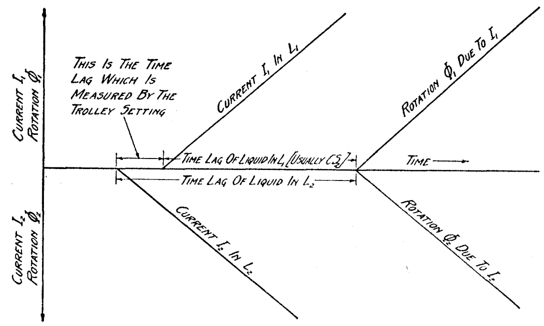

FIG. 4.

millimeters or less. The minima, however, have a greater width than this, for both the eye and the photoelectric cell note the approach to a minimum over a distance of some millimeters."

SURGE TREATMENT

It is of interest to consider the conditions which the currents in the coils must satisfy if such sharp minima as these are to be interpreted in terms of a Faraday effect. The interpretation of sharp minima in such terms introduces the tacit assumption that the growth of current in the coils is so rapid that a change in the time of arrival of the surge at one of the coils by as small an amount as 7×10^{-11} seconds (this corresponds to a displacement of the trolley of 1 cm, which is much more than sufficient if the minima are located to within 3 mm or less) produces sufficient change in the current (and hence in the magnetic field) to make a noticeable rotation of the plane of polarization.

The breakdown of the spark gap results in the production of initial transient currents, or surges, which travel out from the spark gap along the wires with nearly the speed of light. The time of arrival, at a coil, of a surge resulting from the breakdown may be controlled by varying the length of wire path. These initial surges, in the course of their travels on the system, undergo reflections and other changes imposed on them by the various resistances, inductances and capacities of the system, and there is built up in the case of this circuit a train of damped oscillations. These constitute the "steady state," after the initial transients have disappeared. It has been assumed that the initial surge was responsible for the magnetic fields in the coils which gave rise to the minima, and that the oscillations which followed had a negligible effect.

If these sharp minima have their origin in magnetic fields produced by surges, the surges must be quite short in length, in fact comparable with the width of the minima as Fig. 5 will show.

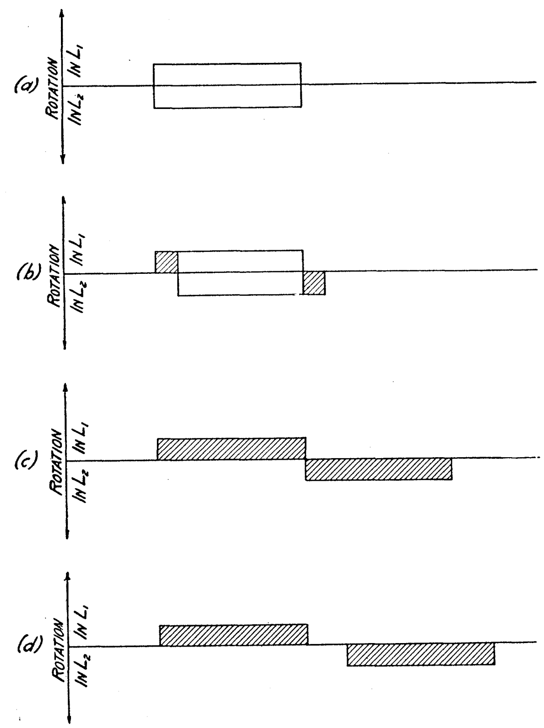

FIG. 5.

The surges in L_1 and L_2 are for simplicity represented as having perpendicular ends; surges of this shape would also produce more pronounced minima. In (a) the condition when the trolley is set on a minimum is shown. (Assuming that a time lag in the Faraday effect does exist, the actual currents may be displaced in time in order that the rotations shall be in phase.) In (b), the trolley T_2 has been displaced, and the shaded area is proportional to the amount of light passing N_2 as a result of one spark breakdown. In (c) the trolley T_2 has been displaced the length of one surge, and the amount of light passing has reached a maximum, for upon further movement of T_2, (d) results, with no increase in amount of light.

For a rapid succession of short light flashes, the visual stimulus increases as the light flash is made longer; hence to the observer of minima the field should brighten as the transition from (a) to (c) is made. If the minima have a width of some millimeters wherein the intensity is seen to change, the surges must be of this length. However, the length of surge comes out to be a matter of many meters, computing from the charge residing in the condenser and the characteristic or surge impedance (some 800 ohms) of the line which drains this charge from the condenser, neglecting any inductive effect of the coils which would further lengthen the surge. In fact, only the first few meters of the surge behave in a simple manner, subject to the control of T_2, because reflections occur at the coils, and at the condenser, and the whole phenomenon becomes a complex disturbance, gradually going into damped oscillations.

Furthermore, if these sharp minima are due to magnetic fields produced by surges, the wave front of the surges as they traverse the coils must be extremely steep, in order that a displacement of 7×10^{-11} seconds along the wave front shall result in a noticeable current change. It is quite conceivable that the wave front of the surge as it exists on the trolley wires would be steep, but it is not so evident that current could grow in a coil having considerable inductance with the requisite speed.

Excellent discussions of the current and voltage phenomena arising when a travelling wave on a line strikes a coil or condenser terminating the line are found in the works of Rüdenberg and Karapetoff.[5] Experimental results on surges have been obtained by Rogowski, Flegler and Tamm[6] who used a special oscillograph.[7]

If a wave of voltage E having a perpendicular wave front travels over a line of characteristic or surge impedance Z and impinges on a lumped inductance L devoid of resistance and capacity terminating the line, the current flowing in the

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[5] Rüdenberg, *Electrische Schaltvorgänge*, pp. 396–407, J. Springer, Berlin; Karapetoff, *A Graphical Theory of Travelling Electric Waves*, Trans. A. I. E. E. **48**, 508 (1929).

[6] Rogowski, Flegler and Tamm, Archiv f. Electrotechnik **18**, 479 (1927).

[7] Rogowski, Flegler and Tamm, Archiv f. Electrotechnik **18**, 513 (1927).

inductance is given by the expression

$$I_L=(2E/Z)(1-e^{(-Z/L)t}), \quad (1)$$

where $t=0$ at the instant the wave reaches the inductance. If L is replaced by a pure capacity C,

$$I_C=(2E/Z)e^{-t/ZC}.$$

The inductance at first acts as an open end to the surge, and no current flows in it. The current builds up gradually, from zero to a value twice that of the line current; the effect of the inductance changes from that of an open end to that of a short circuit. The capacity initially causes the voltage to drop to zero by reflecting an equal voltage of opposite sign, and causes the current to jump to double the value in the line. Its action is opposite to that of the inductance, acting initially as a short circuit, and finally as an open end as it charges up.

The actual operation of a coil such as is used in the magneto-optics experiments is a combination of the above two cases, that is, it presents both inductance and capacity to the surge. If the coil is to have magnetic fields along the axis, and thus rotate the plane of polarization of light passing through it, it is the current I_L which follows the windings which is effective. The current I_C which flows between turns of the coil because of its distributed capacity C drains the energy in the surge without producing a magnetic field which aids the production of minima. Less energy is available as I_L, and unfortunately this diversion of energy into a non-useful path comes with greatest effect just when I_L is zero and growing at its greatest possible rate. Thus it is seen that even granting that the breakdown of the spark gap produces a surge with perpendicular wave front, the current in the coil can only grow at a rate less than that given by Eq. (1). Since Allison reports that a motion of the trolley of 1 cm (or a displacement along the wave front of the surge of 7×10^{-11} seconds) produces a noticeable effect to the eye, when at a minimum, it is of interest to see how much the current in the coil can change in this time. Values which were obtained on the apparatus used in these experiments were: $E=6500$ volts; $Z=790$ ohms; $L=30$ microhenries. Hence from (1) $I_L=0.03$ amp. approx. Thus the unbalanced current impulse, or the current impulse which will flow in one coil without a similar balancing current flowing in the other coil as a result of displacing T_2 one centimeter from the minimum point is an impulse which grows from zero to a maximum possible amperage of 0.03 and then dies away, the life of the impulse being about 10^{-10} seconds. With 60 spark breakdowns per second, the visual sensation would then be the integrated effect of 120 light impulses per second, each light impulse being the result of a rotation of the plane of polarization by one of the current impulses described above, and acting upon a material whose concentration is in some cases less than 10^{-11} g/cc, the length of path in this dilute material being about 10 centimeters.

It is difficult to accept these minute quantities as the cause of any visible phenomenon, despite the numerous proofs cited by Allison in favor of a Faraday time lag as the explanation of such sharp minima.

One assumption which is made in the analytical theory of travelling waves, and their actions on coils, etc., is that the inductance is lumped, or concentrated at one point, so that the current throughout it may be considered constant. This is not strictly correct for the coils in use in these experiments; current must actually flow into the coil for a winding or two before the inductive action of the coil can be built up and applied to the oncoming current. The inductive action does not apply instantaneously because of the finite size of the coil and the finite velocity of propagation of electric and magnetic fields. The net result is that the current in the first windings can produce immediately upon arrival a magnetic field in the right direction to produce a rotation. Nevertheless, it may be said that theory does not predict currents favorable to the production of sharp minima.

Let us suppose that for some unknown reason, the current in this case does not behave as the electrical theory predicts, and let us make assumptions concerning its behavior which we know to be in error on the side of ease of production of sharp minima. Let it be assumed that the spark gap breakdown is instantaneous, that the surge can travel over the trolley system without loss of its perpendicular wave front, that no reflections take place at the coil, that the coil allows current to grow in it instantaneously and that all the current follows the windings ($I_c=0$). In other words, we assume that the surge current in the coil rises from zero to its maximum possible of about 20 amperes instantaneously. With these very obviously one-sided and unrealizable conditions, it is of interest to compute the optical effect which would result.

Since a motion of the trolley of 1 cm (7×10^{-11} seconds) should produce a noticeable optical effect when at a minimum, it follows that the resultant current when the trolley is displaced from the minimum position by 1 cm must be sufficient to produce an effect. If the optical effect has to do with the Faraday effect, then the current surge of 20 amperes (which, we have assumed, grows from zero to full value instantly) lasting for about 10^{-10} seconds must produce a noticeable rotation.

From some work by Beams,[8] on the appearance of a Mg spark photographed with a rapidly rotating mirror, it would seem that the metallic lines exist with good intensity for at least 5×10^{-6} seconds. The eye acts as an integrating apparatus for rapid light pulses, as the use of sector disks in photometry shows, so one can compute a steady current persisting through 5×10^{-6} seconds to which the pulse of 20 amperes and 10^{-10} seconds is equivalent. The value of this equivalent current is 4×10^{-4} amperes.

A number of experiments to determine the amount of d.c. necessary to produce a noticeable rotation led to the conclusion that with the Mg spark as a source of light (but not actuating the coils) and with the usual color filters present, a d.c. of approximately one ampere through one coil was necessary to produce the least detectable change.

The optical effect in this case was produced by one ampere acting during the time the metallic lines were emitted from the spark gap, and this value of one ampere is to be compared with the value 4×10^{-4} amperes obtained above. Even if, for such extremely short flashes, the eye does not integrate in the manner supposed, it does not seem possible to reconcile the widely divergent figures on these grounds. Furthermore, 4×10^{-4} represents an upper limit, due to the assumptions made. The conclusion is that the surge currents cannot be responsible for the sharp minima.

Alternating–Current Treatment

It has been supposed that oscillations in the magneto-optic circuit did not exist, or if they did, had no effect on the production of minima. In the apparatus described herein, damped oscillations of considerable magnitude were found to exist, the peak currents being about 200 amperes; far greater than any currents existing in the initial or transient state. Measurements on the decrement indicated about 30 waves per wave train before the current was reduced to 1 percent of the maximum value.

The wave-lengths were found to lie between 500 and 1100 meters (600–273 kc) depending chiefly upon the value of the condenser C, but to some extent upon the position of the trolleys. In addition to this fundamental frequency, there was found an oscillation of wave-length approximately 70 meters (4290 kc), of much lower intensity. This oscillation seemed to be associated only with the coils and the trolley system, since a variation of the condenser C did not affect it. Moving the trolley T_2 through 36 divisions changed the wave-length from 65 to 73 meters (4615–4110 kc). There is an upper and lower set of wires, connected in series, to increase the wire path length when necessary. Upon disconnecting the upper wire system, the 70 meter wave disappeared. No other oscillations were found in the range from 1 meter to 24,000 meters (300,000—12.5 kc).

To see if these oscillations are responsible for the sharp minima, it is necessary to examine the sort of minimum they are able to produce, assuming the existence of a time lag in the Faraday effect. Let ϕ_1, ϕ_2 represent the angular rotations of the plane of polarization in L_1, L_2. Let the current in L_1 be represented by $I_0 \sin\theta$. Then the current in L_2 is $I_0\sin(\theta-\alpha)$ where α represents the phase difference between the currents due to the change of impedance upon moving the trolley.

When treating the problem with the view that a series of independent surges travel from the

[8] Beams, Phys. Rev. **35**, 24 (1930).

spark gap to the coils, the positions of the trolleys govern the time of arrival of the surges. From the standpoint of oscillations, the positions of the trolleys govern the value of the impedance in series with each coil, and hence the phase relation of the currents in L_1 and L_2. In order to determine this phase relation, it is necessary to know the high frequency resistance and inductance of both the coils and trolley system. These measurements were made at 750 meters (400 kc) which was taken as the normal operating frequency of the system. Resistance measurements were made by the resistance substitution method; inductance measurements by the resonance method. The quantities which determine the phase relations of the currents are shown in Fig. 6.

Knowing these values, one can then obtain the phase relation between the two currents, since

$$\tan\theta=\frac{(L_1+L_{T_1})}{R_1+R_{T_1}}\quad\text{and}\quad\tan(\theta+\alpha)=\frac{(L_2+L_{T_2})}{R_2+R_{T_2}}.$$

The resultant optical rotation $\phi_1-\phi_2$ is proportional to $I_0(\sin\theta-\sin(\theta-\alpha+\beta))$. The quantity β accounts for the time lag difference of the two liquids in use. This expression is a maximum when $\theta=\frac{1}{2}(\alpha-\beta)$. Then the maximum resultant optical rotation is proportional to $2I_0\sin\frac{1}{2}(\alpha-\beta)$ or for small angles to $\alpha-\beta$. β is a constant of the two liquids, independent of the trolley position, so the shape of a minimum with respect to the motion of T_2 is given by examining the case for

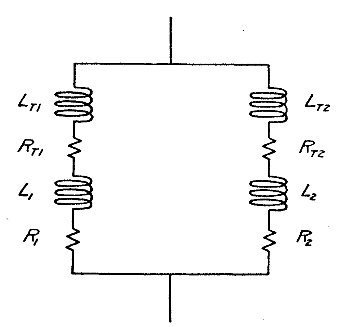

FIG. 6. L_1 and R_1 are the inductance and resistance of coil No. 1. L_2 and R_2 are the inductance and resistance of coil No. 2. $L_1=L_2$ and $R_1=R_2$. L_{T1} and R_{T1} are the inductance and resistance of wire path whose length is governed by position of T_1. L_{T2} and R_{T2} are the inductance and resistance of wire path whose length is governed by position of T_2.

$\beta=0$ (same liquid in each cell). Then the maximum resultant rotation is proportional to the phase difference between the currents in the two coils. If we assume that the visual stimulus is proportional to this maximum resultant optical rotation, then the shape of the minimum is given by plotting α against the position of the movable trolley T_2. Fig. 7 shows the experimentally found

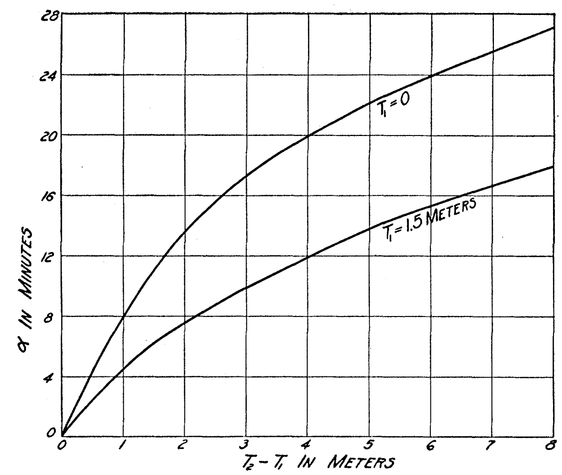

FIG. 7.

values of α for various trolley displacements. If it is assumed that the visual stimulus is more nearly proportional to the time integral of the resultant rotation, the result is again proportional to α for small angles, and therefore the curves of Fig. 7 give a picture of the change of light intensity to be expected upon moving the trolley.

The curves show a gradual and long continued rise; therefore, upon treating the current as oscillatory rather than as a surge the conclusion is reached that the intensity change due to a minimum is broad and gradual, extending over the whole length of the trolley, rather than sharp and localized to a few millimeters. Furthermore, as was the case with the surge currents, the change of resultant current for a one centimeter displacement of the trolley seems far too small to produce a visible effect.

Further evidence that the oscillations can hardly be the cause of the minima is that Allison has found the same minima in practically the same place on five different set-ups of differing electrical characteristics. He also found that the size of coil had no effect on the minimum position. Changing these electrical quantities changes the relation between phase angle and trolley position, so that any minimum due to the oscillations must shift position as the size of coils or spacing of wires is changed.

If, however, the origin of these minima is in these oscillations, then the actual time lags of the various materials are different from those given by Allison, for the time lags have been "computed on the assumption that the electric surges in the wires travel with the speed of light," whereas the true time lags should be obtained from a knowledge of the wave-length and phase angle of the oscillatory currents.

It is now seen that if a time lag in the Faraday effect does exist, two minima are possible for each time lag, one due to the surge currents and one due to the oscillatory currents. Both these minima are extremely broad, and present so gradual an intensity change as to seem impossible of detection. A system could be devised with the right values of inductance and capacity per unit length and right size of coils so that the two minima would coincide. In the apparatus already described they fall widely apart. When they do not fall together, a minimum due to one type of current has superposed on it light resulting from the other type of current, thus rendering still more difficult its detection.

Large Minima Found

A further cause of difficulty in detecting these minima is that a motion of T_2 causes a change in the impedance in series with L_2, and hence a change in the magnitude of the current in the coil. The two coils then allow light to pass not due to any time or phase displacement of the currents but due to a change of current magnitude. The amount of this light is considerable, and can be observed to vary as the trolley is moved. In fact, in the course of some experiments, minima of light intensity have been observed as the trolley was moved, and the positions of these minima have been peculiar to the organic liquid used. Similar phenomena have been observed by Slack and Breazeale.[9] These minima are due to the change in current magnitude mentioned above, and which is illustrated by the graph of a typical set of data—Fig. 8.

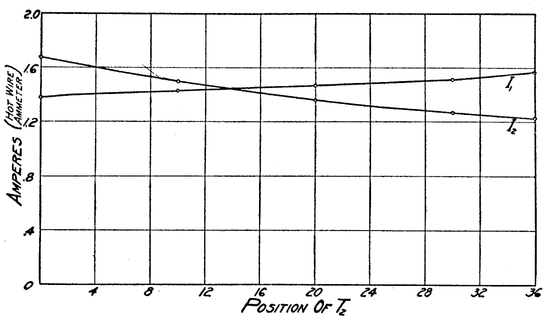

Fig. 8.

Since the position of the trolley affects the amount of current in the coils L_1 and L_2, it follows that the position of T_2 affects the amount of rotation of the plane of polarization of light which passes through the liquids in L_1 and L_2. Suppose first that the same liquid is placed in L_1 and L_2. Since the rotation of light in L_1 is opposite to that in L_2, the light passing the second nicol will be at a minimum of intensity when the currents in the two coils are equal. Remembering that the part of the apparatus containing L_1 and T_1 is symmetrical to that containing L_2 and T_2, it follows at once that a minimum of light passes the second nicol when T_2 is placed at about the same distance from the zero as T_1 and that for any other position of T_2, the rotations in the two cells will be unequal, and more light will pass the second nicol. In Fig. 9 the appearance of some of these minima is shown, as recorded by a photoelectric cell. Note their extreme broadness. The fact that the CS_2 minimum is more pronounced than the $CHCl_3$ minimum confirms the explanation of these minima, for the larger Verdet constant of CS_2 causes greater rotation.

Thus far, then, there is agreement with the Faraday time lag hypothesis in that a minimum is found when the trolleys are approximately equal and the same liquid is in each cell. Leaving aside the explanation of the extreme width of this minimum, one might easily be led to the assumption that its cause is a time lag in the Faraday effect, for in this case of similar liquids

[9] Slack and Breazeale, Phys. Rev. **42**, 310 (1932).

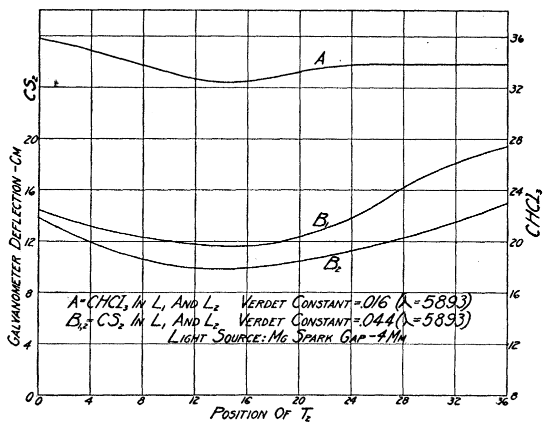

FIG. 9.

such an hypothesis also leads to the conclusion that the minimum is to be found near the point where trolley distances are equal. In cases wherein dissimilar liquids are used, one finds proof that the origin of these broad minima is other than a time lag in the Faraday effect.

Suppose that benzene is placed in L_1 and carbon bisulphide in L_2. The minimum will no longer be at the point where the trolleys are nearly equal. Since carbon bisulphide has a Verdet constant larger than that of benzene, the current in L_2 must be diminished, in order to make the rotations in L_1 and L_2 equal. This diminution of current is accomplished by placing T_2 farther out—i.e., adding impedance to the branch containing L_2. Hence for this case, the minimum will appear further out on the T_2 scale. Fig. 10 is the photoelectric record of this case. The amount of rotation of the plane of polarization is proportional to the product of the Verdet constant of the liquid and the current in

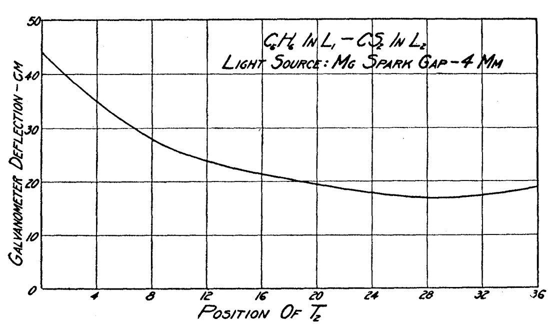

FIG. 10.

the coil surrounding the liquid. Then, for any two liquids, the minimum of light will be found at that point where the rotations are equal (and opposite), namely, at the point where $V_1I_1 = V_2I_2$ where the subscripts 1 and 2 refer to L_1 and L_2. Then $I_1/I_2 = V_2/V_1$, which tells us that if we place a certain liquid (for example xylene) in L_1 and various other liquids in L_2 and locate the minima, the ratio of currents is proportional to the Verdet constant of the liquid in L_2. Fig. 11 shows the experimental results obtained; a striking confirmation of the theory. There is also plotted the relation between the excess trolley and the Verdet constant, showing that it is the Verdet constant rather than a time lag in the Faraday effect which determines where the minimum shall appear. These large minima were

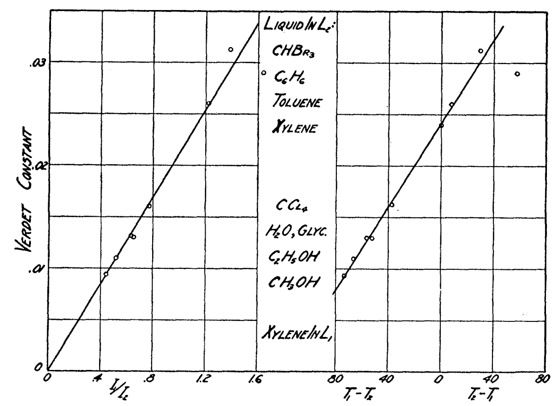

FIG. 11.

observed at wave-length 4481 and also approximately 5890, obtained by using iron electrodes and a filter. This was done so that values of the Verdet constant (which are mostly given for 5893) could be obtained from tables. Fig. 11 refers to minima observed with the yellow light.

It therefore seems improbable that these minima are the phenomena observed by others[1, 10] since:

(1) Their cause is definitely shown to be other than a time lag in the Faraday effect.

(2) Their positions on the trolley scale do not accord with positions calculated for them from data of others.[1, 10]

[10] Allison, Phys. Rev. 30, 66 (1927).

(3) The positions of these minima on the trolley scale are linearly related to the Verdet constant.

(4) These minima are very broad, requiring a trolley movement of several meters to pass through them, and are easily observable, even to an untrained observer. They are large in comparison with the unavoidable fluctuations of the light source (series of sparks between magnesium rods about 4 mm apart).

Faraday Effect Calls for Change in Minimum Appearance with Concentration

It has been assumed that the phenomenon observed by Allison had to do with the Faraday effect; the idea of time lags in the Faraday effect has required discussions and proofs but since the apparatus is built for the purpose of applying magnetic fields to liquids, it has been taken for granted that the optical phenomenon of minima had to do with a rotation of the plane of polarization by the magnetic field. The trolley position merely determines *when* the compound shall rotate the plane of polarization, and the foundation of the theory is that at some time the Faraday effect occurs and plays a part in minimum formation.

The usual procedure has been to have CS_2 in one cell and an aqueous solution of some material in the other. Assuming that we have CS_2 in one cell and an aqueous solution of sodium chloride in the other, let us examine the part that a rotation of the plane of polarization plays in the formation of a minimum.

According to the Faraday-effect time-lag hypothesis, if the trolley is set on the minimum position for NaCl, the rotation produced by the NaCl occurs at the same time as that produced by the CS_2 but is opposite in direction. Since the time lag for water is greater than that for NaCl[3], the light which is acted on by the CS_2 and NaCl will pass to the eye unaffected by the water. If the light surge emitted from the spark is long enough (which is very probably the case, and which is of course the case when a steady source of light is used) the water rotates light which passes through the coil at a little later time.

Verdet himself showed that for a number of salts in aqueous solution the ratio of the Verdet constant of the solute to solvent was independent of the concentration; in other words the ability of any one molecule to rotate is not affected by the proximity of its neighbors. Thus for a given solution, the rotation due to a compound dissolved therein is proportional to the number of molecules of the compound per cc. Furthermore it has been long known that the Verdet constant of a solution varies with the concentration of the solute. Schönrock has given an able treatment of this question[11] and has shown that the correct expression for the Verdet constant of a solution is given by

$$w = w_1 q_1/s_1 + w_2 q_2/s_2$$

where w, w_1, and w_2 are the Verdet constants of the solution, solvent, and solute; q_1 and q_2 the number of grams per cc of solution of solvent and solute; and s_1 and s_2 are the densities of solvent and solute. The term w_2q_2/s_2 thus represents the effect of the dissolved material. The rotation produced by the CS_2 is $\theta(CS_2) = HlV(CS_2)$ where H is the magnetic field strength due to the surge of current, l the length of cell and $V(CS_2)$ the Verdet constant of CS_2. The rotation produced by the NaCl is $\theta(\mathrm{NaCl}) = Hlw_2q_2/s_2$. The resultant rotation, when the trolley is set at the NaCl minimum, is $Hl[V(CS_2) - w_2q_2/s_2]$ which corresponds to a certain light intensity. A displacement of the trolley from the minimum position causes more light to be seen because the two rotations represented by the above terms are put out of phase. It is evident that the intensity of a minimum, or the change of intensity as one moves the trolley through the minimum point, must depend on the relative magnitudes of the two rotations, and hence of the terms $V(CS_2)$ and w_2q_2/s_2. When these two quantities are equal, the intensity change is greatest. From data in the *International Critical Tables*, $V(CS_2)$ and w_2 computed for wave-length 4481 have values 0.0871 and 0.056 respectively. Then taking $q_2 = 0.3$ g/cc (which corresponds to a 25 percent NaCl solution) we have $w_2q_2/s_2 = 0.056(0.3)/2.16 = 0.0078$ which is much smaller than 0.0871 but might possibly result in a noticeable effect to the eye when the rotations of the two coils are put out of phase. But it is evident that the mag-

[11] Schönrock, Zeits. f. Physik **46**, 314 (1928).

nitude of any effect due to the NaCl is proportional to its concentration. Consider its effect when $q_2 = 10^{-10}$ g/cc, a concentration at which there has been no difficulty in seeing minima.

$$w_2 q_2 / s_2 = 0.056(10^{-10})/2.16 = 0.026 \times 10^{-10}.$$

Thus the rotation due to the NaCl is 3×10^{-11} of the rotation due to the CS_2, yet the Faraday time lag hypothesis applied here requires that by shifting the phase of this minute quantity with respect to the rotation in the CS_2, one produces an effect which the eye can detect. Furthermore, we have neglected in the above discussion to include the effect of the water (Verdet constant at $4481 = 0.0237$) which makes the light effect of the NaCl a still smaller fraction of the total.

The above discussion may be applied to any number of cases in which other materials than NaCl are present. Certainly if the phenomenon reported has to do with the Faraday effect, concentration must play an active rôle. Such is not the case, however, as the following descriptions of the minima show: "The minima show no appreciable change in distinctness at various stages of concentration until the dilution at which they vanish is approached, when they gradually fade out, usually between 1 part in 10^{11} and 1 part in 10^{12}."[12] "The photoelectric cell registers but very slight differences in the blackness of the minima with changes in concentrations over wide limits, a result in agreement with visual observations."[13] "The minima are noticeably broader and somewhat blacker in concentrated solutions."[14]

Absolute Value of Time Lag for CS_2

Allison has reported[15] that the time lag for TlCl is 27.65×10^{-9} seconds shorter than the time lag for CS_2, and since then has reported further data on chlorides of tin and rhenium[16] with even greater values, one corresponding to a lag which is 32.54×10^{-9} seconds shorter than the lag of CS_2. This means then that the absolute value of the lag of CS_2 must be greater than 3.25×10^{-8} seconds.

Abraham and Lemoine[17] attempted to measure the time lag of CS_2, and concluded that it must be less than 10^{-8} sec. Allison has reported a change of time lag with wave-length[10] but the wave-length used by Abraham and Lemoine was not far from that used by Allison (4481) for they used blue light, so it does not seem that the divergent conclusions of Allison and Abraham and Lemoine can be explained on these grounds.

A persistent search for sharp minima was made by several observers, especially in the regions in which minima were found by Allison. In the course of these observations, various changes and adjustments were made in the apparatus, such as varying the value of the condenser C, length of spark gap and frequency of spark, in order that optimum conditions would be obtained for the appearance of minima. The sources of light used were the spark between magnesium rods, a mercury arc and a Pointolite (incandescent bead of tungsten). Filters were used to obtain nearly monochromatic light. The only consistent optical phenomena which our observers were able to find were the large minima already mentioned; a series of tests showed that other "minima" reported by the observers were due to random and spurious influences.

To supplement visual search, a high sensitivity photoelectric cell (General Electric Company) was used. The amplifier for the small currents was a two tube Pliotron bridge. The Pliotrons were selected from a number of tubes as the ones showing most similar characteristics. The photoelectric current flowed through a resistance of 10^{11} ohms which constituted the input to the working tube grid. The limit of accuracy in searching for minima was set by the natural fluctuations of the source of light rather than by a lack of sensitivity or stability of the system itself. For the case of the smallest deflections, the system introduced fluctuations amounting to 2 percent of the deflection produced by the light, and in cases when more light was available, this figure dropped accordingly. When using the Pointolite as source, the deflection was not in error by more than $\frac{1}{3}$ of 1 percent.

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[12] Allison, J. A. C. S. **52**, 3803 (1930).
[13] Allison, Phys. Rev. **37**, 1004 (1932).
[14] Allison, Chem. Ed. **10**, 70 (1933).
[15] Allison, J. A. C. S. **52**, 3800 (1930).
[16] Allison, J. A. C. S. **54**, 614 (1932).
[17] Abraham and Lemoine, C. R. **130**, 499 (1900).

The photoelectric cell was not tried in as many cases or over as great a range as was done with the visual observations but was tried in the cases given in Table I. In the first two cases, no effort was made to render the light monochromatic. Allison has reported a variation of the time lag with the wave-length of light used, but when the same liquid is in each cell, the position of the minimum should be unchanged with wave-length. The results obtained with the magnesium spark as light source and the photoelectric cell as observer may be summed up as follows:

TABLE I.

Case	Position of T_2 at which according to Allison the minimum is to be found	Range examined
CS_2 in each cell	11.5	10–12
$CHCL_3$ in each cell	11.5	10–12
CS_2 in L_1, $CHCL_3$ in L_2	3.2	2.5–4.5
C_6H_6 in L_1, CS_2 in L_2	17.0	14–19

The unavoidable unsteadiness of the spark introduced random galvanometer deflections whose mean value was about 5 percent of the total deflection and whose peak values were about 10 percent of the total deflection. These fluctuations were the same in character whether the trolley was moved slowly over the minimum region or was left stationary at any point. No minima were found within the limits of error set by the fluctuations mentioned above, and if minima of light intensity were present in these tests whose origin was not a random or spurious influence but was a Faraday time lag or other property of the compounds being examined, they were too small to be detected.

It was found that when a steady source of light is used (still using the spark as source of oscillations), any optical phenomenon arising in the coils is much less pronounced than when the spark is used as source of light. The explanation seems to be that in the case of the spark, an intense light is emitted at the moment when there is current in the coils; and when the coils are inactive between sparks, no light is present. With a steady source, because of scattering and depolarization, some light passes the crossed nicols even when the coils are inactive. Hence a much smaller fraction of the steady light than of the spark light is affected by the currents in the coils and a motion of the trolley. See Fig. 12.

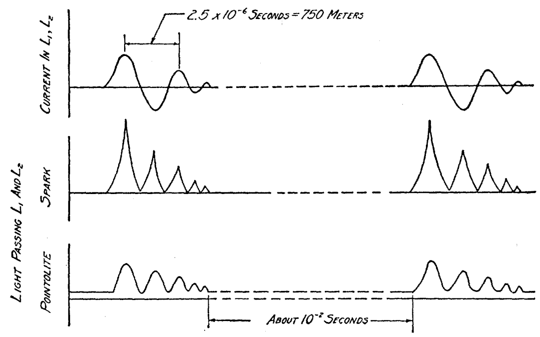

FIG. 12.

With CS_2 in each cell, and a Pointolite as source, no minimum was visible in the region around 11.5. The least detectable change in light intensity was 1/300 of the incident light. Even the large minimum, which in the case of the spark was easily recorded (see Fig. 9), produced an indication scarcely larger than the limit of error. Visual observations also yielded no results, so the steady source was deemed too unsatisfactory to warrant further work with it.

No x-ray tube was in use nearby at the time of making observations; hence the failure to obtain minima other than the broad ones whose explanation has been given cannot be due to any interference of x-rays such as is described by Allison, who reports "it was found in every case that the time lag differences of the Faraday effect between any pair of the liquids vanished so long as the liquids were exposed to the x-rays and that the lags were restored with the screening off of the x-rays." [18]

In view of the results set forth in this paper it appears that a time lag in the Faraday effect cannot be responsible for Allison's results. The conclusion is reached that the results are due to some as yet unknown cause, appearing only in certain apparatus for reasons unknown. Such a cause might be in the nature of a Kerr effect in the coils, as a result of voltage waves travelling

[18] Allison, Nature **120**, 729 (1927).

down the wire paths and building up on the coils in a peculiar manner. A few tests were made with some cells designed to test this Kerr effect hypothesis, but the experiments were not carried far enough to reach a definite conclusion.

Professor J. Papish of the Cornell University Department of Chemistry kindly placed his magneto-optics apparatus at our disposal, and much of the experimental work was done on it. The electrical characteristics were studied both on this system and on a similar wire system at the University of Minnesota.

It is a pleasure to thank Professors Ernest Merritt and J. R. Collins of the Cornell Physics Department for their interest and helpful advice, and Mr. A. C. Shuman of the Chemistry Department for many hours of help in the experimental work.