THE

PHYSICAL REVIEW

A Journal of Experimental and Theoretical Physics

VOL. 44, No. 5 SEPTEMBER 1, 1933 SECOND SERIES

An Effect of Positive Space Charge in Collector Analysis of Discharges

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(Received March 23, 1933)

By use of a collector with mutually insulated sections to which different potentials could be applied, it has been shown, with a low voltage arc in argon, that under the special conditions of short collector and small electron concentration the positive ion sheath which forms on insulating surfaces near a collector in a plasma may lead to serious misinterpretation of the characteristics of the collector. The effect is twofold, first, a general diminution in the apparent concentration of electrons in the plasma, and, second, an apparent preferential suppression of the slowest electrons. Under most working conditions, these are probably inconsiderable at low pressure but perhaps important at a pressure of a few mm and higher. The collector characteristics obtained have been analyzed for Maxwellian distributions and by a method in which no initial assumption is made concerning the energy distribution other than that it is isotropic. A formula is given to facilitate double differentiation in the latter case.

SEELIGER and Hirchert[1] have found that the results obtained with collectors in positive columns at a pressure of a few mm depend on the length of collector used. In particular, a collector characteristic which can be taken to reveal the presence of two Maxwellian groups of electrons and is obtained with fairly long collectors may degenerate with short collectors into the simple form indicating the presence of one group only. It was thought desirable to investigate this effect at lower pressures and to see if it could be accounted for through action of the positive ion sheath which forms on the glass near the collector, since it has been shown that considerable modification of the usual characteristic is produced if part of a collector is exposed to a positive space charge.[2] With this aim a collector, in a low-voltage arc in argon, has been designed which permitted variations of a nearby positive ion sheath with or without change in length of the collector. It has been found that at low pressures distortion of the characteristic through positive ions congregating on insulated surfaces is usually small but that it may be serious with an abnormally short collector.

DETAILS OF EXPERIMENTS

The collector is shown in Fig. 1. A molybdenum wire A (0.11 mm diameter) was arranged so that the length projecting beyond the end of a copper shield could be adjusted by moving an iron slug with a magnet. A glass shield B insulated the

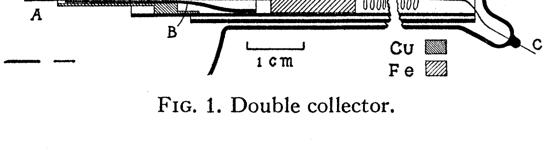

FIG. 1. Double collector.

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[1] Seeliger and Hirchert, Ann. d. Physik **11**, 817 (1931).

[2] Emeleus and Sloane, Phil. Mag. **14**, 355 (1932).

Mo collector from the Cu. The latter could be held at any desired potential by the connecting wire *D*, which passed out as shown between two coaxial Pyrex tubes. Contact was made with the Mo through the iron by the lead *C*. *B* was slightly tapered so that it did not come into contact with the Cu at the exposed end of the latter. The insulation resistance between *C* and *D* was better than 6×10^{11} ohms. The collector was mounted in a low voltage arc tube about halfway between an oxide coated platinum filament cathode $(-)$ and a nickel disk anode $(+)$ (2 cm in diameter, 1.8 cm from the cathode). The tip of the Cu extended a little under the anode. It was found immaterial whether the argon contained 0.5 percent nitrogen or not. The circuits employed were of the usual type for collector analysis of discharges.

The problem can be approached in two ways. (a) The length of Mo projecting from the Cu can be kept constant and the characteristic curve of the former taken for several potentials of the latter; in this method the extent of the positive ion sheath on the Cu is varied. (b) The Cu can be held at a fixed potential and characteristics taken with various lengths of Mo projecting; the Mo then passes to a varying extent through an almost fixed positive ion sheath on the Cu.

In analyzing the characteristics the positive ions have first to be taken into account at small negative potentials. This has been done by making a large scale plot of either the current or the square of the current, to the Mo as a function of voltage, from about -70 to -25 volts. One or other of these plots has invariably been nearly straight and it has been assumed that the extrapolation of this straight line to smaller negative voltages gives the contribution of positive ions to the current there. The electron currents can then be obtained from the differences between this and the total observed current at any potential. In a few cases the positive ion line, when so extrapolated, gave an electron current at the space-potential; this usually occurred only when the Mo was short, with a considerable negative potential on the Cu. The resulting semi-logarithmic electron plots, a good example of which appears in Fig. 3, I*a*, can be analyzed in two ways. In the first, the fall of the plot for the highest negative voltages is ignored and the more negative straight part of the remainder taken to be due to a fast Maxwellian group. From the extrapolation of this straight portion to low negative voltages a difference curve is constructed (I*b*) which is taken to represent a slow Maxwellian group. Electron temperatures and concentrations are calculated from the two straight lines.[3] In the second way, no assumption is made about the electron velocity distribution in the plasma other than that it is isotropic.[4] The distribution is deduced from the second differential coefficient of the electron current *vs.* voltage curve for the same retarding potentials for electrons as before.

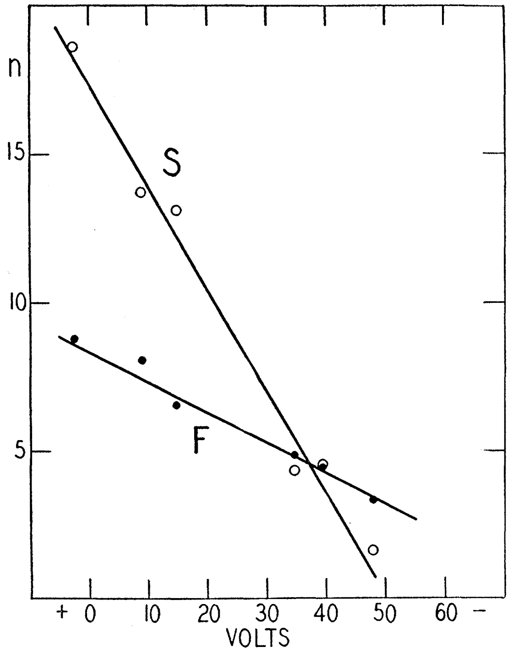

FIG. 2. Electron concentration *vs.* potential of copper (Table I).

RESULTS OF EXPERIMENTS

1. Temperature analysis

Table I shows a set of results obtained by method (a). Discharge conditions were: projecting length of Mo 1 mm, tube voltage about 30, tube current 35 milliamp., argon pressure 0.09 mm Hg. Concentrations have been calculated as if the whole length of Mo projecting were effective. It is difficult to obtain more consistent

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[3] Langmuir and Mott-Smith, Gen. Elec. Rev. **27**, 449 ff. (1924).

[4] Electron reflection is however neglected.

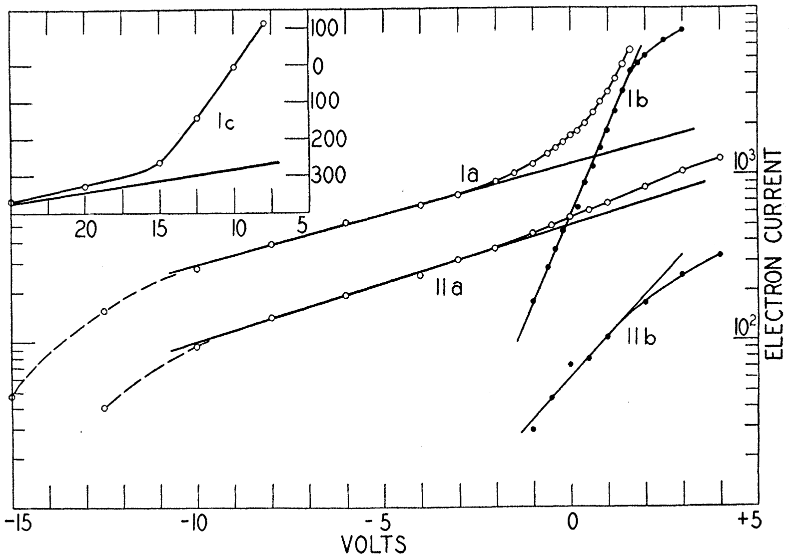

FIG. 3. Part of original characteristic and semi-logarithmic plots.

values for the electron temperatures without more elaborate precautions than the present investigation warranted.[1] Fig. 2 is a plot of the concentration data of Table I. In 1 the Mo appeared entirely covered by the positive ion sheath on the Cu, and in 2 was just projecting. The Cu was at its floating potential in 5. The apparent concentration of the slow group S falls off much more rapidly with increase in the negative potential applied to the Cu and hence with increase in the region occupied by the positive ion sheath on the latter, than does that of the fast group F, a result confirmed by other runs and in this example deliberately exaggerated by use of a short projecting length of Mo.

TABLE I. *Results obtained by method (a).*

Run	Potential of Cu relative to anode (volts)	*Fast Group* Temperature (volts)	*Fast Group* Concentration per cc	*Slow Group* Temperature (volts)	*Slow Group* Concentration per cc
1	−48	9.6	3.35 10^8	0.91	1.66 10^9
2	−39	9.9	4.43	0.78	4.53
3	−35	11.5	4.87	0.84	4.33
4	−15	11.6	6.55	1.13	13.1
5	− 9	10.5	8.04	0.88	13.7
6	2.5	9.5	8.78	0.92	18.6

Two examples from another set, one showing the slow group on the point of complete suppression, are shown in Fig. 3. I*a* and I*b* represent data obtained with the Cu connected to anode, I*b* giving the slow group and space-potential. II*a* shows the effect of applying −60 volts to the Cu relative to the anode. An attempt has been made to analyze the low voltage rise in II*a* for a slow group, II*b*.

The results of method (b) need not be presented since they confirm those obtained by method (a).

2. Differential analysis

By modifying slightly a result due to Druyvesteyn,[5] the energy distribution ($N_v dV$) for electrons in unit volume of the plasma can be obtained from the current-voltage characteristic of the collector for electrons received in a retarding field, from the relation

$$N_v = K V^{\frac{1}{2}}(d^2 i/d V_s^2),$$

where V_s is the potential of the collector, V the difference between the space potential and that of the collector and K a constant, involving numerical and atomic constants, which is of no interest here except that it is inversely proportional to the area of the collector. Double differentiation of slightly erratic experimental data presents difficulty. In the present case the opera-

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[5] Druyvesteyn, Zeits. f. Physik **64**, 781 (1930).

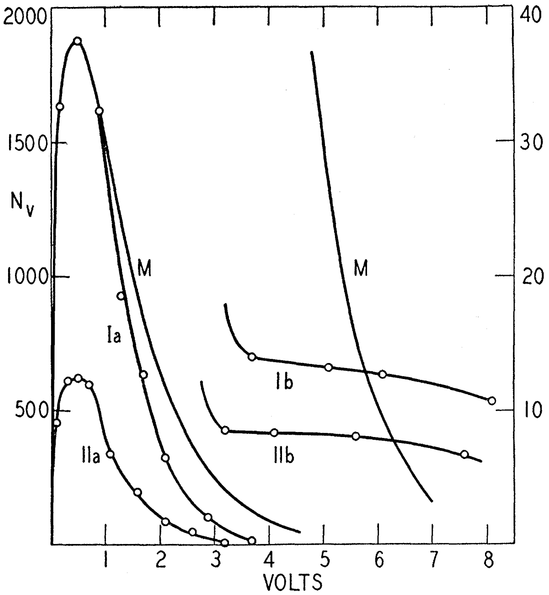

FIG. 4. Electron energy distributions compared with Maxwellian distribution.

tion is simplified by the fact that considerable stretches of the characteristic are straight in a semi-log plot. From the identity

$$y^{-1}(d^2y/dx^2)=(d\ln y/dx)^2+d^2\ln y/dx^2.$$

N_v is then readily evaluated, since the second term on the right vanishes. Even when the second term does not vanish determination of the second differential is often facilitated and it has been used except for the rapidly falling semi-log curve for the highest negative voltages.

Some results are shown in Figs. 4 and 5. In Fig. 4, I and II give the distributions for runs 5 and 2 of Table I. The characteristics were generally similar to those of Fig. 3. The ordinates are in the same arbitrary units for I and II. The low voltage Maxwellian groups of the temperature analysis correspond roughly to I*a* and II*a*, and the high voltage groups to the large scale sections I*b* and II*b*. These results again show an apparent preferential suppression of low speed electrons produced by surrounding part of the Mo with a positive ion sheath on the Cu. Curve *M* in Fig. 4 is the Maxwellian distribution corresponding to the linear portion of the original semi-logarithmic plot I*a* near the space potential. The right-hand part of *M* is on the larger scale of I*b* and II*b*. As pointed out by Druyvesteyn in a similar case,[5] there is a deficiency of medium speed electrons, relative to *M*.

The electron currents for high retarding potentials cannot now be ignored. Fig. 5 shows the complete distribution curve derived from the data used for curve I*a*, Fig. 3, Curve I*c* being a portion of the original current voltage characteristic, which was closely linear for larger negative potentials and extrapolates as shown. Compared with the Maxwellian curves a new high voltage peak in N_v appears, although accurate determination of its position and height is difficult. This peak might be due to electrons, accelerated through the cathode fall in potential, which have made a single resonance collision with argon atoms. The electron energies are of the right order for this explanation to be tenable but the exact fall in potential across the tube was not measured. Presence of such groups is important in the interpretation of the spectrum of the discharge.

DISCUSSION OF RESULTS

From both methods of analysis the effect of application of increasing negative potentials to

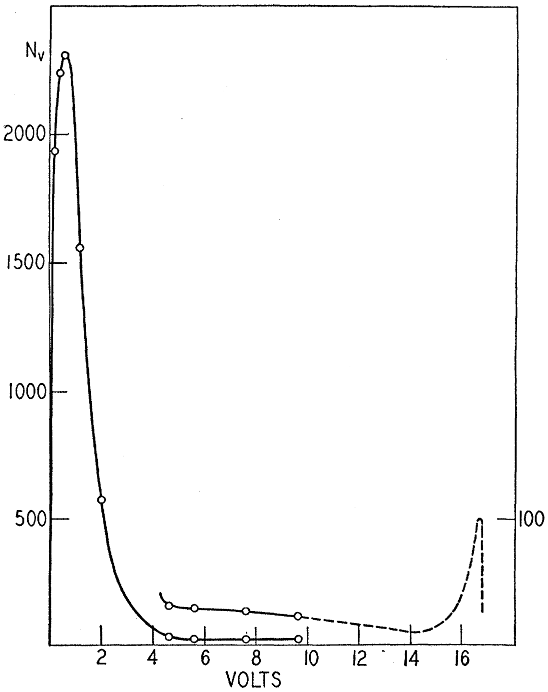

FIG. 5. Electron energy distribution with high voltage peak.

the Cu, with increase in the size of the positive ion sheath on it, is to cause a greater apparent decrease in the slower electrons in the plasma than of the faster. In other words, the effective length of the Mo decreases more rapidly for the slow electrons than for the fast.

A full theory of the effect is impracticable on account of the necessarily complicated conditions of collector geometry and space-charge. An approximate account may be given as follows. Suppose the plasma contains two full Maxwellian groups, with temperatures T_1, T_2, random currents I_1, I_2. To a rough approximation the Mo may be considered to be in two sections. That more remote from the copper will receive electrons when negative to the plasma through a region of positive space-charge in which the potential varies monotonically between that of the plasma and that of the Mo. For this, conditions for the original theory of Langmuir[3] obtain and the currents received at the Mo will be

$$lpI_1 \exp\left(-eV_{\mathrm{Mo}}/kT_1\right)$$

and

$$lpI_2 \exp\left(-eV_{\mathrm{Mo}}/kT_2\right),$$

where l is the length of this section of the Mo, p the perimeter of the Mo, and zero and V_{Mo} the potentials of the plasma and Mo. The other section of Mo, nearer the Cu, will receive electrons through a positive space-charge region in which they pass through a potential minimum $V_{\min}$ lower than V_{Mo}. The magnitude of $V_{\min}$ will increase from the end of the first region—absent for sufficient negative potential on the Cu—towards the inner end of the Mo at the Cu base. Between this minimum and the Mo the concentration of the electrons will not be governed by the Boltzmann relation on account of the drain of electrons to the Mo from it in the local accelerating field but the flux of electrons to the Mo is more nearly determined by the electrons reaching the potential minimum from the plasma, and the two components are nearer

$$pI_1 \int \exp\left(-eV_{\min}/kT_1\right) dl$$

and

$$pI_2 \int \exp\left(-eV_{\min}/kT_2\right) dl$$

along this inner section of Mo. On account of the entry of the electron temperatures through the exponential there will thus be an excess reception of the fast electrons relative to the slow compared with what would have been received had there been no potential minimum. Similar reasoning is applicable to the Druyvesteyn distributions. The approximate linearity of the lines in Fig. 2 must be regarded as fortuitous, although the sense of the changes is that predicted and the approximate constancy of the electron temperatures in the table may be regarded as in part due to the insensitiveness of the semi-log method of analysis.

The arc tube used did not function well for the higher pressures used by Seeliger and Hirchert.[1] It is believed, however, for two reasons, that the effect studied now at low pressures may have been important in their tubes. The first is that with increase in pressure the ionization in the positive column becomes more concentrated towards the axis of a tube; it is thus likely that the ionization near the wall would be small, and the wall sheaths thick. The second is the observation made in the course of the present work that the sheaths which formed on the floating Cu at the higher pressures could be seen to be abnormally thick, which, whether due to an effect of collisions in the sheath, or to the first effect, would tend to lead to conditions similar to those which we have studied.

Our thanks are due to Professor W. B. Morton for the facilities provided for this work and to Professor O. S. Duffendack for discussion of it.