X-Ray Line Intensities and Cathode-Ray Retardation in Thick Targets of Silver

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K-line intensities from thick silver targets have been measured in arbitrary units for voltages up to 180 kv, with emergence angles from 1 to 25°. A practical use of these data is the determination of the voltage which will give the maximum efficiency of line emission at any emergence angle. At 1° the best voltage is 70 kv but at 25° it is well above 180 kv. These data are also used in conjunction with data on thin targets to get information as to the retardation of cathode rays in silver. For this purpose the thick-target data are corrected to eliminate the effects of target absorption, indirect production of line rays through fluorescence and loss of cathode rays by rediffusion. The loss of energy per unit distance along the path of a cathode ray varies as the -1.4 power of the speed, just as it does in light elements according to more direct evidence collected by E. J. Williams.

I. LINE INTENSITIES

DATA on x-ray line intensities as functions of tube voltage have often been used to test theories of ionization probabilities for inner electrons. Taking rays from the thick targets generally used, such tests depend on assumptions about the law of retardation of cathode rays within the targets. While the Thomson-Whiddington law has been used practically universally for this, evidence for a change has appeared from various sources. These include x-rays, in Kulenkampff's[1] comparisons of his continuous-spectrum intensities from thick and thin aluminum, which indicate that the change in the retardation rate with speed is less rapid than that law would make it. Our present purpose is to make similar comparisons for silver, with line intensities, by using our thin-target data already published[2] and thick-target data to be presented here; and to find a more quantitative law of cathode-ray retardation.

Our thick-target data were taken in two groups, with different tubes. The first tube[3] was made of brass and glass, cemented with Dennison's "royal scarlet" sealing wax and carrying the electrodes on greased ground joints. While this tube ran smoothly even at 200 kv, it was discarded for one with no wax or grease[4] and our conclusions on cathode-ray retardation are based on the work with this second tube.

Fig. 1 shows the data of both groups. Here the $K\alpha$ line intensity for constant current is shown as a function of U, the ratio of tube voltage to K-ionizing voltage, and Θ, the angle made by the x-rays with the polished surface of the target. The dependence on Θ is due primarily to absorption in the target but there are also slight differences of scale caused by differences in reflection efficiency at different parts of the crystal, which affect any comparison of line rays from sources of different widths.

In Fig. 2 the same data are shown logarithmically and another graph is added, above those for the angles noted, for the intensity with target absorption entirely eliminated, as described below.

A straight line here would correspond to the formula[5]

$$I(U)=K(U-1)^n, \qquad (1)$$

where K and n are constants. This formula is evidently not accurate even for the case of no target absorption, though for that case it fits fairly well up to about $U=4$ with $n=1.65$.

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[1] H. Kulenkampff, Ann. d. Physik **87**, 597 (1928), and Phys. Zeits. **30**, 515 (1929).

[2] D. L. Webster, W. W. Hansen and F. B. Duveneck, Phys. Rev. **43**, 839 (1933).

[3] D. L. Webster, H. Clark and W. W. Hansen, Phys. Rev. **37**, 115 (1931).

[4] D. L. Webster, W. W. Hansen and F. B. Duveneck, Rev. Sci. Inst. **3**, 729 (1932).

[5] D. L. Webster and H. Clark, Proc. Nat. Acad. Sci. **3**, 181 (1917); A. Jönsson, Zeits. f. Physik **36**, 426 (1926); S. K. Allison, Phys. Rev. **30**, 245 (1927); and others.

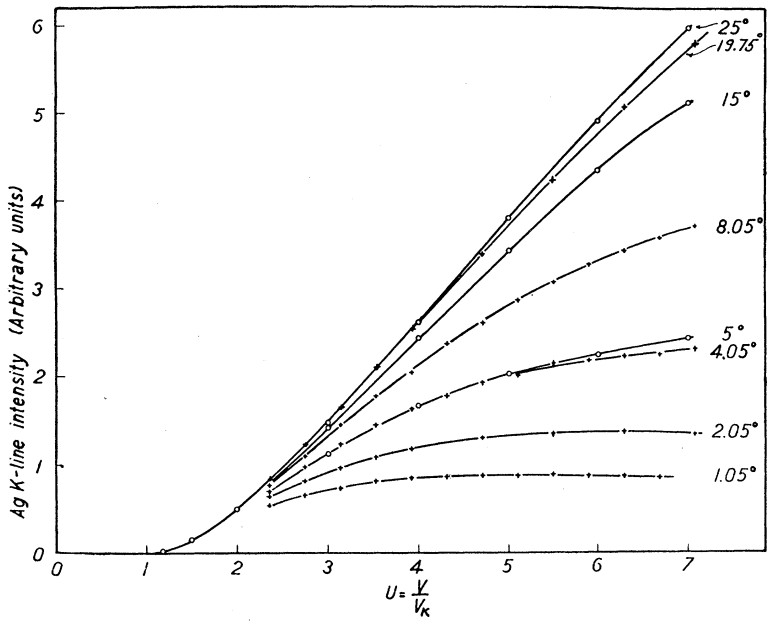

FIG. 1. Ag line intensities for constant current.

For practical uses of line rays, such as crystal analysis, it is often necessary to choose a tube voltage for maximum output at constant power, rather than constant current. In Fig. 3, therefore, we have plotted graphs on this basis. Evidently the best voltage depends greatly on the emergence angle.

II. CORRECTIONS

The comparison of thick and thin targets proposed in Section I would be more direct if the thick targets did not involve certain minor complications. One of these, obviously, is the loss of intensity by absorption of some of the rays to be measured, on their way out of the target. Another is a gain of intensity in the line-spectrum rays by fluorescence absorption of continuous-spectrum rays. A third complication is a loss resulting from the loss of some of the cathode rays by rediffusion. The next procedure is therefore to correct the observed intensities into those of an ideal thick target, not involving these complications.

To eliminate absorption of the line rays, one might perhaps attempt first to calculate the absorption from the depths at which the rays are produced, obtaining these in turn from such data as are already at hand on the retardation of cathode rays in the target material. Practically, however, this is next to impossible because in any element as heavy as silver the depths of penetration of cathode rays are limited much more by diffusion than by retardation and the combined effect of these two processes is very difficult to calculate. We are therefore using the experimental method due to Kulenkampff[6] which is based on a comparison of rays at different emergence angles. Theoretically, if all the rays could be produced at the same depth, X, their intensity would be reduced by absorption in the ratio $\exp(-\mu X \csc \Theta)$ where Θ is the emergence angle measured from the surface of the target. A graph of the logarithm of the intensity against $\csc \Theta$ would then be linear, and could be extrapolated accurately to $\csc \Theta = 0$, eliminating all target absorption. As a matter of fact, however, the rays are not all produced at the same depth and the graph is therefore not quite straight; and with an element as heavy as silver, the depths of production vary even more than the depths of penetration of cathode rays, because about a third of the line-spectrum x-rays

[6] H. Kulenkampff, Ann. d. Physik **69**, 548 (1922).

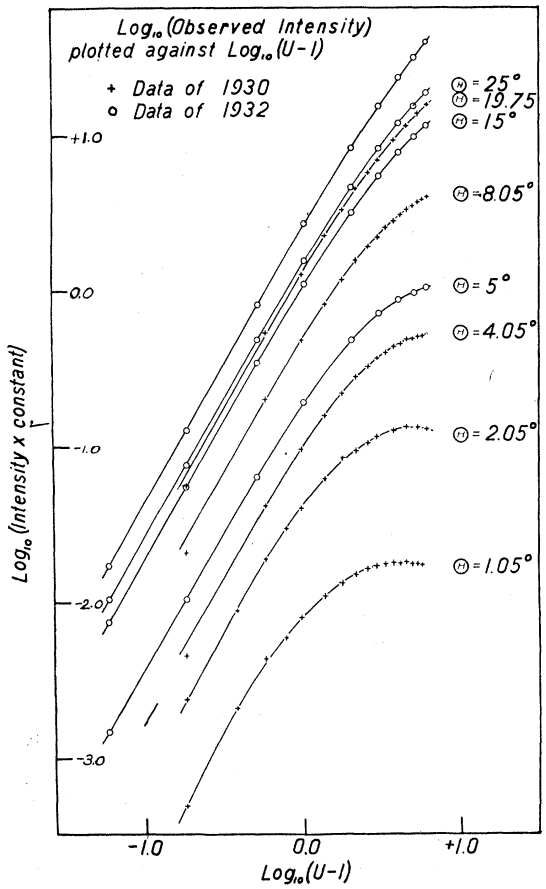

FIG. 2. Intensities on logarithmic scales.

are produced indirectly by fluorescence within the target. This departure from straightness produces some uncertainty in the extrapolation, and another source of uncertainty here is the difficulty of standardizing the units of intensity at different emergence angles. To remove this latter uncertainty, the best plan is to straighten one of the graphs arbitrarily by a proper choice of units for each emergence angle. If there was any theory for the correct curvatures of these graphs, this plan would sacrifice it and thereby lose accuracy. Without any such theory, however, it gains accuracy by reducing the curvatures of the graphs most strongly curved. The one we have chosen to straighten is that for $U=2$. The family of graphs thus obtained is shown in Fig. 4, and the intensities for csc $\Theta=0$ from this figure are those plotted as the uppermost graph in Fig. 2. Incidentally, the spacings of the graphs in Fig. 2 at $U=2$, or $\log_{10}(U-1)=0$, are proportional to csc Θ, except for the lowest two graphs, which would require too much space if plotted this way. The regularity with which the curvatures of these graphs increase with this spacing is therefore an index of the consistency of these data.

The corrections for the other two complications are by methods explained elsewhere. For the elimination of the rays produced indirectly, through fluorescence, we use the data of Hansen and Stoddard.[7] Working with palladium at an emergence angle of 90°, they find the direct rays, produced by impact ionization, to constitute 0.66 of the total line-spectrum rays at 40 kv, 0.67 at 100, and 0.68 at 180. For silver, the ratios are probably slightly lower but since our intensities are in an arbitrary unit, the only important feature of this ratio is its change with voltage. Data on silver up to 80 kv[8] indicate small changes similar to those with palladium. Here, therefore, we are simply using the palladium ratios and neglecting the small effect of target absorption on them.

For the third correction, for rediffusion, the method is that of Webster, Clark and Hansen[3] but with one notable improvement. To calculate the loss of intensity at any voltage by rediffusion, one needs to know the function giving the intensities to be found without rediffusion, at all lower voltages. Fortunately this function need not be known accurately but whereas we used very rough approximations before, we used Eq. (1) this time, with $n=1.73$. The correction is made after those for absorption and indirect rays, by multiplying the intensities as they are then by the factors given in Table I.

TABLE I. *Correction factors for rediffusion.*

U	Factor	U	Factor
1.000	1.00	3	1.26
1.056	1.01	4	1.28
1.178	1.06	5	1.29
1.5	1.16	6	1.30
2.0	1.23	7	1.31

[7] W. W. Hansen and K. B. Stoddard, Phys. Rev. **43**, 701 (1933).

[8] D. L. Webster, Proc. Nat. Acad. Sci. **14**, 339 (1928).

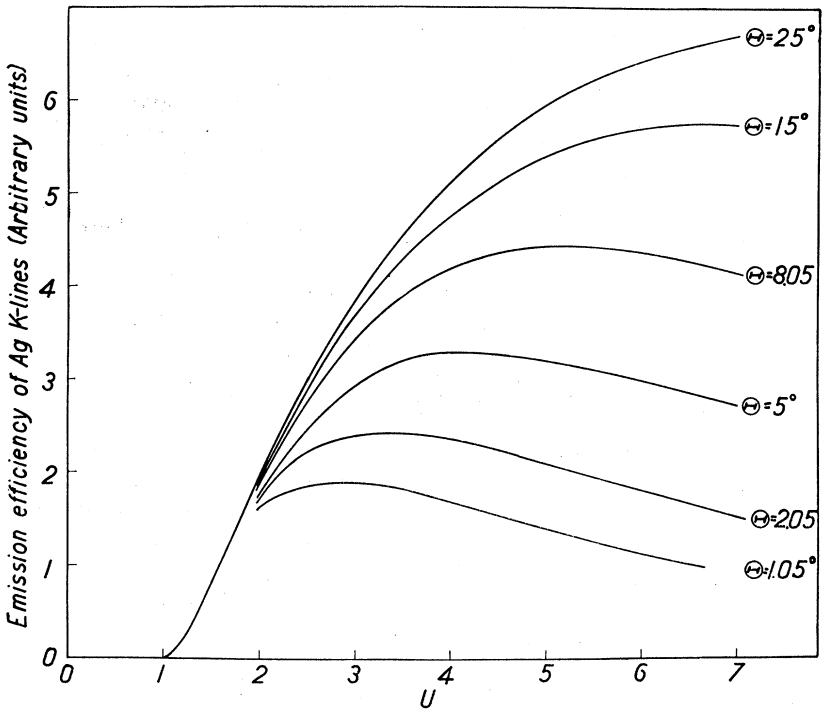

FIG. 3. Ag line efficiencies in arbitrary units.

The resulting intensities are shown logarithmically in Fig. 5. As is evident there, the corrections for indirect rays and rediffusion make the graph steeper than that of Fig. 2, which is repeated here for comparison. In fact, where the data corrected for absorption only were fitted fairly well below $U=4$ by Eq. (1) with $n=1.65$, those corrected for all three complications are fitted about equally well over the same range with $n=1.74$.

It must be noted, however, that Eq. (1) does not fit exactly with any fixed exponent. The best value of n in the immediate neighborhood of any point is of course the slope of the logarithmic graph at that point. Like the slope of any empirical graph, this is somewhat uncertain, especially near $U=1$, where the intensity is weak and also the errors in U show prominently in log $(U-1)$. Fortunately, however, we can get the slope at $U=1$ from our data on thin targets,[2] because the derivative of the thin-target intensity with respect to U is finite and not zero at $U=1$. Whatever the law of cathode-ray retardation may be, provided only that it is continuous, equations to be given below show that this behavior of thin targets requires thick targets to have n approach 2 as U approaches 1. We have therefore used this value as one of the data for plotting n against U, regarding it as more reliable than any other point near $U=1$. Above that, the values of n are computed from secants between the points of Fig. 5 and they are plotted for values of U in the middles of these intervals. The resulting graph is Fig. 6.

III. Cathode-Ray Retardation

Returning to the problem proposed in Section I, we shall now use our data on line intensities from thick and thin[2] targets to calculate the cathode-ray retardation rate. Letting U' represent the kinetic energy in terms of the ionization energy, at a distance s along the path of a cathode ray in the target, as U represents it at $s=0$, it is known that

$$i_o(U)=(dI_o(U)/dU)(-dU'/ds)_{U'=U} \qquad (2)$$

where $I_o(U)$ is the intensity from an idealized thick target, obtained as described above, and

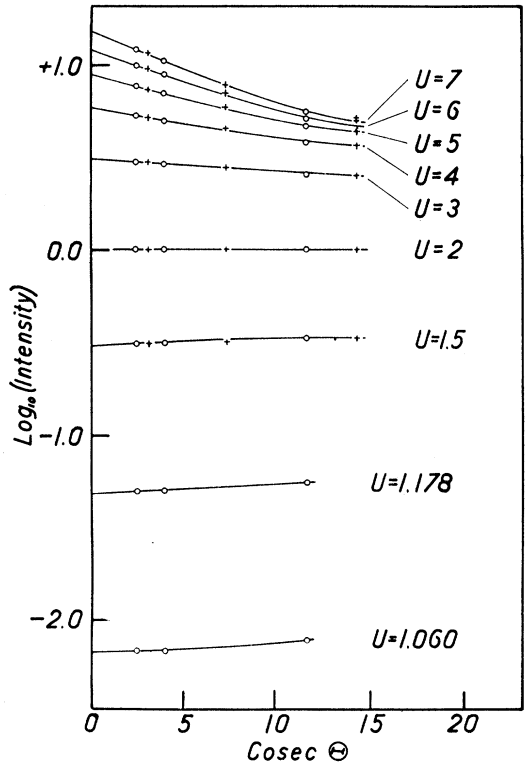

FIG. 4. Eliminating target absorption.

$i_o(U)$ is the intensity per unit thickness from a correspondingly idealized thin target. Both of these intensities are measured here in arbitrary units. Theoretically these units ought to be the same, so that they would cancel in solving for $-dU'/ds$. Practically, however, the difficulties of measuring the thickness of a thin target accurately make i_o even more arbitrary than I_o. Consequently we do not wish to commit ourselves on the absolute value of $-dU'/ds$ here beyond its order of magnitude, which was shown, in our paper on thin targets, to agree well enough with previous measurements. The objective here is rather to find how dU'/ds varies with U'.

While the data for $i_o(U)$ and $I_o(U)$ are fairly accurate, those for $dI_o(U)/dU$ are subject to the uncertainty common to all derivatives of empirical functions. In this case, this uncertainty is increased by the rapidity with which $dI_o(U)/dU$ changes with U. The logarithmic derivative shown in Fig. 6, however, changes much less rapidly and needs only to be multiplied by $I_o(U)/(U-1)$ to give $dI_o(U)/dU$. The best procedure, therefore, seems to be to eliminate the erratic errors as well as possible by reading values of these functions from the graphs of Figs. 6 and 5. Since this gives dU'/ds as a continuous function, it is plotted as such in Fig. 7. In this graph the upper limit of U is 6.5, as in Fig. 6. For the lower limit, an extension of the reasoning about the behavior of these functions near $U=1$ gives a value for dU'/ds for that point but not with the same degree of certainty, so the graph is dotted below $U=1.2$.

In much of the theoretical work done on cathode rays, relativity has been neglected, more or less of necessity, and their behavior has been described in terms of either their kinetic energy or the square of their speed, as if these quantities were equivalent. At the lower end of the range of the present data, this is not very inaccurate but at the upper end the kinetic energy varies practically as the cube of the speed. This raises a question, whether we should expect dU'/ds to be specified most simply in terms of energy or of speed.

For evidence on this point from previous work,

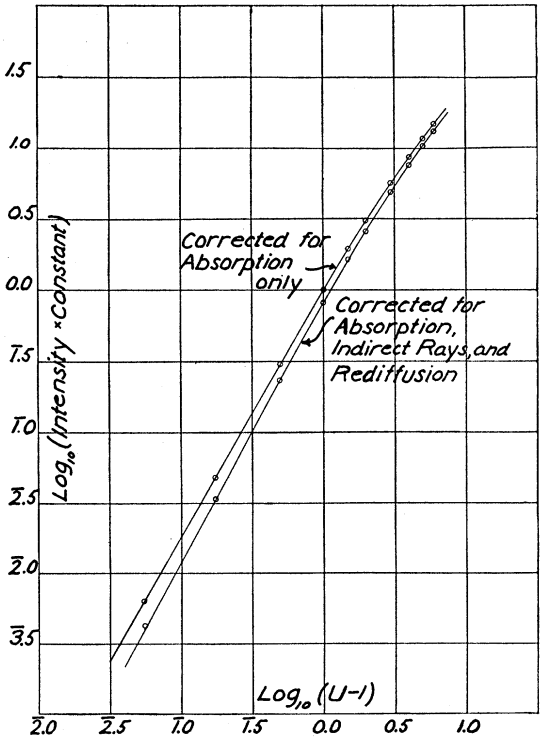

FIG. 5. Intensities after corrections.

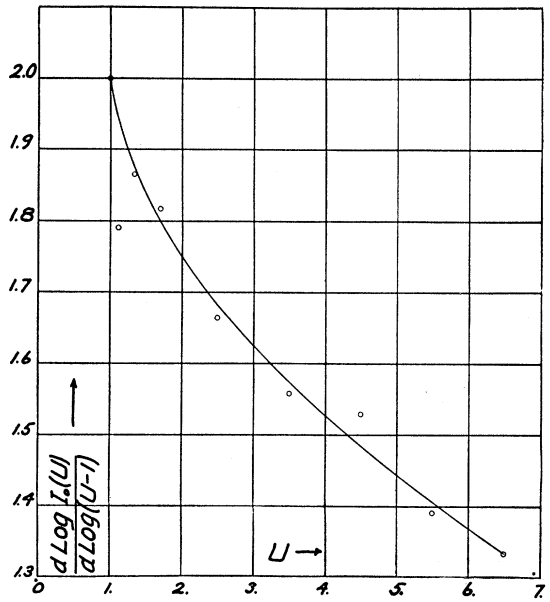

FIG. 6. n as a function of U.

an exceptionally clear case is described by Williams[9] in reviewing the work of Becker[10] on aluminum and White and Millington[11] on mica. Their data begin at $\beta=0.52$, or about 80 kv, in the middle of our range. For them a constant power of the speed, namely the -1.4 power, gives a surprisingly close proportionality to a quantity differing by only a few percent from $-dV'/ds$, even up to $\beta=0.96$. Over most of this range no constant power of the kinetic energy could imitate any constant power of the speed. For this reason we have plotted two graphs in Fig. 7, both logarithmic, against energy and (speed),[2] to show whether any proportionality occurs in our data with any power of either energy or speed.

It is evident that while neither graph is exactly linear, the speed graph is the straighter of the two. Furthermore, while the slope of the straightest part of this graph, in terms of speed rather than its square, is -1.5, slightly steeper than Williams' formula would indicate, the slope of a chord from end to end is almost exactly his exponent, -1.4.

To be sure, a different exponent is advocated by him for very low voltages, below 20 kv, where he puts the law in a form equivalent to an exponent -0.82 for the energy, or nearly -1.7 for the speed. For the region between 20 and 80 kv he has no data, but the -1.4 power holds so well from high speeds down to 80, and the exponent -1.7 differs from -1.4 so little, that it seems reasonable to expect a steady transition between them. This agrees excellently with our data—indeed too well for us to consider the agreement not partly due to chance.

For practical purposes it is often convenient to calculate in terms of voltage rather than speed and a fractional power of V' will sometimes serve for rough calculations. For the lower half of our range the best exponent is -0.67; for the upper half, about -0.45; near the middle or for the range as a whole, about -0.6.

The theoretical significance of our results becomes evident through later papers of Williams.[12] In these he made very careful analyses of the data on light elements and concluded that Bethe's[13] theory, based on nonrelativistic wave mechanics, was the best, especially if modified in certain ways based on relativity. Since most of the energy taken from the cathode rays is taken by the more easily ionized electrons in the atoms, the differences between silver and the light elements ought to be small, probably

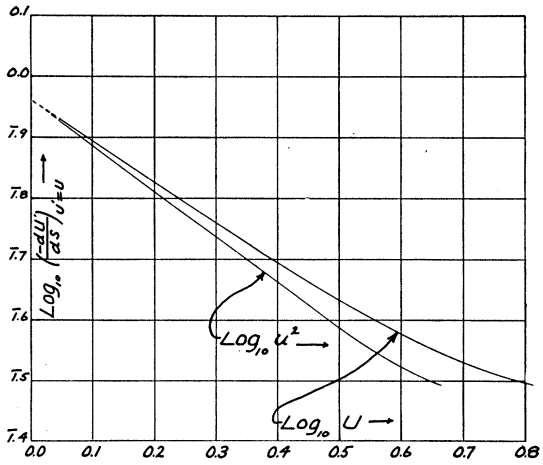

FIG. 7. Cathode-ray retardation in arbitrary units.

[9] E. J. Williams, Proc. Roy. Soc. **A130**, 310 (1931).
[10] E. Becker, Ann. d. Physik **78**, 209 (1925).
[11] P. White and G. Millington, Proc. Roy. Soc. **A120**, 701 (1928).
[12] E. J. Williams, Proc. Roy. Soc. **A130**, 328 (1931); **A135**, 108 (1932).
[13] H. Bethe, Ann. d. Physik **5**, 325 (1930).

within our present limits of error. So the similarity of our retardation function to Williams' indicates that our data, like his, favor Bethe's theory.

Sooner or later our results will probably be supplanted by others based on more direct measurements of retardation. If they confirm the law deduced here, the present measurements will no longer be needed as evidence on cathode-ray retardation but will then become evidence on the theory of thick-target x-ray emission. They will then indicate that the process of line-spectrum emission does, indeed, depend primarily on the laws of thin-target emission and cathode-ray retardation, as assumed here, with minor complications due to absorption, rediffusion and the indirect production of line rays by fluorescence and that all the important factors in the process of thick-target line emission are now recognized and approximately measured.