$$\begin{array}{ccccc} Ne^{20} & + & n^1 & \rightarrow & O^{17} & + & He^4 \\ (19.9967\pm0.0009) & + & (1.0067-E) & \rightarrow & (17.0032-E'\pm0.003) & + & (4.00216\pm0.0004). \end{array}$$

Here E represents the energy converted into a γ-ray in the formation of a neutron from boron and E', the similar value for the formation of O^{17} from N^{14} by the capture of an alpha-particle by the latter. If E and E' are both zero, then 0.002 unit of mass or 1.9×10^6 electron-volts must be supplied. Since E is probably larger than E', the amount of energy which must be supplied is probably greater than 2×10^6 electron-volts.

The velocity v_n of the neutron, its kinetic energy KE_n, the kinetic energy $-\Delta KE$ which disappears in the reaction, and the energy which presumably escapes in the form of a γ-ray are given in Table I.

TABLE I.

	1	2	3	4
$v_n\times10^{-9}$ cm/sec.	3.9	4.5	5.3	5.1
$KE_n\times10^{-6}$ e.v.	7.8	10.6	14.5	13.4
$-\Delta KE\times10^{-6}$ e.v.	5.0	5.9	10.6	6.9
Energy of γ-ray$\times10^{-6}$ e.v.	3.1	4.0	8.7	5.0

The average energies in millions of electron-volts of the neutrons which have been found to disintegrate light nuclei are 5.8 for nitrogen, 7.0 for oxygen, and 11.6 for neon. The value for oxygen is taken from the work of Feather. The number of disintegrations from which the averages were taken is 19 for nitrogen but only 4 each for oxygen and neon. While the number is too small to show definitely what the relations are, it seems probable that the energy necessary for disintegration increases rapidly in the order N^{14}, O^{16}, Ne^{20}. The mass values indicate that the energy needed to supply mass in each of the reactions increases in this order. The actual values are uncertain but those now accepted indicate that the change of mass is equivalent to a decrease of 1.4×10^6 e.v. for nitrogen, a conservation of mass for oxygen, and an increase of $1.9\times10^{+6}$ e.v. for neon.

The kinetic energy which disappears in the disintegration of neon varies from 5.0 to 10.6×10^6 e.v. Of this, from 3.1 to 8.7×10^6 e.v. is presumably converted into a γ-ray. It is of interest that neon seems to follow the rule found for nitrogen and oxygen, which is that in every disintegration by capture kinetic energy is found to disappear, except that with nitrogen it may occasionally be conserved. In no case is there an increase of kinetic energy.

The range-energy relations for O^{17} in neon are somewhat uncertain, and so the values given above should in this respect be considered as preliminary.

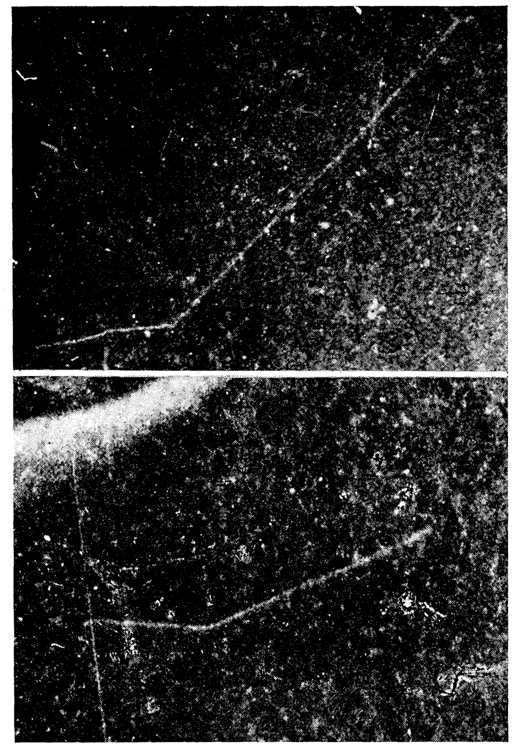

FIG. 1. Stereoscopic views of the disintegration of a neon nucleus by capture of a neutron.

The figure is a reproduction of a photograph of the disintegration of a neon nucleus by a neutron.

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On The Production of Positive Electrons by Electrons

The experimental discovery of the positive electron makes possible, as has recently been observed by Oppenheimer and Plesset,[1] a consistent development of Dirac's theory of the electron, in which the appearance of pairs (electron and anti-electron) plays an important part. It appears that, in the experimental production of the pairs, the Coulomb fields, especially those of nuclei, are most important. In this case the theory shows that radiation of energy greater than $2mc^2$ is required for their production. Oppenheimer and Plesset have made a preliminary study of the production of pairs by γ-rays; we have carried through the analogous work for their production by high energy electrons (and positives). For γ-rays it appears, both from OP and from the experiments of Anderson, that the production of pairs becomes increasingly important as the energy of the γ-rays is increased; we have wanted to see whether this

[1] J. R. Oppenheimer and M. S. Plesset, Phys. Rev. **44**, 53 (1933) (hereafter referred to as OP).

is also true for electrons of high energy and to estimate the relative numbers of pairs produced by the two radiations. We shall see that for electrons of sufficiently high energy the production of pairs is as important as for γ-rays; and we believe that these processes will prove essential to an understanding of the behavior of the corpuscular component of the cosmic rays, in particular their absorption curves and the multiple tracks observed in cloud chambers.

We consider, then, a primary electron (or positive) of high energy passing through the Coulomb field of a nucleus; the primary may lose part of its energy and eject a pair. This process is similar to the photoelectric production of pairs in that a nucleus must take up a small recoil momentum, and differs from it in that the pair need not, and in general does not, receive all the energy of the primary radiation. We have treated the case in which the primary loses but a small part of its energy; this makes it possible to neglect the effect of interchange of primary and secondary and the effect of the nuclear field on the primary. When so formulated the calculation is altogether analogous to that of OP for the photoelectric production of high energy pairs. This parallelism between the two cases was maintained throughout the work. To this approximation primary electrons and primary positives behave alike and our results are equally applicable to both.

We have followed OP in using wave functions for the particles of the pair, correct to the first order only, in the field of the nucleus. This approximation is known to be invalid when these particles have very high energies and for this reason the OP results are unreliable for γ-rays of too high energy. The approximation appears, however, to be just sufficient to answer the question which we have put as to the relative importance of production of pairs for the two radiations. For the absolute number of pairs produced our results, too, are untrustworthy. Nevertheless, in our calculation it is only when the energy of the pair is too large that the results are invalid; the energy of the primary may be as great as one wishes.

On this basis the calculation can be made by a straightforward application of the quantum electrodynamics, which is formally analogous to the problem of the interaction of two electrons in an external field. For the case when the kinetic energies of the particles of the pair are large compared to their proper energies and still small compared to the energy of the primary, the differential cross section for production of a pair is:

$$\sigma d\epsilon_+ d\epsilon_- = (\pi/2)(Z\alpha)^2(e^4/m^2c^4)\{\epsilon_+^2+\epsilon_-^2+(8/\pi^2)\epsilon_+\epsilon_-\}(\epsilon_++\epsilon_-)^{-4}\ln(\epsilon_0^2/\epsilon_+\epsilon_-)d\epsilon_+d\epsilon_-. \quad (1)$$

Here ϵ_0, ϵ_+, ϵ_- are the energies of the primary, positive, and electron in units mc^2, and Ze is the nuclear charge. The distribution in relative energies of positive and negative is of the same general sort as in the photoelectric case, there being a tendency toward unequal energies. The distribution in angle is also similar, the mean angles for positive, negative and deflected primary being of orders of magnitude $1/\epsilon_+$, $1/\epsilon_-$, $1/\epsilon_0$, respectively. Formula (1) shows that most of the pairs have small fractions of the primary energy, which justifies our having regarded this as the important case.

To get the total cross section for production of pairs one should integrate the differential cross section over all ϵ_+ and ϵ_-. We can get a lower limit on the total cross section by integrating (1) over the region for which it is valid. For very large ϵ_0 the result is

$$\int \sigma d\epsilon_+ d\epsilon_- \gtrsim \pi(Z\alpha)^2(e^4/m^2c^4)\ln\epsilon_0\cdot\ln\bar{\epsilon}. \quad (2)$$

Here $\bar{\epsilon}$ is the highest energy of a particle of the pair for which the first order calculation can be applied and is of the order of magnitude $(2\pi^2/Z\alpha)$. (In the case when $\epsilon_0 \gg 1$ but $\epsilon_0 < \bar{\epsilon}$, one must replace $\ln\bar{\epsilon}$ in (2) by $(\ln\epsilon_0)/2$.)

The cross section (2), which gives a lower limit to the production of pairs by high energy electrons (or positives), differs from the OP formula for the case of γ-rays by a factor of about

$$\begin{array}{ll} \pi\alpha\ln\epsilon_0\cdot\ln\bar{\epsilon}, & \epsilon_0>\bar{\epsilon} \\ (\pi\alpha/2)(\ln\epsilon_0)^2, & \epsilon_0<\bar{\epsilon}. \end{array} \quad (3)$$

For primary particles of energy 10^8 volts this factor is roughly 0.3; for 10^{10} volts it is about 1.2. Thus for the range of energies of the cosmic rays the production of pairs is surely as important for electrons and positives as it is for γ-rays.

We wish to thank Professor Oppenheimer for suggesting this problem to us and for much helpful advice.

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The Relative Abundances of Elements of Even and Odd Mass Number and Atomic Number

In matter which is in equilibrium[1] as regards transmutations, the particles which obey the Fermi-Dirac statistics should be, as a rule, less abundant than the particles which obey the Einstein-Bose statistics; this should hold, whatever the ultimate particles are, of which nuclei are composed. The mean number of particles $\overline{X}_s$ of the sort s is always a monotonic increasing function[2] of μ_s for any particular value of the temperature T but there is a marked difference between the dependence of $\overline{X}_s$ upon μ_s in the case of Fermi particles on the one hand and in the case of Einstein particles on the other. For the former, it can be shown that as μ_s increases from 0 to infinity, $\overline{X}_s$ increases from 0 to infinity. For the latter, as μ_s increases from 0, $\overline{X}_s$ increases at first in the same way as before, but as μ_s approaches 1,

[1] T. E. Sterne, Phys. Rev. **43**, 585, 768, 1056 (1933); M.N.R.A.S. in press.

[2] The parameter μ_s defines the distribution of particles of the sort s in the Einstein or Fermi statistics. It always contains the factor exp $[c^2M_s(f_H-f_s)/10{,}000RT]$. c is the velocity of light, R is the gas constant, M_s is the mass number, f_H and f_s are the packing fractions of hydrogen atoms and atoms s.