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The Absolute Saturation of Cubic Cobalt

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With an ellipsoid of cobalt, quenched so as to be in the cubic state, and Weiss' method of extracting the ellipsoid from the field of a powerful electromagnet, the magnetization was measured for a given temperature but increasing field strengths. With Weiss' formula for the approach to saturation the magnetization for infinite field was calculated. The Weiss formula fitted the upper half of the experimental curve within 0.03 percent. When the saturation magnetizations, J_{ST}, obtained for 13 temperatures were plotted against the square of the absolute temperature a straight line was obtained. When this was extrapolated to absolute zero the saturation intensity (J_{SO}) was found to be 1418. By plotting J_{ST}/J_{SO} against $(T/\theta)^2$, θ being the Curie point, the line obtained coincided with that for iron and nickel, but not with the curve for hexagonal cobalt nor the line found for orthorhombic crystals. It is indicated that crystal structure plays an important part in ferromagnetism.

THE purpose of this investigation was to determine the absolute saturation intensity of magnetization for cubic cobalt and the law of approach to saturation as a function of the magnetic field and the absolute temperature. It was also of interest to discover if cubic cobalt shares the same reduced-saturation-*vs.*-reduced-temperature curve with the cubic crystals, iron and nickel, or with the "pseudo-cubic" group such as magnetite and cementite. This problem was suggested by Dr. Francis Bitter.[1]

The method adopted was a slight modification of that employed by Weiss and Forrer[2] in determining similar data for several other substances—particularly iron, nickel, magnetite and cementite. It consisted of a repeated extraction of the substance in the form of an ellipsoid of revolution from a system of coils placed in the field of a powerful electromagnet having conical pole pieces and the measurement of the magnetization by the deflection of a ballistic galvanometer which had been calibrated in terms of a known magnetic moment. Before each extraction the ellipsoid was immersed in a pentane bath of known temperature previously cooled with liquid air. The range of temperatures investigated was from the boiling point of liquid air to that of boiling water. The extraction was accomplished by means of the mechanism indicated in Fig. 1. Operation of a trigger released a brass rod (R), to the lower extremity of which the ellipsoid was secured by a Bakelite clamp (B), thus permitting a tightly stretched spring (S) suddenly to jerk the ellipsoid out of the center of the field, the induction coils (C, C, C', C'), and the Dewar flask (F) containing the pentane (P). An air cushion was arranged to stop the rapid movement of the rod. The throw of the ellipsoid was about 20 cm. Weiss and Forrer used a throw of only 11.8 cm and estimated that the error introduced for failure to extract to infinity amounted to only 0.1 percent in a field of 20,000 gauss.

The ellipsoid of cobalt was obtained from Ward's Natural Science Establishment, Inc. of the University of Rochester and showed the following analysis (in percent): cobalt, 98.00; nickel, 0.75; iron, 0.48; sulphur, 0.10; silicon, 0.05; carbon, 0.25; aluminum, 0.25. To make sure that the cobalt was in the cubic crystalline form, the ellipsoid was very carefully heated above its Curie point (θ) (1413°A) and quenched in oil, precautions being taken to prevent oxidation and other injury.

The dimensions of the ellipsoid were: major axis, 1.00 cm; minor axis, 0.42 cm; volume, 0.0924 cc. The ellipsoidal shape was preferable because of the greater ease afforded for magnetizing it uniformly as well as calculating its demagnetizing effect upon the magnetic field.

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[1] Bitter, Phys. Rev. [2] **39**, 337 (1932); also personal correspondence.

[2] Weiss and Forrer, Ann. de Physique [10] **12**, 279 (1929).

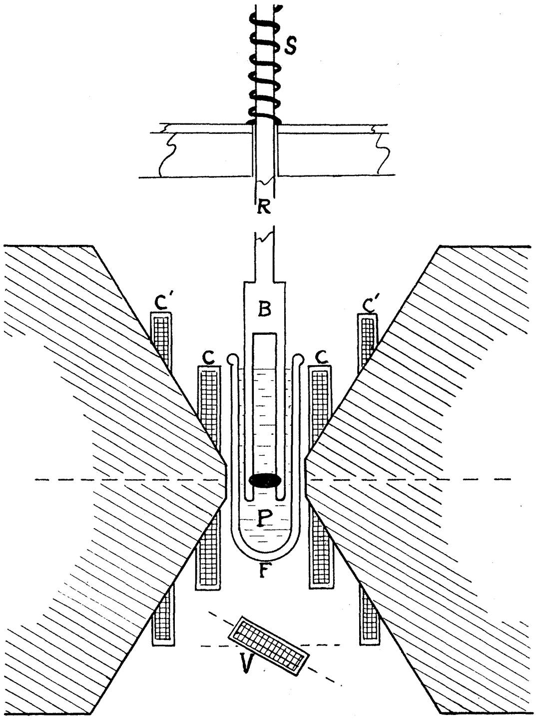

FIG. 1. Experimental arrangement.

Measurements accompanying extraction were possible at the rate of about one in every two minutes, simultaneous readings being taken of the exciting field current, the temperature, and the galvanometer deflection. Over 1500 extractions were observed.

The thirteen temperatures at which extractions were made are as follows: 93, 130, 137, 156, 180, 196, 202, 222, 240, 257, 273, 306, and 373° absolute. These were measured by means of a copper-constantan thermocouple which had been calibrated at the following temperatures: boiling liquid air, carbon dioxide snow and ether, melting ice, room temperature, and boiling water. The "null method" was used in determining the thermocouple readings. The contact junction was mounted in the pentane bath very near the position occupied by the ellipsoid before extraction. Convection in the pentane was sufficient to fully refrigerate both the ellipsoid and thermocouple.

The field strength of the electromagnet was adjusted by a rheostat which regulated the exciting current. Extractions of the ellipsoid were made at the following settings of field current: 0, 4, 9, 12, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30 amperes. By means of a previously calibrated bismuth spiral these amperes of field current were converted into gauss of field strength. It was found that the latter was a linear function of the current between 16 and 30 amperes.

It was necessary to correct these values of the field strength ($\mathbf{H}_0$) on account of the demagnetizing effect of the ellipsoid, by using the following relation given by Ewing:[3]

$$\mathbf{H}_e = \mathbf{H}_0 - \mathbf{N}\mathbf{J},$$

where $\mathbf{H}_e$ is the effective magnetizing force, $\mathbf{J}$ the intensity of magnetization and $\mathbf{N}$ the demagnetizing factor found from the equation:

$$\mathbf{N} = 4\pi\left(\frac{1}{\mathbf{e}^2} - 1\right)\left(\frac{1}{2\mathbf{e}}\log\frac{1+\mathbf{e}}{1-\mathbf{e}} - 1\right).$$

Here $\mathbf{e}$ is the eccentricity of the ellipsoid. The value thus obtained for $\mathbf{N}$ is 1.79, the demagnetizing factor. The corrected values of the field strength for the ellipsoid at room temperature are shown in Table I, column 4. These values approximated closely those for other temperatures as $\mathbf{J}$ did not vary over 1 percent.

INDUCTION COILS AND COMPENSATION SYSTEM

The induction coils to which the ballistic galvanometer was connected are represented by *C*, *C* in Fig. 1. These were in the form of two thin disks, each mounted on opposite pole pieces near the center of the field, their axes being in line with the latter. They were of sufficiently large diameter so that when the small ellipsoid was placed lengthwise in the field and then extracted the change in induction would produce a galvanometer deflection proportional to the magnetic moment of the ellipsoid.

If it had been found possible to maintain the exciting field current perfectly constant, coils *C*, *C* would have constituted a sufficient induction system; but due to slight fluctuations of the field, the water pressure in the cooling system of the electromagnet, the high sensitivity of the induction coils and galvanometer and other

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[3] Ewing, *Magnetic Induction in Iron and Other Metals*, Chapter I.

causes, the resulting effect would produce a continual drifting of the zero position of the galvanometer even though the ellipsoid remained fixed. To overcome this difficulty it was found necessary to use an additional pair of "compensation coils" (C', C'), similar to C, C, except larger and farther away from the ellipsoid. These two compensation coils, their axes being coincident with the main flux, were connected in opposition to the induction coils (C, C) so as to exactly balance out any change of flux caused by the effect just described. On the other hand, due to the relatively close proximity of the interior coils (C, C) to the ellipsoid, any change in flux due to its extraction would produce a predominating influence only upon C, C.

On account of the unequal approach to saturation in different parts of the pole pieces the compensation coils (C', C') were insufficient to give perfect compensation except for a single excitation. It was therefore necessary to connect in series with the coil system and galvanometer a small "vernier compensation coil" (V). This was mounted in one edge of the main field, near the other coils, upon a horizontal axis in such a manner that its effect would add to or subtract from the effect of the compensation coils (C', C'). Table I, column 2, shows the angular settings (as read upon a divided scale attached to the shaft comprising the axle) for complete compensation corresponding to the various values of main flux used. The compensation was adjusted by observing whether any galvanometer deflection accompanied the lowering of the field current by one or two percent, the correct adjustment being that position for which no deflection was obtained.

TABLE I.

Field current (Amp.)	Position of compensation coil (Degrees)	Image correction coefficient	Effective magnetic field ($\mathbf{H}_e$) (Gauss)
0	0	1.000	25
4	0	1.000	1580
9	18	1.118	6500
12	47	1.441	8290
14	59	1.625	9053
16	69	1.773	9525
17	74	1.840	9722
18	79	1.902	9890
19	84	1.952	10045
20	89	2.000	10200
21	94	2.042	10350
22	96	2.085	10500
23	98	2.115	10692
24	99	2.140	10845
25	100	2.155	11005
26	101	2.165	11180
27	101	2.171	11330
28	102	2.175	11500
29	102	2.178	11675
30	102	2.180	11825

MAGNETIC IMAGES

As the pole pieces of the electromagnet approach saturation their permeability diminishes to a marked extent causing the sensibility of the measuring apparatus to decrease proportionately. It was therefore necessary to determine experimentally the sensibility as a function of the excitation, and accordingly correct all galvanometer deflections for each extraction of the ellipsoid. Weiss termed this effect, "magnetic images," by analogy with the electrical images of Lord Kelvin.[4]

This study was made by substituting for the ellipsoid a definite magnetic moment in the form of a little coil which occupied approximately the same volume as the former. Instead of extracting the "image coil," a current of 1.800 amperes was reversed, thereby doubling the resulting galvanometer deflection. By starting with a deflection of 474.0 mm when the magnet was not excited, the deflection decreased to 217.4 mm for the maximum excitation of 30 amperes. Table I, column 3, gives this important "image correction" in the form of a ratio:

$$\frac{\text{image deflection for zero excitation of electromagnet}}{\text{image deflection for specific excitation of electromagnet}}$$

for each value of the exciting current used. These coefficients show the averages of more than 1000 observations of the "image effect."

The image coil consisted of 114 turns, wrapped upon a copper rod 4 mm in diameter. Its length was 1.0 cm and its outer diameter about 6 mm. It was mounted inside a glass tube through which a water current circulated to prevent noticeable rise in temperature during the brief period of time required for commutation of 1.800 amp.

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[4] Weiss, Ann. de Physique [10] 5, 171 (1926).

Obviously, to correct a given galvanometer deflection produced by the extraction of the ellipsoid, it is only necessary to multiply by the "image effect coefficient" corresponding to the particular field excitation used. Fig. 2, curve *B*, is a typical case giving the relation at room temperature between the field excitation and the galvanometer deflection as actually observed when the ellipsoid was extracted. Curve *A* of this same figure shows the same data *after* correcting for the image effect. Similar curves were obtained for the thirteen different temperatures investigated.

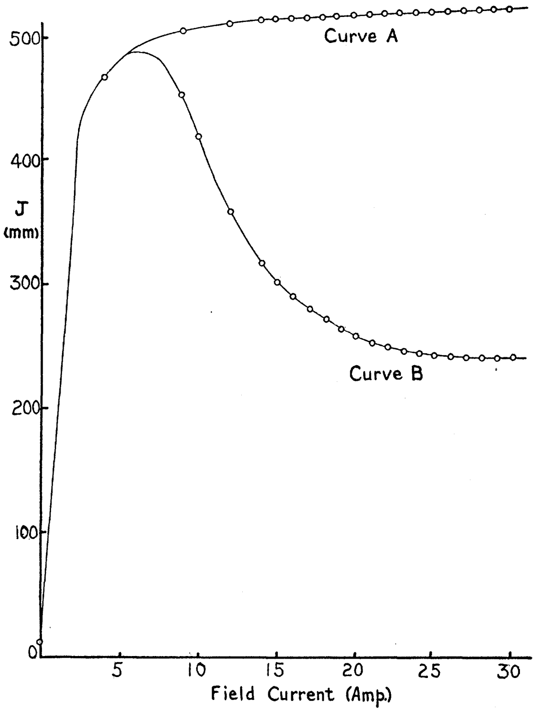

FIG. 2. Typical magnetization curve; Curve *A* corrected and curve *B* uncorrected, for image effect.

ABSOLUTE CALIBRATION

The curves last mentioned were plotted in terms of the image-corrected galvanometer deflection as a function of the excitation. It now remains to determine the constant ratio by which the *Y*-axis may be calibrated in terms of ordinary c.g.s. units of intensity of magnetization (**J**). The calibration of the *X*-axis in terms of **H** has already been mentioned.

With the coil system and other physical conditions kept constant, and with a current of 9 amperes in the electromagnet, the ellipsoid was replaced at the center of the field by a small coil similar to that used in studying the image effect. This "absolute calibration coil" consisted of 395 turns and had an effective area of 101.300 cm^2. When a current of 1.000 ampere was reversed in this coil there resulted a galvanometer deflection of 73.3 mm. Correcting for the image effect (1.118), the deflection amounted to 81.9 mm. Thus, since a known magnetic moment of 20.260 c.g.s. units produced a deflection of 81.9 mm, the ratio obtained is 0.247 c.g.s. units per mm. Now, dividing this ratio by the volume of the ellipsoid (0.0924 cc), the necessary coefficient for calibrating the galvanometer deflections in terms of the desired intensity of magnetization units is determined to be 2.67 c.g.s. units per mm per cc.

EFFECTIVE AREA OF CALIBRATION COIL

In the previous paragraph the effective area of the calibration coil was given as 101.300 sq. cm. This calculation was accomplished by comparing this coil with a larger coil of known area, the latter consisting of three turns wound closely upon a hollow Bakelite cylinder 6.994 cm in diameter, hence 115.257 sq. cm in total area. The small calibration coil was carefully mounted in the center of the larger coil, the two axes being coincident. These two coils were placed in the center of, and with axes coincident to, a uniform and intense magnetic field, the latter being produced by a large solenoid 133 cm long and 9.48 cm in diameter, and wound upon a hollow smooth cardboard cylinder. With the two coils connected in opposition to each other, a current of 16 amperes was commutated in the large solenoid which caused a deflection of 57.6 mm upon a ballistic galvanometer connected in series with the two coils. Again, with the two smaller coils connected in the same sense, a deflection of 893.34 mm resulted. The larger coil had the bigger effective area. From these two deflections and the area of the larger coil that of the calibration coil was obtained.

Results and Discussion

According to the recent theory of ferromagnetism advanced by Weiss,[5] Heisenberg,[6] Bitter[7] and others an unmagnetized sample of cobalt is regarded as made up of numerous regions each containing many atoms. Each region is supposed to be magnetized to saturation by a large "internal" or "molecular field," but the direction of this magnetization varies at random from block to block so that the specimen as a whole appears unmagnetized. Application of an external field merely tends to line up the directions of magnetization of the blocks, saturation being reached when they are all parallel. This saturation magnetization $\mathbf{J}_{ST}$ depends on the temperature $\mathbf{T}$, and investigation of its variation with $\mathbf{T}$ is equivalent to a study of the behavior of the atoms of a single region. Such data should be more valuable in checking theories as to the nature of ferromagnetism than results based on the statistical behavior of many regions, such as hysteresis curves.

It was first necessary to calculate $\mathbf{J}_{ST}$ for each temperature investigated. Although saturation could not be obtained experimentally Weiss and Forrer have established that the approach to saturation at a given temperature is represented very well by the equation

$$\mathbf{J}_{HT}=\mathbf{J}_{ST}(1-\mathbf{a}/\mathbf{H}),$$

where $\mathbf{J}_{HT}$ is the magnetization at the field $\mathbf{H}$, and $\mathbf{a}$ is a constant for a given temperature which measures the magnetic hardness. It was possible in this work to assign such values to $\mathbf{a}$ and $\mathbf{J}_{ST}$ that $\mathbf{J}_{HT}$ would fit the experimental curves within 0.03 percent between field currents of 16 and 30 amperes in the electromagnet. This verified Weiss and Forrer's formula for cubic cobalt. Fig. 3 shows in more detail the portion for high fields of curve *A* of Fig. 2, and similar curves for three other typical temperatures. It was to curves such as these that Weiss and Forrer's formula was applied to obtain $\mathbf{J}_{ST}$ for each of the thirteen temperatures.

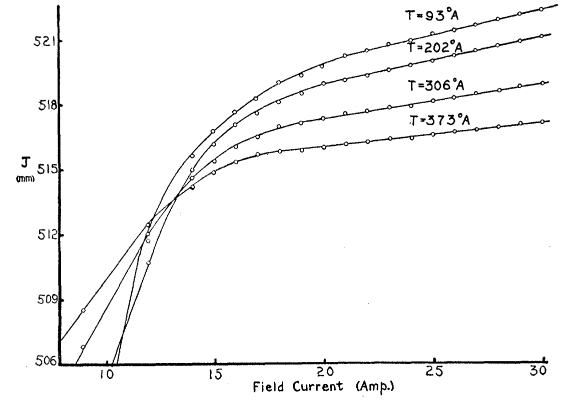

Fig. 3. Typical magnetization curves in region where Weiss and Forrer's formula could be applied.

The thirteen values of $\mathbf{J}_{ST}$ obtained were plotted as a function of the square of the absolute temperature. As in the case of the materials previously investigated a straight line was obtained which was extrapolated to absolute zero, giving $\mathbf{J}_{SO}$, the absolute saturation of cubic cobalt, which was found to be 1418 c.g.s. units per cc. With the weight of the ellipsoid (0.8180 g) and the density of cobalt (8.9) the value obtained was 1426. The error in these values is estimated at 2 percent, due chiefly to the absolute calibration. The error in the points of Fig. 4, which do not involve this calibration, is estimated at less than 0.5 percent.

According to Honda and Masumoto[8] the absolute saturation should be the same for cubic

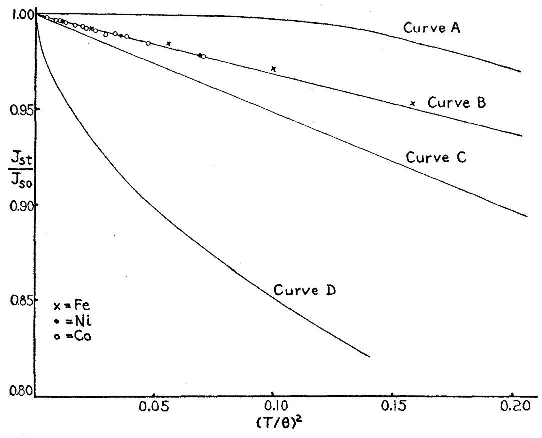

Fig. 4. Reduced magnetization (J_{ST}/J_{SO}) *vs.* square of reduced temperature ($\mathbf{T}/\theta$).

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[5] P. Weiss and G. Foex, Le Magnetisme, 101 (A. Collin, 1926).

[6] W. Heisenberg, Zeits. f. Physik **49**, 619 (1928).

[7] F. Bitter, Phys. Rev. **42**, 697 (1932).

[8] K. Honda and H. Mazumoto, Sci. Reports of Tohoku U. **20**, 323 (1931).

and hexagonal cobalt. A summary of the values of the saturation magnetization found by previous observers, who probably used a mixture of the two forms, is given in Table II for comparison. As most of these values are for room temperature it must be remembered that saturation at absolute zero is about 1 percent higher.

Table II. *Saturation values for cobalt.*

Observer	J	t
Ewing, 1889	1310	20°C
Weiss, 1910 (162, 8.9)	1442	17°C
Stifler, 1911	1421	20°C
Williams, 1915	1504	20°C
Barker, 1916 (from Kerr Effect)	1398	20°C
Honda and Masumoto, 1931 (single crystals)	1446	−273°C

Finally, the reduced saturation intensity ($\mathbf{J}_{ST}/\mathbf{J}_{SO}$) was plotted against the square of the reduced temperature ($\mathbf{T}/\theta$), where θ is the Curie point for cobalt, taken as 1413°A. This curve should be the same for all ferromagnetic substances. Fig. 4 shows the results obtained in this work on cobalt together with that of others for other materials. It will be seen that cubic cobalt lies on the same line (curve *B*) as the cubic crystals, iron and nickel, but that the orthorhombic crystals, magnetite, cementite and Fe_2B, fall on another line (curve *C*), while hexagonal cobalt gives a still different curve. Curve *A* is the one obtained from Langevin's classical formula,

$$\mathbf{J}_{ST}/\mathbf{J}_{SO}=\coth \mathbf{x}-1/\mathbf{x},$$

and curve *D* represents the formula obtained theoretically when quantum modifications are included, i.e.,

$$\mathbf{J}_{ST}/\mathbf{J}_{SO}=\tanh \mathbf{x}.$$

Some modification of the present theories is obviously called for, and it is evident that crystal structure is an important factor which will have to be taken into account in future theories.