Hoffmann Stösse and the Origin of Cosmic-Ray Ionization

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It is generally recognized that whatever may be the nature of the primary cosmic radiation, the actual rays which play the rôle in our experiments are charged particles. We are familiar with the existence of groups of charged particles in cosmic-ray phenomena. These groups seem to divide themselves into two more or less distinct categories, that involving a few rays and larger bursts of rays known as Hoffmann Stösse, in which hundreds and even thousands of charged particles may be emitted. The number of these Hoffmann Stösse emitted per unit *volume* per unit time from solids seems to depend little upon the atomic number (see reference 2). In other words it is the *number* of the atoms in a given volume rather than their weight which is significant. It is the purpose of this article to point out that an appreciable part, and possibly nearly the whole of the observed cosmic-ray intensity, may be attributed to bursts of the Hoffmann Stoss type in the atmosphere. Only in the case of a burst originating very near the apparatus will more than a single ray enter the apparatus, so that the observed characteristics of such rays will be those of single rays occurring at random.

Let q be the number of Stösse emitted per second per atom. If q is proportional to the intensity I of some radiation, a photon component of the cosmic radiation, for example, and if subscript s refers to the earth's surface, $q=q_sI/I_s$.

It may readily be proved that the atmosphere may be replaced by its homogeneous water equivalent in all matters except that the number of Stösse emitted per cc of the equivalent water atmosphere must be taken as Nq_s, where N is the number of atoms of air in one gram of air.

Consider the contribution of the Stoss particles which are emitted downwards in a solid angle $1-\cos\theta_0$, determined by a polar angle θ_0 measured from the vertical. As a first approximation, let the particles be emitted uniformly in this solid angle and let n be their number. The number reaching the ground within the solid angle and coming from the element of volume of unit horizontal area and thickness dh at the altitude h in the homogeneous atmosphere is $nNIq_s/I_s$ if $R>h\cos\theta_0$; $nNIq_s(1-\cos\theta)/I_s(1-\cos\theta_0)$ if $R<h/\cos\theta_0<R/\cos\theta_0$; and zero if $h>R$, R being the range of the particle in water and θ being given by $\cos\theta=h/R$ for $\theta<\theta_0$.

The total number of Stoss particles reaching each square centimeter of the earth per second from solid angles determined by θ_0 is, on the average, equal to the total number of this class contributed to the whole earth's surface per second by a vertical column of unit cross-sectional area extending to the top of the atmosphere and this is $J(\theta_0)$, where

$$J(\theta_0)=nNq_s\int_0^{R\cos\theta_0}\frac{I}{I_s}dh+\frac{nNq_s}{1-\cos\theta_0}\int_{R\cos\theta_0}^{R}\frac{I}{I_s}\left(1-\frac{h}{R}\right)dh. \qquad (1)$$

It will suffice,[1] as a first approximation, to assume $I/I_s=e^{\mu h}$, where μ is an apparent coefficient of absorption related to the true coefficient of absorption appearing in Gold's formula and calculable from it for limited ranges of h. With this understanding and with H written for $R\cos\theta_0$

$$J(\theta_0)=\frac{nNHq_s}{\mu H}\left[\frac{e^{\mu R}-e^{\mu H}}{R\mu(1-\cos\theta_0)}-1\right]. \qquad (2)$$

[1] Strictly speaking one should consider each initial direction of the radiation separately in its capacity of producing a shower within an angle θ_0 about it as axis. The correction involved is small, however, particularly in a calculation which concerns itself only with orders of magnitude.

N has the value $N=4.1\times 10^{22}$. For q_s, we have such data as those obtained by Steinke and Schindler and by Montgomery and the writer. According to the latter data, there is a production of about 1×10^{-8} Stoss per cc per second in iron at the surface of the earth. This gives $q_s=1.16\times 10^{-31}$. Since $e^{\mu R}$ and $e^{\mu H}$ really represent, respectively, the ratios of the production of Stösse per atom at altitudes R and H to that at the surface of the earth; and since R and H are the logs of these corresponding ratios, formula (2) really gives $J(\theta_0)$ in terms of *experimental* measurements of the variation of rate of production of the large Stösse with altitude, without even any implication as to their origin from photons. Only the exponential law is involved. Unfortunately, there are no data on variation of the large Stösse with altitude sufficiently complete for the purposes of direct calculation. Pending the accumulation of such data, we are driven to make some hypotheses as to the production of the Stösse founded upon their relation to cosmic-ray intensity. It has been suggested by Steinke[2] that the softer photon components of the cosmic radiation are mainly responsible for the production of the large Stösse. At first sight such a view is at variance with the idea that the Stösse arise from the transformation of the energy of the photon into positive and negative particles. For a Stoss giving even 100 rays of energy 10^9 volts would require a photon of energy 10^{11} volts to produce it. The older theories which attributed cosmic-ray photon absorption to the extranuclear electrons would have prohibited the association of a 10^{11} volt photon with the softer components of the cosmic-ray photons; and, while absorption theories have had to suffer changes in the light of the now known importance of the nucleus in such phenomena, one hesitates to take advantage of the slackening of the control of those theories to make assumptions which might well be vetoed by their successors. If, however, we assume that the energy of the Stoss comes from a single atom, we are again in difficulty; for, the complete annihilation of the mass of an iron atom, for example, would provide only about 5×10^{10} electron-volts energy; and, for lighter atoms, the amount would be proportionally smaller. Then, as pointed out by Montgomery and the writer,[3] the number of particles emitted in a large Stoss is larger than that contained in any single atom. A way out of some of these difficulties is suggested by some recent experiments of G. L. Locher made in this laboratory. As a result of observations on cloud photographs of Stoss particles, Locher has been lead to conclude that many atoms may participate in the production of a shower through the intermediary agency of neutrons. If we add to this conclusion the supposition that the Stoss particles and their energy come from the atoms, we are able to invoke the energy of many atoms to account for the Stoss; and such a view enhances the probability that the initiation of the act of production of the Stoss may come about through the agency of a soft photon. We shall investigate the consequences of assuming for the true coefficient of absorption of the radiation concerned the value 0.0055 per cm of water quoted by Millikan[4] as the coefficient for the radiation which is responsible for 90 percent of the cosmic-ray ionization in the atmosphere. The μ in our formula is obtainable from this as an approximation and comes out as $\mu=0.012$ between sea level and an altitude of 250 cm in the homogeneous water atmosphere. We shall consider, tentatively, a range of 250 cm for R.

According to some unpublished measurements of C. G. Montgomery, at least 100 particles are emitted within a cone of vertical angle $\pi/4$, i.e., $\theta_0=\pi/8$. Inserting these values and the values of N and q_s already given, we find, for this case, which we shall call Case (1),

$$J(\theta_0)>6.8\times 10^{-4}. \qquad \text{Case (1)}$$

If we should assume that the particles are emitted in the whole angle $\pi/2$ with the same solid angular density as that with which they are

[2] E. G. Steinke, A. Gastell and H. Nie, Naturwiss. **21**, 560 (1933).

[3] W. F. G. Swann and C. G. Montgomery, Phys. Rev. **44**, 52 (1933).

[4] I. S. Bowen, R. A. Millikan and H. V. Neher, Phys. Rev. **44**, 246 (1933). It is realized that to the extent that the photon radiation originates outside the atmosphere, very little of this soft radiation could penetrate to the earth; and the value is used merely by way of illustration. As will be seen later, a value one-fifth this amount, or even a value zero still leaves the Stoss particles a potent agency in contributing to the cosmic-ray intensity.

emitted in the $\pi/8$ angle, we should have[5] $n=100/(1-\cos\pi/8)=1300$. Under these conditions, we find

$$J(\theta_0)>0.32\times10^{-2}. \qquad \text{Case (2)}$$

From measurements of Johnson and Street, the number of cosmic-ray particles given by Geiger-counters as passing through one square centimeter of the earth's surface per second is 1.2×10^{-2}. It will be seen that the value of $J(\theta_0)$ for Case (2), amounts to one-quarter of this value. A total emission of 5000 Stösse particles would be sufficient to make $J(\theta_0)$ equal to the cosmic-ray intensity.

Perhaps one of the weakest points in the foregoing development lies in the assumption of such a large value for μ. However, if, retaining the other hypothesis of Case (2), we take $\mu=0.003$ which represents a true absorption coefficient of 0.0011 and corresponds to the experimental data for the total radiation for the lowest two meters of water equivalent of the atmosphere, we still find $J(\theta_0)>0.10\times10^{-2}$ which corresponds to ten percent of the measured cosmic-ray intensity at the earth's surface. Even the assumption $\mu=0$, which amounts to assuming no increase of Stösse emission with altitude, gives $J>0.08\times10^{-2}$. An increase in the value of R would enhance still further the value of $J(\theta_0)$. We are limited here, however, by the fact that the range of the particles must not be assumed so great that appreciable fractions of them come from altitudes comparable with the total height of the atmosphere as such an assumption would result in a variation of cosmic-ray intensity with altitude which was inconsistent with the facts.

The main purpose of this note is not to insist on any specific values of the quantities taken for illustration in the calculation, but merely to call attention to the order of magnitude of the effect of the Stoss particles in contributing in appreciable amount to the observed cosmic-ray intensity as obtained by Geiger-counters.

[5] Stösse containing as many as 4000 rays have been reported by Steinke (reference 2) and Montgomery has observed Stösse with 5000 rays in this laboratory.