Measurement of the Townsend Coefficients for Ionization by Collision

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The photoelectric current between parallel plates in dry air was measured as a function of plate distance for constant field strength and pressure over plate distances ranging from 1 to 7 cm and at a pressure of 1 mm of mercury. The simple Townsend relation $i=i_0\epsilon^{\alpha d}$ was found to give excellent agreement for values of X/p from 40 to 110. The least squares values of α/p plotted as a function of X/p lie on a smooth curve which does not fit either the equation $\alpha/p=N\epsilon^{-NV_0p/X}$ suggested by Townsend, nor the empirical relation $\alpha/p=A\epsilon^{bX/p}$, found by the writer to hold for values of X/p from 20 to 36.5. Voltages and plate distances were measured to within 1/10 of 1 percent and pressures to 1/2 of 1 percent or better. For X/p's from 120 to 160 the current was found to increase with plate distance more rapidly than the simple exponential relation would indicate, as was observed by Townsend for plate distances less than 1 cm at much greater X/p's. The early appearance of this deviation may be ascribed to the greater sensitivity given by larger plate distances. Both of the relations suggested by Townsend:

$$i=i_0(\alpha-\beta)\epsilon^{(\alpha-\beta)d}/[\alpha-\beta\epsilon^{(\alpha-\beta)d}],$$

which was derived on the assumption that each positive ion produces β new pairs of ions by collision in 1 cm of path, and

$$i=i_0\epsilon^{\alpha d}/[1-\gamma(\epsilon^{\alpha d}-1)],$$

which assumes γ new electrons liberated by each positive ion which strikes the cathode, fit the experimental curves equally well. The values of β/p were observed to increase with X/p as found by Townsend for higher X/p's. The coefficient γ, which is approximately equal to β/α was observed to increase only very slightly over this small range of X/p's.

INTRODUCTION

IN a recent paper[1] the writer describes in detail the results of an investigation on the value of the Townsend coefficient for ionization by collision in air at large plate distances and near atmospheric pressure. In that investigation the value of the Townsend coefficient α was obtained at or near atmospheric pressure for a range of values of X/p (the ratio of the field strength in volts per cm to pressure in mm) from 20, below which point the value of α began to be indistinguishable from the natural unsaturation of the ionization curve, up to $X/p=36.5$ which was within two percent of spark breakdown. The results obtained gave a relation for α/p as a function of X/p of:

$$\alpha/p=2.67\times10^{-8}\epsilon^{0.350X/p}, \qquad (1)$$

which was in disagreement with the equation:

$$\alpha/p=N\epsilon^{-NV_0p/X} \qquad (2)$$

deduced by Townsend[2] and found applicable by him to values of X/p above 300 in air. Paavola[3] concluded that for values of X/p above 40 in air at atmospheric pressure the currents required not only ionization by electrons giving the value of α but also by positive ions when the field was within three percent of sparking. This was not observed in the writer's investigation even within two percent of sparking. The original investigations of Townsend were confined to low pressures and small plate distances (below 1 cm) and did not extend much below $X/p=100$ although one or two points were obtained down to $X/p=40$. In Townsend's measurements[4] the deviation from the law for ionization by electron impacts only was not observed below $X/p=175$. In view of the importance of ionization by positive ions in the mechanism of spark breakdown it seemed of considerable importance to make a complete investigation in the range from $X/p=40$ to $X/p=160$ or thereabouts with the high resolving power and increased accuracy possible with the modern techniques now available in the writer's laboratory.

[1] F. H. Sanders, Phys. Rev. **41**, 667 (1932).
[2] J. S. Townsend, Phil. Mag. **1**, 557 (1902); **1**, 389 (1903); *Electricity in Gases*, Chapter VIII.
[3] M. Paavola, Archiv f. Elektrotechnik **22**, 443 (1929).
[4] J. S. Townsend, Phil. Mag. **2**, 598 (1903); **2**, 738 (1904); *Electricity in Gases*, Chapter IX.

Apparatus and Experimental Procedure

The apparatus was essentially the same as that used in the earlier investigation. For values of X/p from 70 to 160 the measurements were carried on at pressures very near 1 mm so that a bank of B-batteries giving a maximum potential of 900 volts replaced the high-tension set previously used. The positive side of the battery was connected to ground and the negative side to the illuminated plate of the ionization chamber. To set the voltage more closely than the 2 volt intervals between successive cells, five or ten cells were connected across the fixed terminals of a slide-wire rheostat and the sliding contact connected to the ionization chamber.

The potential across the chamber was first set to within one percent or so with a voltmeter after which it could be set to one-tenth of one percent with a ratio box, potentiometer and standard cell. The ratio box and potentiometer resistances and the e.m.f. of the standard cell were accurate to within a few hundredths of one percent. Plate distances were measured with a cathetometer and were of the same order of accuracy.

For the pressure measurements two McLeod gauges were constructed, one reading from 0.5 to 1.5 mm and the other from 3.0 to 40 mm. The capillary tubes of these gauges were accurately calibrated over their complete operating length and it is believed that the pressure readings are accurate to one-half of one percent or better.

The same ionization chamber was used as in the earlier experiments but it was found possible at these lower pressures to operate over a range of plate distances from 1 to 7 cm. A 220-volt quartz mercury-arc lamp was used as before but considerably greater constancy of operation was obtained as follows:

It was found that although the current through the arc was sensibly constant, the potential drop would suddenly increase from time to time even after the arc had reached its equilibrium temperature. By decreasing the current by ten percent or so below its normal operating value for a few minutes the potential drop could be brought back to its equilibrium value after which the current was increased to normal and the arc again operated steadily. As all measurements were taken over periods varying from ten to thirty seconds, this method of obtaining steady photoelectric emission worked very well.

Because of the steadiness of the battery potential the currents were measured by the "rate of deflection" method with a quadrant electrometer of a sensitivity of 500 mm per volt. Capacities varying from 350 to 6970 cm were used in parallel with the electrometer to give a considerable range of sensitivities. By this means, currents ranging from 1×10^{-13} ampere to 6×10^{-9} ampere could be measured.

For values of X/p of 40, 50 and 60, it was found advisable to work at higher pressures than 1 mm. As the approximate values of α/p were known from early data taken at 1 mm, pressures were chosen in each case which would give α a value of about 0.5 as this had proved to give a rate of current increase which could be measured with the greatest accuracy. At these pressures it was necessary to use voltages greater than could be obtained with the batteries and hence the high tension set was used. In this case the potential was measured with a 50-meghom Taylor high voltage resistor unit, accurate to one-half of one percent. The grounded end of the resistor was tapped at 25,000 ohms and connected to the same potentiometer used with the batteries. The current was measured with the same electrometer used at the lower pressures but the rate of deflection method was replaced by the one used in the earlier high pressure investigations where a relatively large capacity was allowed to charge for a known time and its potential measured with the electrometer. This procedure necessitated a correction of one percent or less to the potential because of the fact that the measuring condenser and the ionization chamber formed a set of capacities in series, but was advisable as a protection to the electrometer in case of sparkover.

It was found impossible at these higher pressures to go to the large plate distances used at 1 mm because of the geometrical design of the chamber. For instance at $X/p=40$ and $p=25$ mm, the greatest distance obtainable was 3.5 cm. At 4.0 cm sparkover to the walls of the chamber took place. The spark was observed to jump from the mercury contact in the split insulator to the grounded chamber and as this portion of the chamber was electrically screened from the working section by a grounded metal plate of diameter almost that of the chamber, there was no possibility of distortion of the field just previous to the spark.

The air was dried and admitted to the chamber as before. A slightly oxidized cathode gave much greater constancy of photoelectric emission than a polished plate. The pressure was read in each case before and after a run but in no case was a variation of more than one-tenth of one percent observed. The field was adjusted to give the desired value of X/p for each run and the observations taken by varying the plate distance and voltage to give constant field strength. As in the earlier experiments, the plate distances were chosen at random so that a consistent variation in i_0 would not introduce a consistent error in the value of α.

Experimental Results

The curves giving current as a function of plate distance are shown in Fig. 1. For purposes of assembling the data it was found convenient to plot $(1/p)\log(i/i_0)$ against plate distance in cm. Hence the slopes of the curves give α/p directly. The values of α and i_0 were determined by the method of least squares from the actual observations, the reduced values being used for purposes of plotting only. For X/p's of 120 and higher, where the curves depart from the linear form, the values of α and i_0 were computed from the lower portions of the curves where they are sensibly linear. It will be noted that for X/p's of 40, 50 and 60, the curves are shown on an enlarged scale. This was done to avoid crowding and to indicate their linear form. Up to $X/p=110$ the curves are strictly linear indicating that within the limits of accuracy of these measurements, the ionization by collision is overwhelmingly due to electron impacts and hence proving the validity of the simple Townsend relation:

$$i=i_0\epsilon^{\alpha d} \tag{3}$$

for these lower X/p's. At $X/p=120$ definite evidence for the existence of ionization either by positive ions or by some mechanism other than pure electron impact begins to appear. An

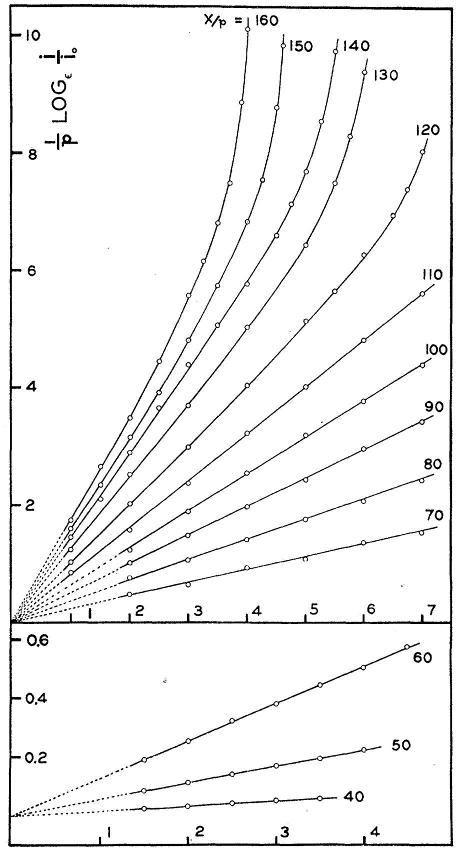

FIG. 1. Current as a function of plate distance.

analysis of the curves using either the classical Townsend relation:

$$i=i_0(\alpha-\beta)\epsilon^{(\alpha-\beta)d}/[\alpha-\beta\epsilon^{(\alpha-\beta)d}], \tag{4}$$

in which β is the number of ion pairs produced by one positive ion in 1 cm of path, or the Townsend equation:

$$i=i_0\epsilon^{\alpha d}/[1-\gamma(\epsilon^{\alpha d}-1)], \tag{5}$$

in which γ is the number of electrons liberated by one positive ion on impact with the cathode, showed that both the equations fit the experimental data equally well. The reason for this is quite apparent. By means of a simple substitution Eq. (5) may be reduced to the form:

$$i=i_0(\alpha-\beta')\epsilon^{\alpha d}/[\alpha-\beta'\epsilon^{\alpha d}]. \tag{6}$$

Now at $X/p=160$ the value of β obtained from Eq. (4) is 0.00165 while the corresponding value of α is 1.759. The difference between α and $\alpha-\beta$

is hence only one part in one thousand, which is considerably less than the probable error in α, so that Eqs. (4) and (5) become absolutely identical for these lower X/p's. In fact it can be shown that all the equations relating current to plate distance which have been suggested, by using various mechanisms of secondary ionization or combinations of these mechanisms, lead to an equation which can be reduced to one of the above forms. Hence these curves give no clue as to the nature of the secondary increase.

The values of α/p are shown as a function of X/p in Fig. 2. The experimental data lie on a very smooth curve, but it is obviously not a simple exponential relation either of the Townsend form nor of that found by the writer to fit below $X/p=36.5$. In Fig. 3 these same data are shown plotted on a logarithmic scale, as are also the data obtained earlier at high pressure. It is seen that from $X/p=20.0$ to 36.5 the observations lie on a very good straight line but that for $X/p=40$ and higher the curve falls off rapidly. Several attempts were made to fit this curve by the use of combinations of exponentials, but apparently no simple combination will fit the data at all well. The Townsend form of equation which gives a rectangular hyperbola when plotted on such a logarithmic scale fits very approximately, but the deviations are well outside of the maximum allowable error of the observations. When the Townsend values of the constants are used: $N=14.6$ and $V_0=25$, the deviation is exceedingly great. The values of X/p, p and the corresponding α/p with its probable error are given in Table I. In the same table are shown the experimental results of K. Masch,[5] who has recently investigated the same region at small plate distances, ranging from 1 to 6 mm. The agreement is excellent in the region where the two sets of data overlap. Masch, with his small plate distances, was apparently unable to carry his investigations below $X/p=31$, while the writer's work was discontinued at $X/p=160$ as there appeared no point to carrying the investigations into the region of higher X/p's which has been thoroughly investigated by Townsend. It was impossible to compare the writer's data accurately with those of Townsend, as the latter are obtainable only by interpolation from Townsend's curves, but the agreement with these interpolated values is very good.

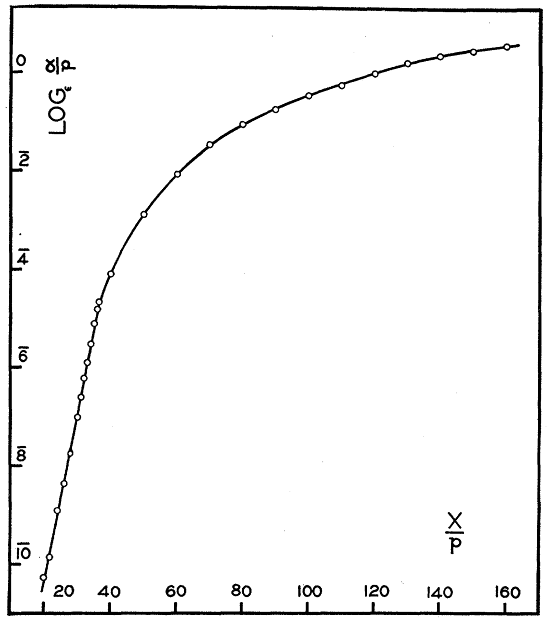

FIG. 3. Logarithm of α/p as a function of X/p.

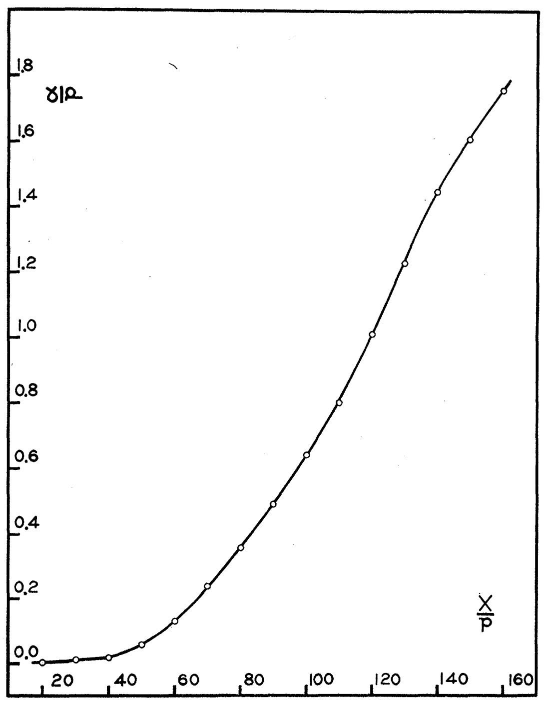

FIG. 2. Values of α/p as functions of X/p.

[5] K. Masch, Archiv f. Elektrotechnik **26**, 589 (1932).

Table I. *Values of X/p, p and α/p.*

X/p (v/cm/mm)	p (mm)	α/p (writer)	Probable error	α/p (Masch)
20.0	380.0	0.000034	±0.000004	
22.0	380.0	0.000052	0.000011	
24.0	380.0	0.000134	0.000001	
26.0	380.0	0.000234	0.000028	
28.0	380.0	0.000430	0.000015	
30.0	380.0	0.000910	0.000019	
31.0	380.0	0.00136	0.00002	0.00152
32.0	380.0	0.00201	0.00002	0.00204
33.0	380.0	0.00305	0.00003	0.00309
34.0	380.0	0.00459	0.00005	0.0044
35.0	380.0	0.00605	0.00008	0.0059
36.0	380.0	0.00820	0.00005	0.0076
40.0	25.00	0.0167	0.0001	0.0168
50.0	9.95	0.0554	0.0006	0.057
60.0	4.90	0.127	0.001	0.130
70.0	1.000	0.224	0.005	0.235
80.0	0.980	0.340	0.003	0.365
90.0	0.970	0.491	0.003	0.51
100.0	0.960	0.637	0.006	0.68
110.0	0.975	0.806	0.004	0.85
120.0	0.975	1.007	0.006	1.05
130.0	0.973	1.236	0.019	1.23
140.0	0.950	1.477	0.019	1.40
150.0	0.990	1.602	0.017	1.60
160.0	1.000	1.758	0.038	1.83

Table II. *Values of γ, β and β/p.*

X/p	p	γ	β	β/p
120	0.975	0.00058	0.00057	0.00059
130	0.973	0.00062	0.00075	0.00077
140	0.950	0.00040	0.00056	0.00059
150	0.990	0.00063	0.00099	0.0010
160	1.000	0.00094	0.0016	0.0016

The values of γ, β and β/p are shown in Table II. Both coefficients increase with X/p but not in any very consistent fashion. As these data were obtained under the most favorable conditions, i.e., large plate distances which give high resolving power and the very steady source of potential given by batteries, it is extremely doubtful if this increase can be accurately examined at higher pressures and lower X/p's. In this region it would be necessary to use a kenotron-rectifier high-tension set-up from which it is practically impossible to eliminate fluctuations of one-fifth of one percent or so. Since the coefficient α is increasing exponentially with the field strength and the current is itself an exponential function of α, the variation in current for even a one-fifth percent change in field strength becomes exceedingly great for electron ionization alone. If the secondary mechanism now comes into play, the variation of the current becomes enormous and sparkover is likely to occur. As mentioned earlier, Paavola, working at atmospheric pressure, small plate distances and X/p's near 40, apparently observed this secondary increase, but his values of β were found to decrease with increasing X/p. In view of the difficulties just mentioned, this is by no means surprising.

Conclusion

Thus it is seen that these two investigations of the writer, having a resolving power which is probably as high as can conveniently be obtained with this method of attack, thanks to the modern developments in technique, have yielded a continuous and, it is hoped, useful set of values of α/p for values of X/p where ionization by collision can just be detected up to values at which the excellent measurements of Townsend have given us reliable data. These data should be of considerable value to all theoretical investigators of the mechanism of spark breakdown, in view of the continuous and accurate set of values covering all ranges of X/p involved in a study of the breakdown mechanism in air. In conformity with Townsend's earlier investigations, they have shown that current studies of this nature are incapable directly of differentiating between the various theories of the mechanism of electron production by positive ions and it appears from an analysis of the results obtained in this and earlier investigations that there is little hope of obtaining the solution to this problem by ionization current measurements.

In conclusion the writer wishes to acknowledge his indebtedness to Dr. L. B. Loeb at whose suggestion the original problem was continued and under whose direction it was carried out. The writer also desires to express his thanks to Miss Florence Ehrenkranz and Mr. W. E. Bowls for their assistance in setting up the apparatus and taking readings. This investigation was made possible through the use of the apparatus, the cost of which was in part defrayed by a grant in aid from the National Research Council.