Electron Orbits in Crossed Electric and Magnetic Fields

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The focussing properties of crossed electric and magnetic fields for electrons have been investigated for the case of circular orbits, and it has been found that this combination of fields provides extremely sharp focussing, which is in very good agreement with the theoretical predictions. Measurements of the electric field intensity have revealed the presence of polarization layers which form on the plates of the electric field. These layers reduce the effective potential which is used to deflect the electron beam. The absolute magnitude of these layers has been measured for gold and bronze, and it has been found to depend upon; (a) material of the plates, (b) gas pressure, and (c) electron intensity. These layers are found to be constant under a fixed set of conditions and to vary in a reversible manner as conditions are altered. In addition, permanent insulating layers may be formed if electrons bombard a metal surface. Errors in e/m determinations may be traced to these layers and tests are developed for their elimination.

I. INTRODUCTION

It has been shown by Bartky and Dempster[1] that both velocity and direction focussing of a beam of charged particles can be obtained with crossed electric and magnetic fields. Recently Bondy and Popper[2] have investigated the characteristics of this type of combination of fields for the case of positive ions of sodium and potassium. The present paper is an account of the behavior of electrons in crossed fields.

In Fig. 1 is shown a schematic representation of a cylindrical condenser in which a beam of charged particles is free to move under the action of a radial electric field and a magnetic field which is normal to the plane of the orbit.

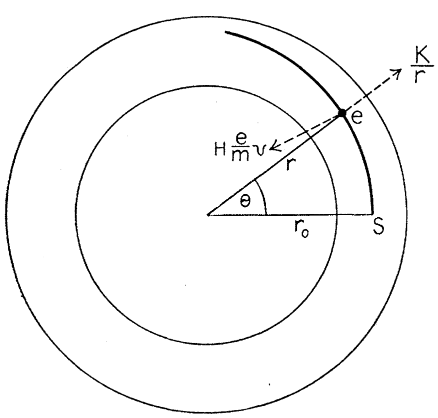

FIG. 1. Section through cylindrical condenser showing forces acting on an electron in crossed electric and magnetic fields.

A charged particle entering the field at S describes an orbit under the influence of a radial acceleration K/r, due to the electric field, and of an acceleration Hev/m, due to the magnetic field which acts at right angles to its direction of motion at every point of the orbit. The solution[1] of the differential equations of motion gives a general equation for the radius vector of the path at the point of focus, *viz.*, $\theta=\pi/2^{\frac{1}{2}}=127°\ 17'$, as follows,

$$r=\rho\left(1-c_0+\frac{c_0^2}{3}+\frac{c_1^2}{3}+\delta^2+\frac{\pi}{2^{\frac{1}{2}}}c_1\delta\right). \tag{1}$$

This equation contains the initial conditions associated with the entering slit, that is where $\theta=0$, we have for the point of entry, $r_0=\rho(1+c_0)$, and for the angle of entry, $(dr/\rho d\theta)_{\theta=0}=c_1$, and for the angular velocity at entry, $(d\theta/dt)_{\theta=0}=\frac{1}{2}H(1+\delta)e/m$, where c_0, c_1 and δ are small. For a first approximation, that is when c_1 and δ are very small and $c_0=0$, $r_{\theta=\pi/2^{\frac{1}{2}}}=r_0=\rho$, where $\rho^2=4K/(He/m)^2$. If K be evaluated in terms of the applied potential V and the radii of curvature of the condenser plates r_1 and r_2, we find,

$$\frac{e}{m}=\frac{4V}{\rho^2H^2\log_e(r_2/r_1)}, \tag{2}$$

[1] W. Bartky and A. J. Dempster, Phys. Rev. **33**, 1019 (1929).

[2] H. Bondy and K. Popper, Ann. d. Physik **17**, 425 (1933).

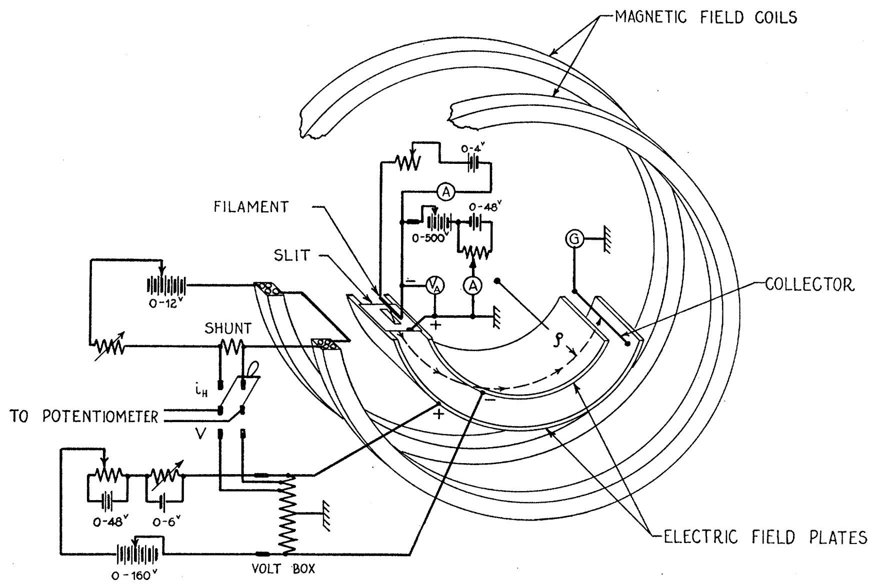

FIG. 2. Schematic diagram of the electric and magnetic fields and associated circuits.

where H is the intensity of the magnetic field. The significance of this equation lies in its being independent of the velocity and hence uncertainties in the accelerating potential do not enter.

II. General Experimental Arrangement

The schematic diagram of the entire apparatus is shown in Fig. 2, where the electric and magnetic fields are represented in oblique projection.

The source of electrons is a coated cylindrical filament, 0.025 mm in diameter which is mounted like a monochord string under the slight tension of a spring, in order that its expansion when heated will not throw it out of the line of the slit. The filament is operated with currents of 0.04 to 0.06 ampere to minimize its magnetic field. The slit shown serves to collimate the electron beam. Directly beneath this slit, a distance of 2.5 mm, is a pair of fine molybdenum wires, not shown in Fig. 2. These wires, which determine the width of the beam used, are supported on two fine pitch radial screws in such a manner as to be parallel to each other and to have their common plane radial. The effective slit width of these wires was varied from 0.025 mm to 0.060 mm. These two wires serve as the initial slit, the inverted image of which is focussed onto the collector, located 127° 17′ away. The use of these two wires for the slit reduces the distortion of the radial electric field. The filament was placed in such a position that the electrons, under the influence of the accelerating field and the magnetic field, entered the slit normally.

The electron beam is picked up at the collector which is joined to a galvanometer. It was found that the collector acquired a net positive charge under the electron impact. This is due to the fact that the ratio of secondary electrons to primary electrons is greater than unity. When operating at pressures of the order of 10^{-7} mm Hg and exceedingly small beam intensities, the deflections are very constant.

(a) Electric field

The electric field is formed between two concentric cylindrical surfaces, which are held concentric by means of an accurately turned glass cylinder and upon the front face of which is engraved the center of curvature. The effective axial length of the field is about 30.0 mm, whereas the axial width of the beam is only 2.0 mm. The field plates were made of bronze with a low magnetic susceptibility with provision for mounting upon their adjacent surfaces thin metallic layers for the study of the polarization phenomena which are described later. The complete cylindrical condenser is mounted inside a glass tube which can be exhausted. The glass tube is supported on six points, four of which are rigid and the other two flexible. This provides two degrees of freedom in angle, which, together with an additional degree in translation, allows the electric field to be located symmetrically inside the magnetic field.

The grounded wire slit and the wire collector are located midway between the two plates of the condenser. One plate was charged positively and the other negatively to such potentials that the slit and collector were located on a circular equipotential surface at zero potential.

(b) Magnetic field

The magnetic field is furnished by two coils, each of which has 119 turns. The entire physical structure of the apparatus was cast from a low magnetic susceptibility bronze whose susceptibility is less than 10^{-5}. The common median plane of the two coils is coincident with the median plane of the electron beam.

The magnetic field coils are shown schematically in Fig. 2. The absolute value of the field of these coils is obtained, not by computation, but by null comparison with two accurate, concentric, single-layer coils not shown in the figure. These two single-layer coils consist of seven turns each and are wound in an 80° helical groove on the surface of a bronze cylinder inside the larger coils. The field of the accurate pair is arranged to oppose the field of the large pair and a small oil-damped magnetometer, placed at the radius of curvature of the trajectory, namely $\rho = 31.00$ mm, and in the median plane of the beam, indicates when they are equal. The magnetometer is so arranged that it can be placed in different parts of the field in order thus to explore the field over the entire volume of the electron ribbon.

The field of the single pair was determined, (1) by series computation, (2) by a new method[3] for the approximation of the values of elliptic integrals, and (3) by comparison with a standard, single coil having a single layer of 24 turns. This coil is arranged on a double eccentric so that its axis can be made coincident with the trajectory. When this occurs, all terms of the series vanish except the first, thus providing an additional check on the rapidity of the convergence of the series. It is of interest to note that the results of these different methods are self-consistent to one part in 10,000.

As is evident from Fig. 2, the absolute value of the applied potential and the magnetic field current can be read directly on the same potentiometer.

III. Experimental Results

(a) Focussing curves

In Fig. 3 are two curves which show the changes in the current to the collector, for variations in either the applied potential or the magnetic field. Fig. 3a illustrates the case when the beam is moved past the collector by a variation of the applied potential, with a constant magnetic field, whereas 3b shows the case for a variation of the magnetic field, with a constant applied potential. The points were plotted from the experimental data, whereas the curves represent

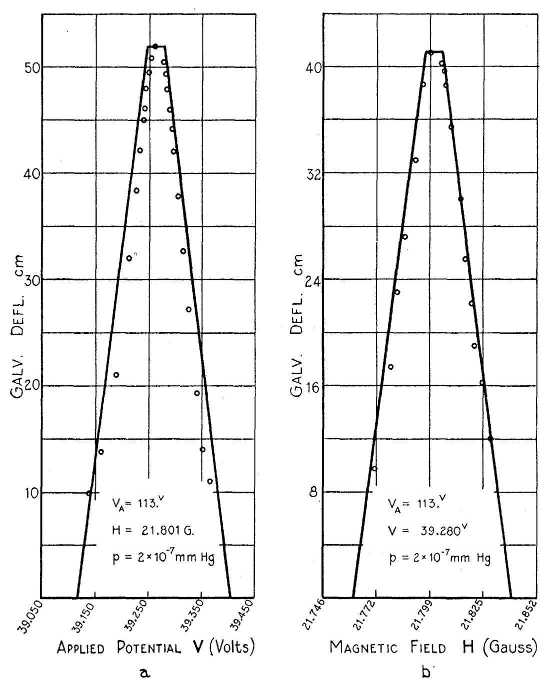

FIG. 3. Typical curves illustrating the great sharpness of focus of electrons obtained with crossed electric and magnetic fields. The theoretical curve in either case is represented by the solid lines and the experimental values are shown as points.

[3] This new method is due to Professor W. Bartky and will be published shortly.

the theoretical values computed from the geometrical fraction of the beam intercepted by the collector.

The two experimental curves show exceedingly sharp focussing. In fact, the relative changes in either the electric or the magnetic field are found to be slightly less than would be anticipated from the widths of the slit and the collecting wire used. The flat top of these curves is due to the use of a slit which is slightly wider than the collector.

(b) Surface polarization phenomena

The value of e/m was calculated from the peak values of the experimental curves in Fig. 3. It was found that instead of being a constant, the value of e/m varied with the intensity of the electric field. For low intensities of the applied electric field, e/m was greater than for higher field values. This apparent difference was finally ascribed to the formation of surface charges upon the plates of the electric field which changed the force acting on the electrons from that computed in terms of the applied potentials.

The equation from which the absolute magnitude of these surface charges can be determined is developed as follows. If there be added to the actual deflecting potential E, an extraneous potential ϵ, such that the applied potential is given by $V=E+\epsilon$, it is found from Eq. (2) that,

$$\frac{e}{m}=\frac{e}{m_0}\left(\frac{V}{V-\epsilon}\right)=\frac{e}{m_0}\left(1+\frac{\epsilon}{V}\right), \text{ for } \epsilon \ll V; \quad (3)$$

where e/m_0 is the true value, and e/m the apparent value, as computed from the applied potentials. Eq. (3) represents the slope-intercept form of the equation of a straight line, where $\epsilon e/m_0$ is the slope and e/m_0 is the intercept.

In order to examine the validity of Eq. (3), preliminary experiments were carried out by using aluminum, gold and platinum. At pressures of the order of 2×10^{-4} mm Hg, and collector current $i=10^{-8}$ ampere, aluminum showed surface potentials of 11.8 volts. Gold under identical conditions developed a surface layer of only 0.446 volt. At pressures of 6×10^{-5} mm Hg, the layer on aluminum fell to 6.0 volts. Platinum was so awfully erratic in its behavior—there was no provision made for outgassing—that no reliable data could be secured.

Provision was made to secure gas pressures and electron intensities as low as possible. In order to detect small electron currents in the collector, an FP-54 Pliotron,[4] operated at current sensitivities of 10^{-14} ampere, was joined into the collector circuit between the galvanometer and the collector. Fig. 4 shows the experimental results for

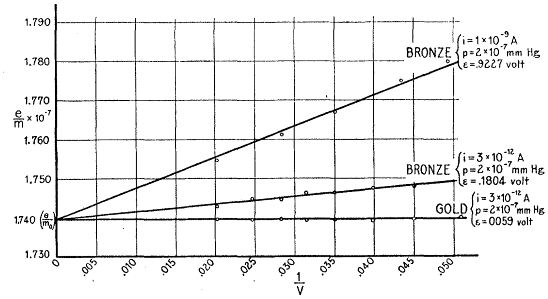

FIG. 4. Curves showing the magnitude of surface polarization for bronze surfaces and gold surfaces, under the conditions indicated.

bronze and gold. The data for each of these curves were taken under the same pressure conditions, namely 2×10^{-7} mm Hg, as read on an ionization gauge. The first curve for bronze was taken at $i=10^{-9}$ ampere. Under these conditions a surface layer of 0.9227 volt developed, whereas when the current i was reduced to 3×10^{-12} ampere, the same bronze showed a surface layer of only 0.1804 volt. Gold was tried under these conditions of low current and low pressure. A thin layer of pure gold foil, 0.025 mm thick, was stretched over each of the bronze plates of the electric field, correction being made for the change in the effective radius of curvature of the plates of the field. Under the conditions indicated, gold developed a surface layer of 0.0059 volt, as shown by the last curve of Fig. 4. That these layers are transient is shown by the fact that when focussing is resumed under some particular set of conditions, after having focussed under different conditions of pressure or of intensity, the original value of the surface layer is obtained.

Thus far we have made no suggestion as to the possible nature of these polarization layers. They behave somewhat like a space charge, which in this case may be due to the ions formed by the

[4] L. A. DuBridge, Phys. Rev. **37**, 392 (1931).

electron beam. That these layers depend upon the gas pressure and the electron intensity lends some support to this hypothesis.[5]

(c) Surface changes under electron impact

In addition to the surface layers discussed in the preceding paragraph, it was found that permanent insulating layers are formed under direct electron bombardment. The polarization layers that were considered under (b) are formed without electron impact, the beam merely passing between the plates of the field.

It has previously been observed[6, 7, 8] that when a beam of electrons bombards a metal surface an alteration of the surface takes place. Electrons of 100 equivalent volts and beam intensities of 10^{-9} ampere, striking the surface of a metal such as platinum, gold, aluminum, tantalum or molybdenum, cause a noticeable change in the surface in about 10 minutes, at pressures of the order of 10^{-5} mm Hg in air. One characteristic of this type of layer is its high resistivity. Even the layers which develop in 10 minutes are definitely insulating, as shown by the progressive decrease of the deflection of the galvanometer joined to the collector wire. If the bombardment be continued for several hours, an optically visible layer is formed, and the deflection may vanish entirely. These heavier layers are dark toward the center of the deposit and out near the edges they show bright interference colors.

The changes that take place on a metal surface under electron impact represent a permanent alteration of the surface. The layer which developed on Pt after several hours of bombardment, was found to be insoluble after 30 minutes in concentrated HNO_3 at 21°C, or in a mixture of concentrated HCl and concentrated HNO_3 of equal proportions, at the same temperature. The layer which formed on Pt vanished very slowly in air at 320°C, and almost instantly at 1000°C, the surface being restored to its original, bright luster. Under extremely good vacuum conditions and a low intensity beam, the collector used showed no indication of a surface alteration after a period of 4 or 5 hours.

Webster, Hansen and Duveneck[9] have called attention to the carbon deposits which formed on the targets of their x-ray tubes. They suggest that these carbon deposits are due to the breakdown, under electron bombardment, of the organic molecules which collect on the targets due to the presence of the vapors of stopcock grease, n-butyl phthalate, etc. They find that when the x-ray tubes are baked out and good vacuum conditions prevail no visible deposit is formed.

(d) Dependence of path upon velocity

Although Eq. (2) does not contain the velocity explicitly, electrons of a certain preassigned voltage equivalent must be used with given values of the electric and magnetic fields. Let us consider the case of the dependence of the path upon the velocity only, and suppose the beam to have no divergence, that is $c_1=0$ in Eq. (1). Suppose further that the slit and collector are adjusted to the same distance from the center ($c_0=0$). Then at the collector, $r=\rho(1+\delta^2)$, where the velocity of the rays is defined in terms of δ by, $v=\rho\theta_0'=\frac{1}{2}H\rho(1+\delta)e/m$. Thus the beam reaches the collector at the minimum distance ρ when $\delta=0$, or $v=\frac{1}{2}H\rho e/m$. Introducing the accelerating potential V_A in place of the velocity v, and substituting for $H\rho$ in (2) we find,

$$V=2\,V_A \log_\epsilon (r_2/r_1). \tag{4}$$

This value of V_A gives a minimum value to r, so that a slight variation in V_A moves the beam outward a very small amount. It may be calculated that a change in V_A of 2 percent gives a change in r of 10^{-4}. However, with the sharpness of focus, as exemplified by the curves in Fig. 3, a variation of this amount could nevertheless be detected.

It was found that the values of V_A, which were required to give this adjustment for minimum distance, did not agree with the values computed from Eq. (4). To illustrate the differences noted in the values of V_A, it was found experimentally, for example, that for a given value of V and H, the measured value of V_A was 115 volts, whereas

[5] Analogous effects are observed in cathode-ray tubes. See P. P. Eckersley, J. Inst. E. E. (London) **66**, 513 (1928); J. T. MacGregor-Morris and H. Wright, J. Inst. E. E. (London) **71**, 57 (1932).

[6] E. Gehrcke and R. Seeliger, Vehr. der Deutschen Phys. Gesell. **15**, 438 (1913).

[7] J. E. Henderson, Phys. Rev. **29**, 360 (1927).

[8] R. Suhrmann, Phys. Zeits. **30**, 939 (1929).

[9] Webster, Hansen and Duveneck, Rev. Sci. Inst. **3**, 736 (1932).

from Eq. (4), V_A should have been 99.5 volts. Likewise for the next higher value, V_A was 129 volts experimentally and only 113 volts theoretically. Throughout the entire range of values, there is practically a constant difference between successive values of V_A obtained experimentally and theoretically. A possible explanation of this difference is that surface charges had formed on the two wires of the initial slit which had been subjected to considerable bombardment. These charges give rise to a retarding field which is followed by an accelerating field just beyond. This results in the path immediately beyond the slit having a large radius of curvature.

(e) e/m values

From Eq. (3), the intercept e/m_0 in Fig. 4 should be the true value of the specific charge. This is given as 1.740×10^7, a value which is low compared with the values generally accepted at present. This disagreement may be due to the presence of the retarding field mentioned in the last paragraph, which, with its subsequent acceleration, gives a ρ which is initially too large to intersect the collector. Hence, to secure a maximum, a larger H or a smaller V is used. This results in a relatively low value of e/m. A test for the absence of this experimental difficulty is the agreement of the accelerating potential, as calculated from Eq. (4), with the applied potential, as measured externally.

The great sharpness of the focussing obtained with this combination of crossed fields should allow a very accurate determination to be made of the specific charge for electrons, and as soon as this one point concerning the possibility of surface charges on the wire slit is cleared up completely, we hope to proceed with the determination.

It is a great pleasure to express my appreciation to Professor A. J. Dempster, under whose general direction this research was carried out, for considerable help and encouragement throughout the course of the work; and to Professor W. Bartky, for his kind interest. I am very grateful to Mr. Joe Getzholtz, instrument maker, who contributed his great skill to the construction of the electric and magnetic fields.