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Measurements of Intensity of Compton Modified Radiation by Means of Filters

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A new method of measuring the ratio of the intensity of the modified to the unmodified scattering by using a triple balanced filter has been developed. With the K_α radiation of molybdenum three balanced absorption filters containing zirconium, yttrium and strontium, respectively, make it possible to absorb or transmit either the modified or unmodified K_α lines or both if the angle of scattering is sufficiently large so that the shortest wave-length of the modified line is greater than the critical absorption wave-length of yttrium. This method makes possible measurements of the modified radiation not only for the scattering by solids but also for the scattering by gases for which case the theory and experiment can be directly compared. Measurements of the ratio of intensity of the modified to the unmodified scattering have been made at $\varphi=120°$ for oxygen, nitrogen and argon as well as for several solid elements and compounds. The results for gases are found to be in good accord with the wave-mechanics theory of scattering as developed independently by Wentzel and Waller.

INTRODUCTION

THE use of a double balanced filter for isolating a given wave-length band or spectral line was introduced by Ross[1] and has since been found very useful in experiments on the scattering of x-rays by gases and liquids. Ross has also attempted to study the structure of the modified line by the use of a properly selected double filter.

A double balanced filter for isolating the K_α lines of molybdenum has been previously used and described by Barrett,[2] the author[3] and others.[4] It consists of one filter of ZrO_2 and one of Sr O. The critical absorption limit of Zr lies at $\lambda=0.6872$A and that of Sr at 0.7693A. The K_α lines of molybdenum ($\lambda=0.71$A) lying between these limits are readily transmitted by the former and strongly absorbed by the latter. The thickness of these filters can be so chosen that the amount of general radiation transmitted by each is very nearly equal. Hence if readings of the intensity of scattering are made by putting these filters alternately in the primary beam the difference between the readings thus obtained gives the effect due to the band of radiation lying between the critical absorption wave-lengths of the filters which in this case includes the intense K_α lines as well as a small amount of general radiation. The K_β and K_γ lines are almost entirely absorbed by either filter. A more ideal double filter for this case would be obtained if yttrium whose absorption limit lies at $\lambda=0.7255$A were substituted for the strontium. Such a filter would give a narrower band of general radiation accompanying the K_α lines. The use of strontium has been resorted to because of the difficulty of obtaining a sufficiently pure and suitable compound of yttrium. Fortunately, a relatively pure sample of Yt_2O_3 has been obtained and in the present experiment a triple balanced absorption filter has been constructed from the oxides of the three above-mentioned elements, zirconium, yttrium and strontium. Such a system of filters makes possible not only the isolation of the K_α lines but also the isolation of a wave-length lying between the critical absorption wave-lengths of yttrium and strontium. With the aid of this triple filter, measurements of the ratio of intensity of the modified to the unmodified scattering by gases and solids have been made.

THE TRIPLE BALANCED FILTER

The relative position of the critical absorption wave-lengths of zirconium, yttrium and strontium and the wave-lengths of the K_{α_1} and K_{α_2}

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[1] P. A. Ross, Phys. Rev. **28**, 425 (1926).

[2] C. S. Barrett, Phys. Rev. **31**, 159 (1928).

[3] E. O. Wollan, Phys. Rev. **37**, 862 (1931).

[4] W. C. Pierce, Phys. Rev. **38**, 1409 (1931).

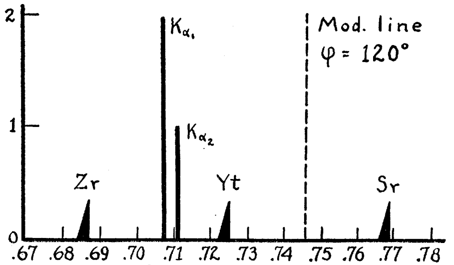

FIG. 1. Relative position of absorption edges and modified and unmodified lines.

lines of molybdenum are shown in Fig. 1. If the K_α lines are scattered from a relatively light element a part of the scattered radiation will consist of the unshifted lines of average wave-length (0.71A) while the remainder consisting of the modified radiation will have a larger wave-length whose value depends on the scattering angle. The wave-length of the modified lines is given by the well-known Compton formula

$$\lambda_{\text{mod.}} = \lambda_{\text{unm.}} + (h/mc)(1 - \cos \varphi), \qquad (1)$$

where $h/mc = 0.0243$ and φ is the scattering angle. From this relation one can calculate the angle φ at which the wave-length of the modified K_{α_1} and K_{α_2} lines will be greater than the wave-length of the critical absorption limit of Yt. This works out to be 74° 14′ for the K_{α_1} line and 63° 23′ for the K_{α_2} line. In view of the considerable breadth of the modified lines as predicted some time ago by Jauncey[5] and recently shown by DuMond,[6] the whole of the modified radiation will not be displaced past the absorption edge until some angle larger than 75° has been reached. At $\varphi = 120°$ the center of gravity of the modified line should have a position as shown in Fig. 1, at which undoubtedly the whole of the modified line will be included in the region between the Yt and Sr absorption limit.

If the three filters under consideration are properly balanced and are alternately placed in the scattered beam for which the scattering angle is sufficiently large, say 120°, one has in effect two double filters, one for the unmodified line and one for the modified line. The use of these filters should then make it possible to make separate measurements of the intensity of the unshifted and shifted lines.

CONSTRUCTION AND CALIBRATION OF THE FILTERS

The zirconium filter is one of the type obtained from the General Electric Company. It is made from ZrO_2 from which the impurity hafnium has been removed.

The strontium filter was made by finely powdering SrO and suspending the powder in paraffin. The filter was made homogeneous by kneading the paraffin and SrO mixture with the fingers, the warmth of which keeps it malleable. It was then pressed into a flat plate.

Yt_2O_3 is difficult to obtain in a pure state since it contains a considerable amount of erbium, the removal of which is very difficult. Fortunately the author was able to obtain some relatively pure Yt_2O_3 and from this a filter was constructed in the same manner as for SrO.

The filters of Yt and Sr were balanced against the Zr filter by successively scraping away small amounts from the surface. It was also found necessary, however, to pad the filters slightly with aluminum, this being apparently due to a small amount of impurity which the Yt_2O_3 still contained.

The calibration curve of the filters is shown in Fig. 2. The curve represents the intensity of the

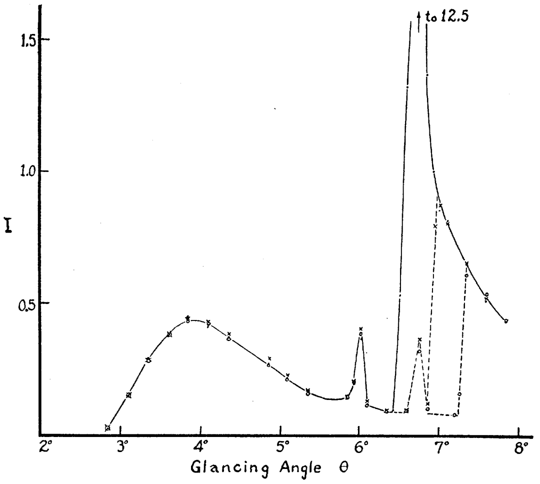

FIG. 2. Calibration curve of absorption filters.

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[5] G. E. M. Jauncey, Phil. Mag. **49**, 427 (1925).

[6] J. DuMond, Phys. Rev. **37**, 136 (1931).

various wave-lengths in the spectrum of the x-ray tube as reflected from a calcite crystal and absorbed by the various filters in turn. The transmission by the Zr filter is represented by the solid points, the transmission of the Yt by the crosses and the transmission of the Sr by the circles. The balance achieved is seen to be quite satisfactory.

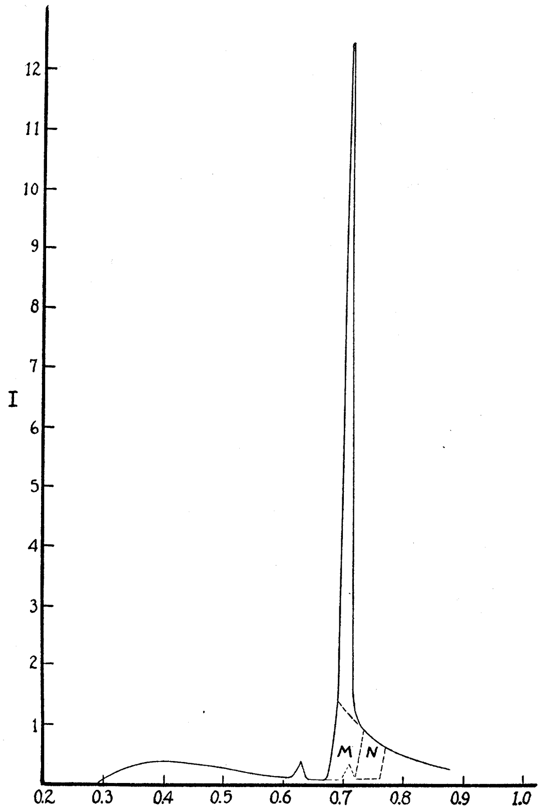

Fig. 3.

Fig. 3 shows the curve as a whole and one sees the approximate relation between the intensity of the K_α lines to the general radiation both outside and between the absorption limits.

Determination of General Radiation Between Absorption Limits of Filters

The presence of general radiation between the absorption limits of the filters presents a difficulty in regard to the measurement of the ratio of intensity of the modified and unmodified radiation. Because of the presence of this general radiation Fig. 2 does not represent a complete calibration of the filters.

For convenience the general radiation between the Zr and Yt absorption limits is referred to as M and that between the limits of the Yt and Sr filters is referred to as N. When the filters are used in the scattered beam a certain amount of the radiation in the region to the left of M will be moved into the region M, a part of the radiation M will be displaced into N while a part of N goes into the region to the right. There will always be a small amount of intensity due to the general radiation between the absorption limits of the filters which must be subtracted from the total readings to obtain the intensity of the characteristic line radiation. That part of the K_α lines which is modified, however, will be displaced bodily into the region N.

The value of M can be obtained if one scatters from a substance, for which all the radiation is modified. When scattering from such a substance, that part of the intensity which still remains between the absorption limits of the Zr and Yt filters will be the value of M desired. Scattering by lithium should very nearly satisfy this condition.

To determine the value of N one might scatter from a body for which all the radiation remains unmodified or at a scattering angle sufficiently small so that none of the modified radiation is displaced into the region N. It should also be possible to obtain N from the difference of intensity between the Yt and Sr filters when these filters are used in the primary beam. Insofar as it is difficult to obtain a suitable scatterer to satisfy the conditions mentioned in the first of these three methods the second two methods have been here used. Fig. 4 shows the scattering by lithium between 35° and 120°. At all angles the total intensity has been taken equal to 1000. Curve I represents the percent of the total radiation which lies between the Yt and Sr filters, while curve II represents the radiation lying between the Zr and Yt filters. Curve I is seen to begin with a constant but relatively small amount of intensity at small angles and then gradually rises until an approximately constant value is reached at large angles. At 35° both the modified and the unmodified K_α radiation lies between the Zr and Yt

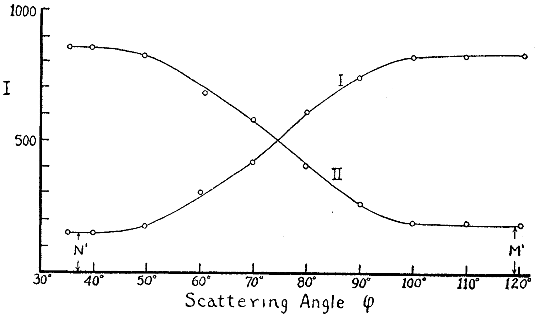

FIG. 4. Scattering curves for lithium as a function of scattering angle. Curve I represents the difference of intensity between Yt and Sr filters and curve II represents the difference between the Zr and Yt filters.

filters and hence the height of curve I at this point is a measure of N, the general radiation between the Yt and Sr limits. The fact that M and N are primed in the figure indicates that certain corrections have not yet been made. At 120° the greater percent of the radiation lies between the Yt and Sr filters. If we assume that all the radiation from lithium at large angles is modified, the intensity corresponding to curve II at 120° would then be a measure of M. If, however, we assume that the scattering by lithium at such large values of $(\sin \varphi/2)/\lambda$ is the same as the scattering by the element in the gaseous state, that is, that the interference between neighboring atoms is negligible, one can obtain the theoretical value of I_m/I_u for lithium. Referring to Table II we see that the calculations for lithium give $I_m/I_u = 64$. On this basis a correction corresponding to 1.56 percent of the modified K_α radiation must be subtracted from the value of M obtained on the assumption that $I_m/I_u = \infty$. The average of a number of measurements of the scattering by lithium at $\varphi = 120°$ gives $M' = 187$. After applying the above-mentioned correction together with the corrections for differential absorption as discussed below we obtain the value $M = 172$.

A value of $N' = 148$ was obtained by scattering from lithium at 35°. As mentioned above N should also be obtained by measurements of the intensity of scattering at 120° when the filters are used in the primary beam. In this way a value of $N' = 144$ was obtained. This value was found to be approximately the same regardless of the atomic number of the scatterer. The average of these two values when corrected for the differential absorption gives $N = 151$* and this is the value which is used in the calculations of I_m/I_u. In view of the fact that in most of the following measurements the value of I_m/I_u is greater than unity the accuracy of the determination of the value of M is of greater importance than that of N.

EXPERIMENTAL ARRANGEMENT

A molybdenum target x-ray tube of the Siemens type was used. The filament current was supplied by lead storage cells and the current was controlled by a fine adjustment rheostat, which could be controlled from the workers position while the tube was in operation. In this way the current and voltage supplied to the tube were kept quite constant.

A Compton electrometer was used for measuring the ionization current and this was operated at a sensitivity of about 8000 mm per volt at a distance of a meter. The ionization chamber was 30 cm long and contained argon at 1 atmosphere.

MEASUREMENTS OF THE SCATTERING BY SOLIDS

The materials to be used as scatterers were finely powdered, except in the cases of paraffin and lithium, and were pressed into a briquet of convenient size. In the case of lithium the metal was cleaned and crushed under oil and pressed into a brass ring. The oil was wiped away and the sample again scraped clean under hot paraffin. On removing, a very thin coat of paraffin protected its surface from oxidizing. The layer of paraffin was so thin that its contribution to the scattering was negligible.

These samples were placed on the center of the spectrometer table such that the face exposed to the radiation made an angle of $\varphi/2$ with the primary beam. A comparatively wide slit[7] was used in the ionization chamber, the angular divergence being about 5°.

The filters were carried on a slide in front of the ionization chamber slit such that each filter could be conveniently moved into a position in front of the slit. Readings were then taken alternately for each of the three filters. Calling the total intensity

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* These values of M and N are reduced to correspond to a total intensity of 1000.

[7] A. W. Coven, Phys. Rev. **41**, 422 (1932).

(i.e., the difference between the readings for the Zr and Sr filters) *C*, the intensity corresponding to the difference between the Zr and Yt filters *A'* and that corresponding to the differences between the Yt and Sr filters *B'*, the results of these measurements are given in the first two columns of Table Ia. For convenience the total intensity, *C*, has been taken equal to 1000 in all cases. By applying the corrections for differential absorption (as discussed below) to the values of *A'* and *B'* the resulting values are given in the third and fourth column as *A* and *B*. These values have also been reduced so that their sum is again equal to 1000.

TABLE Ia.

Element	A'	B'	A	B	$I_{\text{mod.}}/I_{\text{unm.}}$	
					Wollan	Woo
KCl	534	466	505	495	1.03	
Sulphur	499	501	466	534	1.30	
Aluminum	403	597	376	624	2.31	1.45
NaF	362	638	335	665	3.15	
MgO	347	653	322	678	3.51	
Carbon	286	714	276	724	5.56	2.26
Boron	257	743	246	754	8.15	
Paraffin	222	778	216	784	14.30	5.49

It can be readily seen that the ratio of the modified to the unmodified scattering is given by the following formula

$$I_m/I_u=(B-N)/(A-M). \tag{2}$$

The final values of I_m/I_u for solids are given in the fifth column of Table Ia. In column six of this table are given the values of I_m/I_u as measured by Woo.[8] It is seen that his values are all considerably lower than those obtained here. It would seem that an explanation of this can be found in connection with the measurements of DuMond on the width of the modified lines in which it was found that the modified lines are considerably broader than the unmodified. Since Woo used a spectroscopic method of measuring the intensity of the modified and unmodified lines the evaluation of the ratio of the integrated intensities would be considerably affected, it would seem, if the width of one line was much greater than that of the other. The intensity of the narrower unmodified lines would be overestimated and hence he would obtain a too small value of I_m/I_u.

One would expect that the value of I_m/I_u for solids would vary according to the physical condition of the scattering body, i.e., depending on the amount of crystal reflection one took in. In using powdered crystals as scatterers the result should, however, be a constant for a definite angle and angular divergence.

SCATTERING FROM GASES

In order to increase the intensity of the scattering by gases a special scattering chamber was designed to admit of the use of higher pressures than was possible in the previous work of the writer on scattering by gases. The general scheme is shown in Fig. 5. A rectangular brass block 10 cm long was milled out with a rectangular passage 1 cm wide by 2 cm high. A circular opening was made in the center of one side 1.5 cm in diameter. The ends and side opening were fitted with windows of celluloid 0.7 mm thick. Rubber gaskets were used between the celluloid and the chamber and the windows were held in place by screws and brass plates. It was found possible to use this chamber at a pressure of 30 atmospheres with practically no leak. The x-rays traversing the gas were scattered through the window on the side. This type of construction does not admit of measurements at angles much greater than 40° on each side of the normal to the x-ray beam.

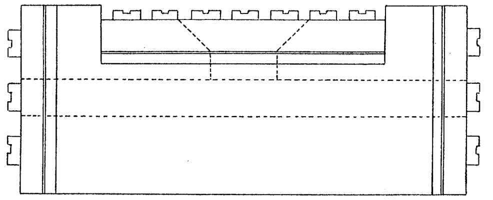

FIG. 5. Gas scattering chamber.

By using this chamber, measurements of the ratio of modified to unmodified scattering have been made for nitrogen, oxygen and argon at $\varphi=120°$. The values of *A'* and *B'* thus obtained as well as the corrected values *A* and *B* are given in Table Ib. The values of I_m/I_u are calculated as for solids according to Eq. (1), the value of the correction factor being slightly different for this

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[8] J. H. Woo, Phys. Rev. **27**, 119 (1926).

TABLE Ib.

					$I_{\text{mod.}}/I_{\text{unm.}}$		
Element	A'	B'	A	B	Exp.	Wave mechanics	Classical
Argon	484	516	470	530	1.27	1.44	1.58
Oxygen	308	692	295	705	4.49	4.33	4.71
Nitrogen	294	706	283	717	5.10	5.08	5.41

note $C=A+B=1000$

case as is pointed out below. The final values of I_m/I_u are given in the fifth column of Table Ib.

CORRECTIONS

On account of the difference in wave-length of the modified and unmodified lines the measured intensity ratio will have to be corrected for the different absorption which these lines experience when traversing any matter interposed in their path. In consideration of this, the following corrections will have to be made.

(1) The differential absorption within the filters themselves

The zirconium filter was found to transmit 36 percent of the unmodified K_α lines. Since the yttrium and zirconium filters transmit equally at the wave-length corresponding to the modified line the transmission of the modified line can be calculated from its wave-length. By taking the absorption coefficients as proportional to λ^3 it is found that the ratio of absorption of the modified line to that of the unmodified line is equal to 1.18.

(2) The differential absorption in the ionization chamber

Since the ionization chamber contained argon at one atmosphere pressure not all the radiation was absorbed. Taking the mass absorption coefficient of argon as $\mu_\rho=14.4$, for a chamber 30 cm long the ratio of the absorption of the modified to the unmodified line comes out as 1.12. Here the greater absorption of the modified line, however, tends to increase its apparent intensity while in the above case (1) the modified line was absorbed away and hence weakened in intensity. Combining corrections 1 and 2 gives a factor 1.04 by which the values of B' must be multiplied.

(3) A calculation of the differential absorption within the scattering body

This gives the relation

$$\frac{I_m}{I_u}=\frac{I_m'}{I_u'}\left(\frac{\mu+\mu_1}{2\mu}\,\frac{1-e^{-2\mu b/\sin\varphi/2}}{1-e^{-(\mu+\mu_1)b/\sin\varphi/2}}\right)=\beta\frac{I_m'}{I_u'},^{*} \qquad (3)$$

where the primes represent the uncorrected value, μ and μ_1 represent the linear absorption coefficients of the unmodified and modified lines, respectively, and b is the thickness of the scattering body.[9] Eq. (3) represents the case where the face of the scatterer makes an angle $\varphi/2$ with the primary beam. Taking the absorption coefficients as proportional to λ^3 we find $\mu_1=1.16\mu$ for this case. This simplifies β to

$$\beta=1.08(1-e^{-\mu b})/(1-e^{-1.08\mu b}). \qquad (4)$$

It can also be seen that when the thickness b of the sample is sufficient to absorb all the radiation the latter term reduces to unity and $\beta=1.08$. For the scattering bodies MgO and heavier, this latter condition is fulfilled. Correction (3) is negligible for the case of scattering by gases at the pressures here used.

(4) The differential absorption in the windows of the ionization chamber and of the scattering chamber

The former of these corrections is negligible and the latter is about 2 percent and of course only applies to the scattering by gases.

THEORY OF SCATTERING BY INDIVIDUAL ATOMS

The theory of scattering of x-rays by a free atom has been worked out independently by Wentzel[10] and Waller[11] on the basis of wave mechanics and independently by Raman[12] and Compton[13] on the basis of classical theory.

To a first approximation the theories as worked out by Wentzel and by Waller are identical, giving for the intensity of the modified and unmodified scattering the relations

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* I_m' and I_u' correspond here to the total measured intensities before M and N have been subtracted and hence the various corrections are applied to B' and the resulting $B+A$ reduced to 1000.

[9] Jauncey and De Foe, Phil. Mag. **1**, 711 (1926).

[10] G. Wentzel, Zeits. f. Physik **1**, 43 and 779 (1927).

[11] I. Waller, Phil. Mag. **4**, 1228 (1927).

[12] C. V. Raman, Ind. J. Phys. **3**, 357 (1928).

[13] A. H. Compton, Phys. Rev. **35**, 925 (1930).

$$I_{\text{unm.}} = I_e\left(\sum_1^Z f_n\right)^2$$

$$I_{\text{mod.}} = I_e\left[Z - \sum_1^Z (f_n)^2\right], \qquad (5)$$

where I_e is the intensity of scattering by a free electron as obtained by the Thomson classical theory, f_n is the structure factor for the nth electron which is defined by the relation

$$f_n = \int_0^\infty u_n(r)\frac{\sin(xr)}{xr}dr, \qquad (6)$$

where $x = 4\pi/\lambda \sin \varphi/2$, and $u_n(r)$ is the charge for the nth electron lying between the spherical shells of radius r and $r+dr$ and expressed in electron units.

Eq. (5) has been worked out on the assumption that the incident wave-length is sufficiently long so that relativity effects are negligible. For wavelengths shorter than 1A the relativity effect becomes quite appreciable and it is necessary to apply a correction to the incoherent part of Eq. (5). For the case of the molybdenum radiation ($\lambda = 0.71$A) which was used in the above described experiments, the relativity effects should be accurately accounted for if the modified part of Eq. (5) is divided by the Breit[14]-Dirac[15] factor $(1+\gamma \text{ vers } \varphi)^3$ where $\gamma = h/mc\lambda$. Waller's derivation of Eq. (5) includes a small additional factor which arises from the consideration of the Pauli exclusion principle. This term has the form $-\sum(f_{nm})^2$ where the sum is taken over all pairs of L electrons of the same spin and $n \neq m$. Including this term the modified intensity given by Eq. (5) will now have the form

$$I_{\text{mod.}} = I_e[Z - \sum(f_n)^2 - \sum(f_{nm})^2]. \qquad (7)$$

The inclusion of this term has no effect on the intensity as calculated for nitrogen and oxygen at this large value of $(\sin \varphi/2)/\lambda$ used, but it does have a small effect on the intensity of the modified radiation from argon.

In order to calculate the values of I_m and I_u from Eq. (5) or (7) it is necessary to know the values of the structure factor, f_n, corresponding to all the electrons in the atoms in question. Hartree[16] has made calculations of the Schrödinger charge distribution for a few atoms by his method of self-consistent fields and from this one can calculate f_n by means of Eq. (6). From the few cases for which the charge distribution has been completely worked out James and Brindley[17] have shown that it is possible by a method of interpolation to obtain reliable values of f_n for all elements of atomic number less than 18 and they have given a table from which the values of f_n for the various electrons can be directly read. From these tables the values of f_n at $(\sin \varphi/2)/\lambda = 1.22$ $(\varphi = 120°)$ have been taken and are given in the first three columns of Table II. From these data the values of I_m/I_u have been calculated for a number of elements by means of Eqs. (5) and (7) and with correction for the Breit-Dirac factor. These values are given in the next to the last column of Table II and are plotted to give the curve in Fig. 6.

In the classical theory of scattering as developed by Raman and by Compton the atom has been assumed to be of the form of a degenerate electron gas. On this assumption the classical theory gives the following expression for the total scattering.

$$I_{\text{tot.}} = I_e[f^2 + Z - f^2/Z], \qquad (8)$$

where f represents the structure factor of the atom as a whole and hence $f = \sum f_n$.

In view of the fact that the classical theory does not distinguish a modified or unmodified scattering, Compton compared Eq. (8) with Eq. (5) and hence concluded that the term f^2 represents the unmodified scattering and the term $[Z - f^2/Z]$ represents the modified part of the scattered radiation. If we write this term in the form $[Z - (\sum f_n)^2/Z]$, a comparison with the same term of Eq. (5) shows that the two expressions are identical only if the values of the structure factors f_n are identical for each electron.

Taking the values of f_n given in Table II which were used to calculate I_m/I_u by means of Eq. (5) one can now calculate this quantity on the basis of Eq. (8) where, as mentioned above, $f = \sum f_n$. The values so calculated are given in the

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[14] G. Breit, Phys. Rev. **27**, 242 (1926).

[15] P. A. M. Dirac, Proc. Roy. Soc. **A111**, 405 (1926).

[16] D. R. Hartree, Proc. Camb. Phil. Soc. **24**, 89 (1928).

[17] James and Brindley, Phil. Mag. **12**, 89 (1931).

TABLE II.

At. No.	Element	$f_{(1s)}$	$f_{(2s)}$	$f_{(2p)}$	$\left[\sum_1^Z f_n\right]^2$	$\frac{Z-\Sigma(f_n)^2-\Sigma(f_{nm})^2}{(1+\gamma \text{ vers } \varphi)^3}$	$\Sigma(f_{nm})^2$	$I_{\text{mod.}}/I_{\text{unm.}}$ Wave mechanics	Classical
3	Lithium	0.10			0.04	2.56		64.00	
4	Beryllium	.225			.203	3.34		16.50	
5	Boron	.320			.409	4.11		10.10	
6	Carbon	.440			.774	4.82		6.24	
7	Nitrogen	.523			1.093	5.56		5.08	5.41
8	Oxygen	.600			1.44	6.25		4.34	4.71
9	Fluorine	.658			1.73	6.99		4.04	
10	Neon	.698			1.95	7.70		3.95	
11	Sodium	.750			2.25	8.48		3.77	
12	Magnesium	.790			2.49	9.23		3.70	
13	Aluminum	.815	0.01	0.04	2.82	10.00		3.54	
16	Sulphur	.876	.03	.12	5.25	12.36	0.042	2.35	
17	Chlorine	.888	.062	.16	7.30	12.97	.155	1.78	
18	Argon	.900	.097	.18	9.45	13.65	.298	1.44	1.63
19	Potassium	.920	.134	.23	12.20	14.12	.503	1.16	

last column of Table II for the gases under consideration. It is seen, as has been previously pointed out by Herzog,[18] that the assumption that the atom scatters as a degenerate gas leads to considerably larger values of I_m/I_u than one obtains from a Schrödinger atom as calculated by Hartree. The classically calculated values are shown in Fig. 6 as short lines above the solid curve.

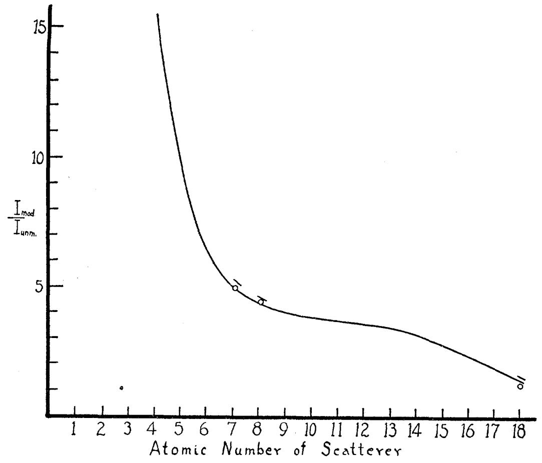

FIG. 6. Ratio of modified to unmodified scattering as a function of atomic number. Solid curve calculated according to Waller, short lines calculated according to Compton and circles represent measured values. Scattering angle, 120°; wave-length, 0.71A.

The measured values of I_m/I_u seem to be in better agreement with the calculations based on wave mechanics.

CONCLUSIONS

The present measurements of the ratio of the intensity of the modified to the unmodified scattering by gases, together with previous measurements of the total scattering of x-rays by gases as made by the writer for neon and argon and by Herzog for argon, are in good accord with the theory of scattering as developed by Wentzel and Waller and the calculations of the Schrödinger charge distribution by Hartree's method of self-consistent fields. Taken as a whole the measurements are in better accord with the wave-mechanics theory than with the classical derivation on the basis of an atom of the form of a degenerate electron gas.

In view of this fact measurements of the total intensity of scattering by a gas are not, in general, sufficient to determine the structure factor of the atoms of which the gas is composed. This can be seen to be the case from Eq. (5) since the total intensity corresponding to the sum of the modified and unmodified scattering is a function of $\sum(f_n)^2$ and hence cannot be solved for the atomic structure factor $f=\sum f_n$ in terms of the measured total intensity. This is the case not only for the scattering by gases but for the intensity of the diffuse scattering of x-rays by crystals as measured by Jauncey[19] and his collaborators.

There are, however, special cases in which the theories represented by Eqs. (5) and (8) are identical. In the case of scattering by helium, the

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[18] G. Herzog, Zeits. f. Physik **69**, 207 (1931).

[19] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **38**, 1071 and 1925 (1931); **42**, 453 (1932).

example discussed by Compton in his initial paper, the theories give identical results because the values of f_n for the two electrons are identical and hence f can be directly obtained. For the extreme case of scattering by a very heavy gas or vapor the modified radiation is negligible and hence the atomic structure factors according to either Eq. (5) or (8) are related to the total intensity by

$$I_{\mathrm{tot.}} = I_e f^2. \tag{9}$$

The measurements of the scattering of x-rays by mercury vapor as made by Scherrer and Staeger[20] are an example of the case where the modified radiation is negligible and hence f can be directly calculated according to Eq. (9).

For cases in which the atomic number lies between these limits, measurements of I_m/I_u, together with the measurements of the total scattered intensity, make it possible to solve for the unmodified radiation and hence for the atomic structure factor. Since $I_{\mathrm{tot.}} = I_{\mathrm{mod.}} + I_{\mathrm{unm.}}$ multiplying by I_m/I_u and substituting the value for $I_{\mathrm{unm.}}$ from Eq. (5) we obtain

$$f^2 = [1/(R+1)] I_{\mathrm{tot.}}/I_e, \tag{10}$$

where R represents I_m/I_u and I_e is the Thomson scattering by a single electron and f is the atomic structure factor equal to $\sum f_n$.

Since the present method of measuring I_m/I_u cannot be applied to all values of $(\sin \varphi/2)/\lambda$ one is not able in this way to obtain a complete structure factor curve. The above facts have been pointed out, however, to show the difficulties involved in the determination of the atomic structure factor from measurements of x-ray scattering involving the modified radiation.

The present experiment seems to furnish additional evidence of the correctness of the wave-mechanics theory of the scattering of x-rays by gases as well as giving another method of obtaining the ratio of the modified to the unmodified scattering at certain values of $(\sin \varphi/2)/\lambda$.

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[20] P. Scherrer and A. Staeger, Helv. Phys. Acta **1**, 518 (1928).