Elastic Electron Scattering in Neon

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The *distribution in angle* of electrons having energies ranging from 10 to 800 volts scattered by *neon* atoms was measured over a range from 7° to 150°. For the higher energies, above 600 volts, the scattering curves fall monotonically with increasing angle. For smaller energies, an increase of scattering with increasing angle in the large angle region becomes more and more accentuated as the energy is reduced to 35 volts. Below this the curves flatten out. The position of the minimum in the scattering curves shifts steadily to smaller angles as the electron energy is increased. The *elastic cross section* obtained by integration of the scattering coefficient multiplied by the sine of the scattering angle is computed for electron energies between 10 volts and 800 volts, and is compared with direct absorption measurements. Mott's formula is inadequate to describe the scattering curves even at the highest voltage, a result in accordance with a criterion set up by Morse. The low-voltage curves can be fitted satisfactorily to a formula of the type developed theoretically by Allis and Morse.

INTRODUCTION

SEVERAL investigations on the elastic scattering of electrons by atoms of neon have been published in the last few years. Arnot[1] studied the elastic scattering, by neon atoms, of electrons having energies between 29 and 412 volts over a range from 10° to 120°. (Results were also given for 830 volt-electrons over a more limited angular range, 10° to 40°.) He found that the scattering fell off sharply with increasing angle up to a certain angle somewhere between 90° and 120° beyond which the scattering increased, especially for the lower electron energies. Bullard and Massey[2] investigated the scattering within the energy range 6 to 30 volts, and within the angular range 30° to 130°. Ramsauer and Kollath[3] devised an experimental method considerably different from that employed by other workers in the field to measure the scattering over the angular range 15° to 167° for electrons having energies between 0.61 and 15.9 volts. Recently Mohr and Nicoll[4] published a paper giving curves for the scattering of 50, 84, and 150 volt-electrons between 20° and 155°. The foregoing investigations were made over different angular ranges and different energy ranges. The agreement, when they overlap, is not always satisfactory. The present investigation was undertaken because it is desirable to have accurate experimental data, taken under identical conditions, over as wide a field as possible, in order to test quantitatively advances which are being made in the theoretical treatment of electron scattering.

APPARATUS AND METHOD

The apparatus is the same as that used in our recent work on helium but which was not described in the published paper.[5] In our earlier work on hydrogen and argon,[6] the apparatus was constructed of metal, made air-tight by sealing wax, and therefore could not be outgassed by heating. It was found that slow electrons (energies below about 50 volts) would not pass through narrow slits in such an apparatus, possibly because of adsorbed gases on surfaces near the slits. To permit some degree of outgassing by heating, the present apparatus (Fig. 1) was so designed that all of it could be enclosed in

* The investigation was made possible by assistance to the first named author from a grant made by the Rockefeller Foundation to Washington University for research in science.

[1] F. L. Arnot, Proc. Roy. Soc. **A133**, 615 (1931).

[2] E. C. Bullard and H. S. W. Massey, Proc. Roy. Soc. **A133**, 637 (1931).

[3] C. Ramsauer and R. Kollath, Ann. d. Physik **12**, 837 (1932).

[4] C. B. O. Mohr and F. H. Nicoll, Proc. Roy. Soc. **A138**, 469 (1932).

[5] A. L. Hughes, J. H. McMillen and G. M. Webb, Phys. Rev. **41**, 154 (1932).

[6] A. L. Hughes and J. H. McMillen, Phys. Rev. **39**, 585 (1932).

a glass tube 5.5 inches in diameter. A small electrostatic analyzer[7] containing the deflecting plates P and R (radius 19 and 21 mm) was made of Monel metal. Only those electrons travelling within a small angle with the normal to the slit S_1 could enter the analyzer and reach the collector E. This small angle was determined by the slit S_1 and the distance apart (2 mm) of the plates P and R. For most of the small angle scattering, increased resolution was secured by the addition of slit S_2, so that S_1 and S_2 now defined the angle of entry. The apparatus was erected on a flat

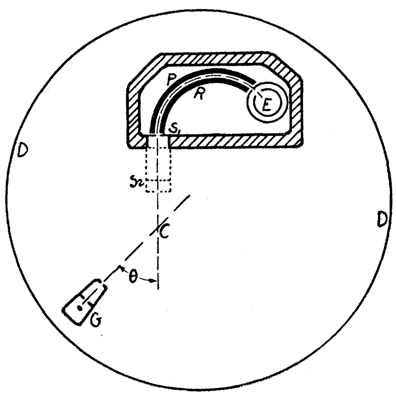

FIG. 1. Apparatus.

brass base plate through which all the necessary wire connections and gas, pump and gauge tubes passed. The electron gun G was rotated by means of a ground glass joint mounted in the base plate. To outgas the apparatus, a furnace was lowered over the glass tube and extended sufficiently far down to allow heating the analyzer and gun. The setting, θ, of the electron gun could be changed from 0° to ±150°. At very small angles, however, the results were unsatisfactory, partly because some of the electrons could pass straight into the analyzer from the gun. Another source of error in the scattering measurements at small angles arises from the fact that, though above a certain small angle, no primary electrons can pass directly from the gun into the analyzer, yet the finite angular width of the electron beam from the gun may cause it to straddle the slit S_2 and so prevent parts of the electron beam from contributing to the electrons scattered into the analyzer. For reasons such as these we do not consider that our apparatus measures satisfactorily the scattering at angles less than about 7°. (This was determined by measuring the width of the electron beam in a subsidiary experiment, in which the beam was swept across a fine slit and the number of electrons passing through it measured for different setting of the gun.)

For any given electron energy, scattering curves were obtained by taking observations at small angular intervals, usually 4° or 5°. To get the scattering coefficient, each observation was multiplied by sin θ to correct for the change in length of the scattering path effective at the different angular settings. Then to make all the values strictly comparable with each other, the scattering coefficients were measured at one angle (30°) for all the electron energies selected for investigation. Thus the results for all angles and for all energies are given in terms of the same unit. It was not possible to make a satisfactory experimental determination of the absolute value of the scattering coefficient, although, in principle, this could be found from the geometry of the apparatus, nor could we follow the procedure we used in our helium paper.[8]

EXPERIMENTAL RESULTS

The scattering coefficients measured in this investigation are given in Table I. The results in all columns are strictly comparable with each other.

The scattering curves are shown in a series of separate figures (Figs. 2–6) as it would be too confusing to plot them all on the same diagram. The only curves which show a regular diminution in the scattering coefficient with increasing angle are those for 800 and 625 volts. For all the others

[7] A. L. Hughes and J. H. McMillen, Phys. Rev. **34**, 291 (1929); A. L. Hughes and V. Rojansky, Phys. Rev. **34**, 284 (1929).

[8] We found for helium that, when the electron energies exceeded 400 volts, the experimental curves fitted perfectly the theoretical curves given by Mott's theory. It was then *assumed* that Mott's theory not only gave the shape of the curve, but also its absolute magnitude. Since the relative values of the curves for all energies were measured, it was possible to compute their absolute values. Such a procedure was not possible for neon, since within the range of energies used by us, no agreement between an experimental curve and the corresponding theoretical curve could be found.

TABLE I. *Scattering coefficients for electrons scattered elastically by neon atoms (arbitrary units). θ = angle. V = energy of electrons in volts.*

θ \ V	10	15	25	35	50	75	100	150	225	300	400	625	800
5.0°	..	..	..	..	(3827.0)	(6792.0)	(4695.0)	(6040.0)	(6855.0)	....	....	(5021.0)	....
7.5°	..	..	..	..	(2798.0)	(5330.0)	(4010.0)	(4640.0)	(4535.0)	(3158.0)	(4400.0)	(3610.0)	(3200.00)
10.0°	..	..	..	1423.5	(2363.0)	3782.0	3283.0	3358.0	3392.0	(2552.0)	(2958.0)	2560.0	1994.00
12.5°	..	..	..	1289.0	2161.0	2730.0	2539.0	2382.0	2385.0	1895.0	1957.5	1850.0	1301.00
15°	..	..	..	1198.0	1878.0	2341.0	2057.0	1977.0	1774.0	1491.0	1463.0	1424.0	918.00
20°	419.0	524.6	..	1044.0	1434.0	1720.0	1488.0	1345.0	1164.0	893.5	836.0	769.3	502.50
25°	417.0	535.0	863.0	910.0	1150.0	1273.0	1072.0	988.8	803.0	587.0	491.0	431.8	308.30
30°	414.0	524.0	731.0	800.0	951.0	956.2	796.6	682.1	514.4	401.2	298.8	250.3	187.20
35°	413.9	527.0	682.0	719.0	795.0	730.0	605.5	479.2	330.8	260.7	195.7	156.7	125.25
40°	428.0	533.0	636.0	664.0	675.0	566.3	506.6	351.0	243.7	160.8	140.6	108.4	87.08
45°	432.8	530.8	593.0		577.7	436.8	326.3	255.3	173.0	113.7	99.2	81.6	65.60
50°	432.8	526.4	570.5	566.5	468.5	359.0	240.6	189.3	128.6	88.8	69.7	64.2	52.81
55°	419.8	506.2	544.4		398.0	290.8	188.3	150.3	97.4	65.5	49.3	50.6	41.95
60°	407.8	498.3	515.5	474.0	344.2	245.5	151.6	116.3	76.3	50.8	41.0	38.7	34.27
70°	367.3	444.0	434.2	383.5	241.2	165.0	90.9	74.6	55.3	39.9	35.2	27.2	20.84
80°	307.7	360.0	344.8	292.0	177.3	96.6	47.4	49.8	47.4	32.9	29.3	23.8	15.44
90°	244.6	266.2	261.2	182.4	104.1	44.2	25.1	44.8	41.4	29.9	24.2	21.1	13.13
100°	184.7	193.0	181.4	118.2	63.3	28.1	24.5	46.4	42.2	28.6	21.2	18.2	11.24
110°	136.1	135.2	147.3	84.0	60.8	44.9	41.1	63.0	48.3	29.5	20.9	16.0	9.41
120°	101.9	103.8	144.2	127.8	107.4	102.4	74.7	88.3	55.3	31.5	20.3	13.6	8.10
130°	80.2	95.5	177.4	230.0	196.4	185.6	128.0	112.9	63.9	33.5	19.4	12.3	7.29
140°	75.4	110.5	306.8	347.0	288.9	283.0	187.7	138.0	73.2	41.6	19.2	11.3	6.65
150°	74.8	124.6	380.5	569.0	353.0	348.8	236.3	157.5	77.6	64.0	20.2	10.7	6.24

a more or less pronounced minimum develops in the region 90° to 135°. It will be noticed that relatively high values of small angle scattering persist down to 50 volts (Fig. 6), while below this value, the small angle scattering tends to become of the same order as that at much larger angles. The *increase* in the scattering coefficient with *increasing angle*, at large angles, first becomes clearly noticeable at 300 volts (Fig. 3) and becomes most marked at 35 volts (Fig. 5). For still smaller energies the curves in the large angle region flatten out once more. Attention is called to a definite, though small, maximum at 50° for 10 volt-electrons and one at 45° for 15 volt-electrons (Fig. 5). A hump on the 25, 35 and 50 volt curves in the vicinity of 70°, suggests the superposition of a maximum on a steeply falling curve (Fig. 5).

The position of the minimum shifts in a regular way with the primary energy of the electrons as shown in Fig. 7.

On comparing our results with those of previous investigators in the regions where they overlap, we find good agreement in several cases, poor in other cases. By "good agreement" is meant that, if two curves are fitted together at some arbitrary point, they do not deviate

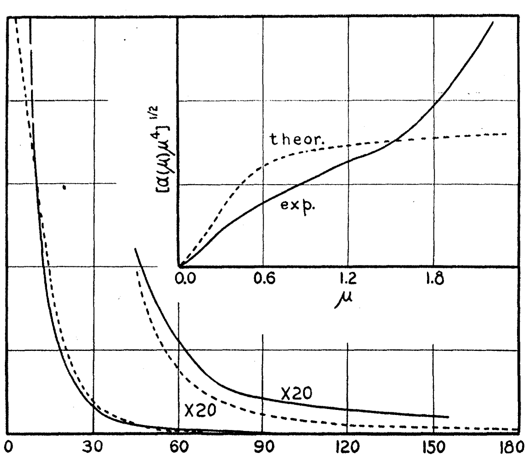

FIG. 2. Scattering of 800 volt-electrons: *continuous line*, experimental values; *broken line*, theoretical values. *Inset*. Comparison of the experimental and theoretical values of $[\alpha(\mu)\cdot\mu^4]^{\frac{1}{2}}$.

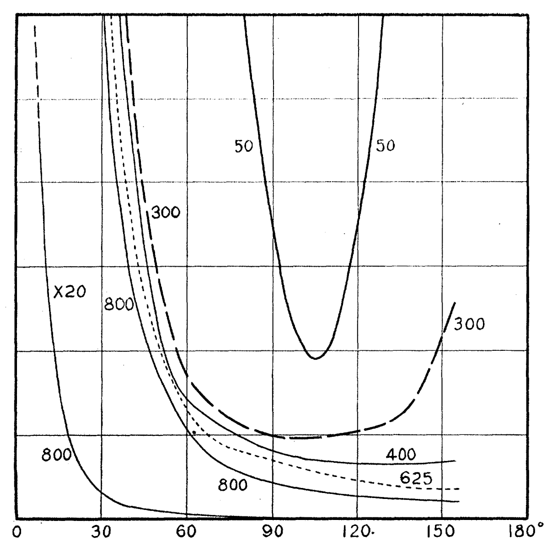

FIG. 3. Scattering of 800, 625, 400, 300 and 50 volt-electrons.

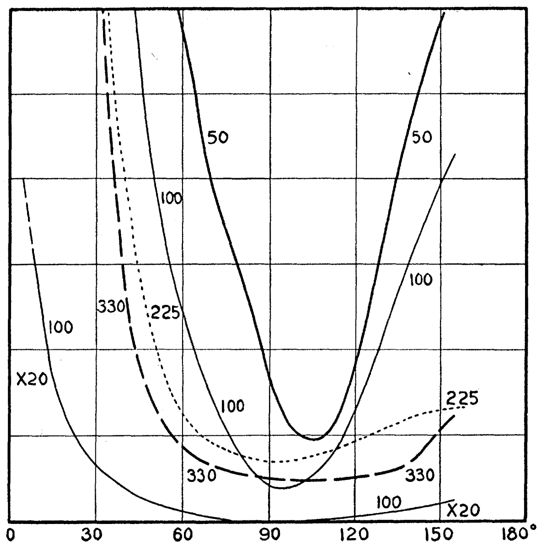

FIG. 4. Scattering of 300, 225, 100 and 50 volt-electrons.

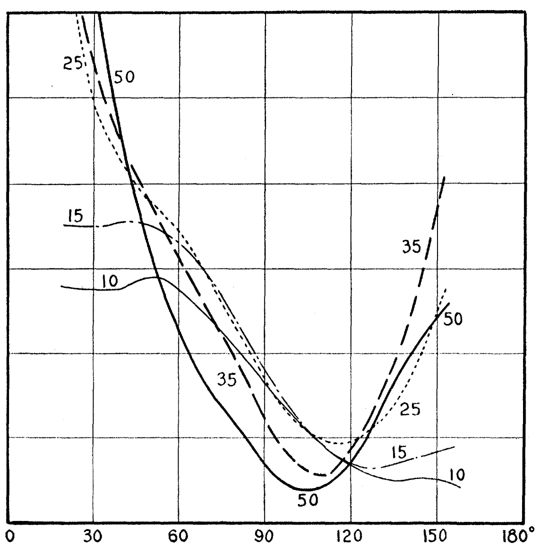

FIG. 5. Scattering of 50, 35, 25, 15 and 10 volt-electrons.

from each other anywhere by more than 10 percent. Our 10 volt curve shows good agreement with that of Bullard and Massey, but only fair agreement with that of Ramsauer and Kollath. The agreement between our 50 volt curve and that of Mohr and Nicoll is fair, that between our 150 volt curve and theirs is good. Our 50 volt curve agrees well with that of Arnot. We find only fair agreement between our 404 volt curve and his 412 volt curve.

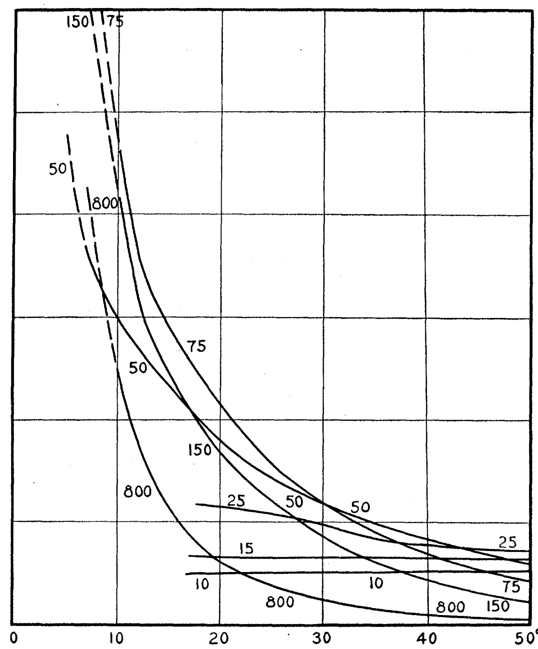

FIG. 6. Small angle scattering of 10 to 800 volt-electrons.

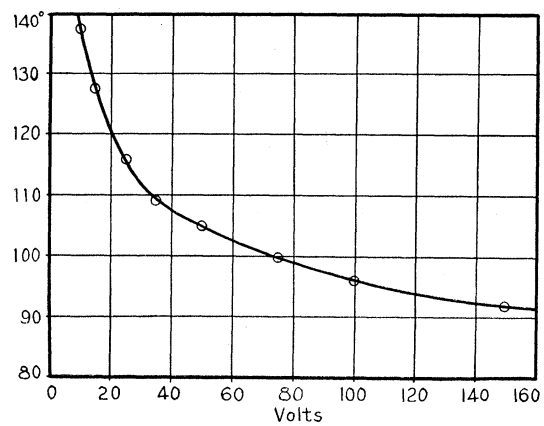

FIG. 7. Change in position of the minimum scattering with electron energy.

DISCUSSION

The first approach to a theoretical treatment of the scattering of electrons by atoms is to suppose that the effect of the electrons within the atom on the incoming electron may be neglected. The problem then becomes one of the interaction between an electron of charge, $-e$, and a nucleus of charge, $+Ze$. This leads, as is well known, to a Rutherford scattering distribution, according to which the scattering coefficient is proportional to $\operatorname{cosec}^4(\theta/2)$, where θ is the angle of scattering. Such a formula did not represent the experimental facts, and it became evident then that the modification in the interaction forces between the atomic electrons and the incoming electrons must be taken into account. The problem has been attacked from the point of view of wave mechanics by several investigators. The formulas given by Mott,[9] using a method first advanced by Born,[10] for the scattering coefficient of electrons, colliding elastically, are

$$\alpha(v, \theta) = [(e^2/2mv^2)(Z-F) \operatorname{cosec}^2(\theta/2)]^2 \quad (1)$$

and

$$\alpha(\mu) = [(e^2m/2h^2)(Z-F)(1/\mu^2)]^2 \quad (2)$$

where e, m, and v are the charge, mass and velocity of the scattered electron, Z the atomic number of the atom, θ the angle of scattering, and $\mu = \sin(\theta/2)/\lambda$, where $\lambda = h/mv$. F, which is a function of μ, or of v and θ, is the atomic structure factor and measures the contribution of the atomic electrons to the scattering. Its value has been calculated by James and Brindley.[11] In Fig. 2, we have plotted the value of $\alpha(v, \theta)$ as given by Eq. (1) together with the experimental values. If we make the curves fit together at $\theta = 10°$, we see that the experimental curve is steeper than the theoretical curve at small angles ($<20°$) and less steep at large angles ($>40°$). This is precisely the kind of departure which occurs between the corresponding curves for the scattering by helium atoms when the speed is not high enough to give a perfect fit.[12]

The failure of Mott's theory to represent the scattering of 800 volt-electrons is shown more clearly in the following way. According to Eq. (2), $\mu^2[\alpha(\mu)]^{\frac{1}{2}}$ should, when plotted as a function of μ, give a graph of $Z-F$ and therefore of F. In the inset in Fig. 2, we have plotted $\mu^2[\alpha(\mu)_{exp.}]^{\frac{1}{2}}$ in arbitrary units, where $\alpha(\mu)_{exp.}$, the *experimental* scattering coefficient, is calculated from the 800 volt column in Table I. We have also plotted $Z-F$, using James and Brindley's values for F. It is evident that there is no similarity in shape between the theoretical and experimental curves. The conclusion to be drawn is that the simplifying assumptions made in deriving the above equations do not hold for collisions between neon atoms and electrons with energies as low as 800 volts. Morse[13] gives a curve for $Z-F$ for neon and shows that Arnot's scattering coefficients for 412 volt-electrons fit on it. The reason for this apparent agreement is that only a few points corresponding to small angle scattering were taken. On examining the inset in our Fig. 2, it will be seen that if we adjust the experimental curve to coincidence with the theoretical curve at $\mu = 0.7$, then the two curves will practically coincide for *all* values of μ less than 0.7, but will diverge tremendously for greater values. From $\mu = 0$ to $\mu = 0.45$ (on allowing for the fact that Morse's μ is 4π times ours) is just the limited region over which Morse found agreement between Arnot's results and theory. Morse[14] gives as a criterion of the minimum energy at which one would expect the formulas given in Eqs. (1) and (2) to hold, a value of 50 Z^2 volts. (This value comes from the fact that the approximations made in deriving the formula are satisfactory only when the distance of closest approach of the particle to the scattering center is much smaller than the wave-length associated with the electron.) For the inert gases this leads to the values in Table II.

TABLE II.

Gas	Z	$50Z^2$	Experimental
He	2	200 volts	>350 volts, <500 volts
Ne	10	5000 "	>800 volts
A	18	16,200 "	?

The results in the "Experimental" column for helium are taken from our previous paper, and those for neon, from this paper. It would be

[9] N. F. Mott, Proc. Roy. Soc. **A127**, 658 (1930).

[10] M. Born, Zeits. f. Physik **38**, 803 (1926).

[11] R. W. James and G. W. Brindley, Phil. Mag. **12**, 81 (1931). See also D. K. Froman, Phys. Rev. **36**, 1339 (1930).

[12] It is to be admitted that there is a certain degree of arbitrariness in selecting a point at some distance from either end of the experimental range for fitting the curves together. Had either end point been chosen, the curves would not have crossed a second time. In view of the trend observed in the comparison between the experimental and theoretical curve for helium, it was reasonable, however, to look for a similar trend in the neon curves.

[13] P. M. Morse, Rev. Mod. Phys. **4**, 597 (1932); Fig. 9.

[14] P. M. Morse, *Private communication.*

interesting to carry out experiments with neon up to 10,000 volts, to see if Morse's criterion holds also for this gas. Considerable modification in our apparatus would be necessary to enable us to use voltages higher than about 800. It is evident then that Mott's formula is not adequate to describe the scattering of electrons of energies of the order of 800 volts in neon.

The formulas given by the Born-Mott theory are approximate, because, in deriving them, it has been assumed that the waves representing the electrons are not distorted by the atomic field. Only when the electron wave is sufficiently short is the assumption justified by agreement between theory and experiment. This distortion of the electron wave is not neglected in another method used by Mensing,[15] Allis and Morse,[16] Faxen and Holtzmark.[17] The method is similar to that used by Mie and Debye[18] in the problem of the scattering of ordinary light waves by very small particles, where the distribution in angle of the scattered light contains maxima and minima which move in towards small angles as the wave-length is diminished. Allis and Morse find that the angular distribution of the scattered electrons is given by the expression

$$\frac{d\omega}{k^2}\sum_{\lambda,\,\lambda'=0}^{\infty}(2\lambda+1)(2\lambda'+1)P_\lambda(\cos\theta)P_{\lambda'}(\cos\theta)\sin\gamma_\lambda\sin\gamma_{\lambda'}\cos(\gamma_\lambda-\gamma_{\lambda'}),$$

where $d\omega$ is the solid angle, θ the angle of scattering, k^2 is the kinetic energy of the electrons, λ and λ' are integers, the P's are Legendre polynomials, and the γ's are certain angles. These angles, γ_λ and $\gamma_{\lambda'}$, are the shifts in phase of the wave representing the electron when in the field of force of the atom. The above expression is exact as it stands. Before anything can be done with it, however, the γ's must be calculated for a given electron wave-length and for the potential field representing the atom. This is where an approximation has to be made, for the calculation has been carried out only for very simple types of field. Morse calculates the γ's for the case of an atom represented by a nucleus Ze surrounded by a spherical shell of charge $-Ze$ and having a radius r_0. On inserting these into the formula, it is possible to compute the scattering coefficients for various angles and for various electron energies. The results, however, do not agree with our experimental values. We therefore attempted to use the expression as an empirical formula for electron scattering. Expanding the formula as far as the first three terms, we get, for the scattering, a quantity proportional to

$$(1/v^2)(\sin^2\gamma_0+6\cos\theta\sin\gamma_1\sin\gamma_0\cos(\gamma_1-\gamma_0)+9\cos^2\theta\sin^2\gamma_1)$$

where v is the velocity of the particle. The γ's were determined, for each velocity, from the scattering at two arbitrarily chosen angles. Then with the γ's so determined (Fig. 8*A*) the complete curves were computed by means of the above expression. The results are shown in Fig. 8. It will be seen that, if we ignore the small angle scattering ($<30°$), the agreement between the experimental and theoretical curves is fairly good. The curves shown in Fig. 8 are all plotted on the same scale, i.e., the experimental and theoretical curves have not been separately fitted together in each of the plots *C*, *D* and *E*, except insofar as the γ's in plot *A* are determined for each electron speed at some arbitrarily chosen angle. (Mohr and Nicoll[4] have already shown that certain large angle scattering curves resemble in shape some of the Legendre coefficients.)

Massey and Mohr[19] have developed a theory of electron scattering in which the distortion of the electron wave by the atom and electron exchange are taken into account. Unfortunately they have

[15] L. Mensing, Zeits. f. Physik **45**, 603 (1927).
[16] W. P. Allis and P. M. Morse, Zeits. f. Physik **70**, 567 (1931); see also P. M. Morse, Rev. Mod. Phys. **4**, 578 (1932).
[17] H. Faxen and J. Holtzmark, Zeits. f. Physik **45**, 307 (1927).
[18] See M. Born, *Optik*, p. 275 (Springer, 1933).
[19] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. **A136**, 289 (1932).

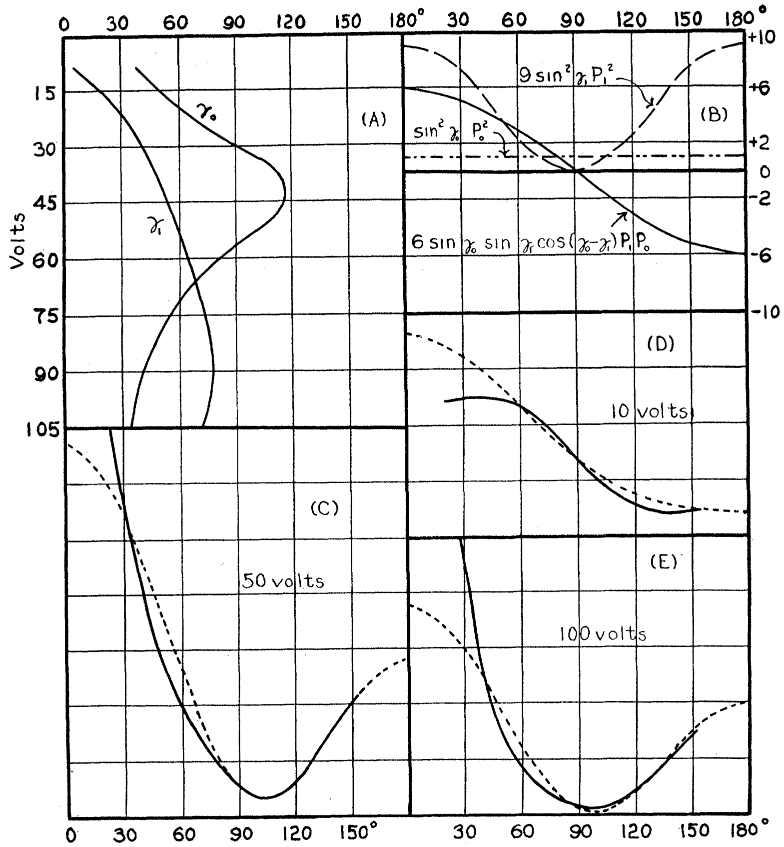

FIG. 8. Comparison of experimental with theoretical scattering curves: A, phase angles, γ_0 and γ_1. B, Legendre coefficients. C, D and E, *continuous lines*, experimental values for 10, 50 and 100 volt-electrons; *broken lines*, theoretical values.

not computed numerical results for neon which could be checked by our experimental values.

The absorption coefficient of a gas for electrons is frequently interpreted in terms of the "cross section" of the atom for interception of the electrons. If a single atom be placed on an area 1 cm^2, and if that area be bombarded by a beam of electrons uniformly distributed over the area, then the ratio of the number of electrons diverted from the original direction to the total number measures the cross section of the atom. (Another type of cross section would involve, in addition, the number of electrons losing speed but not changing direction on passing through the atom. This does not concern us here.) The cross section includes the "elastic cross section" and the "inelastic cross section." The former refers to the number of electrons diverted from the beam without loss of energy, while the latter refers to those diverted and losing energy. It is possible to use our scattering curves to compute the elastic cross section. Since the number of electrons scattered down a solid angle $d\omega$ is proportional to $d\omega \cdot \alpha(v, \theta)$, by the definition of the scattering coefficient $\alpha(v, \theta)$, then the number deviated from the main beam of electrons, through an angle larger than a certain small angle $\delta\theta$, by elastic scattering, is proportional to

$$\int_{\delta\theta}^{2\pi} \alpha(v, \theta) \cdot 2\pi \sin \theta d\theta.$$

This integral can be evaluated graphically from the values given in Table I after each value is properly multiplied by $\sin \theta$. According to theory, the value of $\alpha(v, \theta) \cdot \sin \theta$ goes to zero, as θ

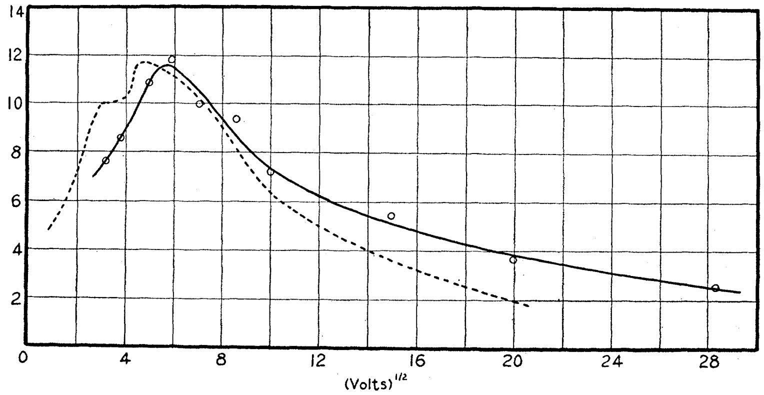

FIG. 9. Elastic cross section. *Continuous lines*, experimental values; *broken line*, calculated from Normand and Smith's results.

changes from a small angle to zero angle. This tendency was very evident in our curves, even though the smallest angles at which measurements were made was usually 7°. Consequently an obvious extrapolation of our curves to $\theta=0°$ gives us the total number of electrons scattered between the extreme limits $\theta=0°$ and $\theta=180°$. It is believed that the elastic cross section, so computed, is more accurate than any values which can be inferred from direct measurements of absorption of electrons in gases, for it has been found[20] that the total cross section depends considerably on the geometry of the apparatus, which is another way of saying that the effective minimum angle, $\delta\theta$, through which the electrons must be deviated in order to be counted, is not small enough to have no effect on the measured cross section. The elastic cross sections so calculated for different energies are plotted as a heavy line in Fig. 9. A comparison with direct measurements of the absorption coefficient can be made in this way. Normand[21] measured the absorption coefficient for electrons in neon, which gives us the total cross section. If we make the assumption that the inelastic cross section is accounted for chiefly by collisions resulting in ionization, thus neglecting the inelastic collisions resulting in excitation, then, on subtracting the ionization yield from the total absorption, both in the proper units, we should get a quantity proportional to the elastic cross section. Smith[22] has determined the ionization yield. The dotted curve is that obtained by subtracting the ionization yield from the total absorption. It will be seen that there is fairly satisfactory agreement between the shape of the elastic cross-section curve determined by the proper integration of the elastic scattering coefficients and that obtained from total absorption and ionization experiments. The nature of the deviation between the two is accounted for qualitatively when we remember that, as the energy of the electrons is diminished, the relative number of inelastic excitation collisions increases. This is the factor which was left out of consideration in computing the dotted curve, as we have no quantitative data available as to the relative number of collisions leading to excitation.

[20] R. R. Palmer, Phys. Rev. **37**, 70 (1931).

[21] C. E. Normand, Phys. Rev. **35**, 1217 (1930).

[22] P. T. Smith, Phys. Rev. **36**, 1293 (1930).