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On Compton's Latitude Effect of Cosmic Radiation

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By considering the influence of the earth's magnetic field on the motion of charged particles (electrons, protons, etc.) coming to the earth from all directions in space, it is shown that the experimental variation of cosmic-ray intensity with magnetic latitude, as found by Compton and his collaborators, is fully accounted for. The cosmic radiation must contain charged particles of energy between limits given in the paper. The experimental curve may be represented by a suitable mixture of rays of these energies, but it is not at all excluded that a part of the radiation may consist of photons or neutrons. For predominantly negative particles there must be in the region of rapidly varying intensity a predominant amount of rays coming from the east, and conversely for positive rays. Because of the fact that in regions near the magnetic equator there is a predominance of rays coming nearly horizontally, the absorption by the atmosphere may be increased. Finally the fact that Compton's result definitely shows that the cosmic rays contain charged particles gives some support to the theory of super-radioactive origin of these rays advanced by one of the present authors.

I.

IN the course of a survey of the intensity of cosmic radiation at a large number of stations scattered all over the world, A. H. Compton and his collaborators[1] discovered the remarkable fact that while the intensity is nearly constant for latitudes north of 34° in the American continent and south of 34° in Australasia, it drops sharply between these latitudes to a value about 87 percent as great as that for high latitudes, reaching a minimum at or near the magnetic equator. A close correlation with magnetic latitude was also found. These results are in agreement with those of J. Clay and H. P. Berlage.[2] This discovery rules out the hypothesis that the cosmic radiation consists of photons alone and suggests that it is made up at least partly of electrons, protons or other charged particles. The question as to the origin of these particles remains as yet unanswered; it is very likely bound up with general cosmogonical problems, an hypothesis as to which has already been advanced by one of the present authors.[3]

It is clear that the latitude effect discovered by Compton is attributable to the charged components of the cosmic radiation alone, so that the problem arises as to whether the experimental results can be accounted for by considering the influence of the earth's magnetic field on the motion of such particles.[4] This influence has

[1] A. H. Compton, Phys. Rev. **41**, 111 (1932); **41**, 681 (1932); also a paper presented at the Chicago meeting of the American Physical Society, November 25, 1932 (Abstract in Bull. Am. Phys. Soc. **7**, 13 (1932)). See also R. D. Bennett, J. L. Dunham, E. H. Bramhall and P. K. Allen, Phys. Rev. **42**, 446 (1932).

[2] J. Clay and H. P. Berlage, Naturwiss. **20**, 687 (1932). Reference is also made in this letter to a paper by A. Corlin which unfortunately is unavailable to the authors.

[3] G. Lemaitre, Nature **128**, 704 (1931).

[4] Qualitatively, the latitude effect was predicted by W. Heisenberg at the end of his paper *Theoretische Überlegungen zur Höhenstrahlung*, Ann. d. Physik **13**, 430 (1932).

already been extensively treated by Carl Störmer[5] in connection with his investigations on the origin of the aurora borealis. It will be shown in the present paper that the experimental variation of intensity with latitude is fully accounted for if the cosmic radiation consists at least in part of electrons (or protons) of energy of the order of 10^{10} electron-volts, coming to the earth from all directions in space.

II.

Since the force is perpendicular to the path of a charged particle moving in a magnetic field, the kinetic energy is constant and the speed is constant. The relativistic mass is therefore also constant and the motion may be treated by the methods of classical dynamics. Since a Hamiltonian exists (the relativistic Hamiltonian for motion in a magnetic field), Liouville's theorem is applicable.[6] If now we assume that the intensity distribution of the cosmic radiation at infinity is homogeneous and isotropic, the intensity in all allowed directions at any point in the earth's magnetic field is, by Liouville's theorem, the same. Thus the question of calculating the intensity at any point on the earth's surface reduces to that of finding out in which directions particles coming from infinity can reach that point. There are, as we shall see, three possibilities: either all directions are forbidden, or all directions are allowed, or only certain directions are allowed and the rest forbidden. At all points belonging to the last category there is a cone which encloses all directions in which trajectories issuing at infinity can reach the point in question. For particles of any given kinetic energy our main problem is thus the determination of this cone, which in the cases of points on the earth's surface belonging to the first two categories may be completely closed or completely open. After the cone is found the intensity at the corresponding point can be immediately calculated by computing the solid angle of the cone.

[5] Carl Störmer, Zeits. f. Astrophys. **1**, 237 (1930). References to his previous work are given at the end of this paper. See also Zeits. f. Astrophys. **3**, 31 (1931) and a discussion by E. Brucke, Phys. Zeits. **32**, 31 (1931).

[6] See for example E. T. Whittaker's *Treatise on Analytical Dynamics*, 2nd edition, p. 284.

III.

We use spherical coordinates r, φ, λ, where r is the distance from the center of the earth, φ the longitude counted positively towards the east, and λ the magnetic latitude. We assume as a first approximation that the earth's magnetic field may be represented by the field of a dipole of moment M at the center of the earth, with its axis towards the magnetic poles.[7] The components of the earth's magnetic field in the direction of r increasing and λ increasing are, respectively,

$$H_r=(2M/r^3)\sin\lambda,\quad H_\lambda=-(M/r^3)\cos\lambda.$$

The equations of motion are then,

$$(m/eM)(r\ddot{\lambda}+2\dot{r}\dot{\lambda}+r\dot{\varphi}^2\sin\lambda\cos\lambda)=-(2/r^2)\dot{\varphi}\sin\lambda\cos\lambda, \quad (1)$$

$$(m/eM)(\ddot{r}-r\dot{\lambda}^2-r\dot{\varphi}^2\cos^2\lambda)=-(\cos^2\lambda/r^2)\dot{\varphi}, \quad (2)$$

$$\frac{m}{eM}\frac{1}{r\cos\lambda}\frac{d}{dt}(r^2\dot{\varphi}\cos^2\lambda)=\frac{2\dot{\lambda}}{r^2}\sin\lambda+\frac{\dot{r}}{r^3}\cos\lambda, \quad (3)$$

where m is the relativistic mass at the constant speed corresponding to the kinetic energy of the particle, e is the charge on the particle and the dots have their usual meaning. The last equation is immediately integrated, yielding as its first integral,

$$-(m/eM)r^2\dot{\varphi}\cos^2\lambda=2\gamma+(1/r)\cos^2\lambda, \quad (4)$$

where γ is an integration constant which in our physical problem is proportional to the φ-component of the moment of momentum of the particle at infinity. Since the particle may be moving there in any direction, γ may have all values from $-\infty$ to $+\infty$. It follows further that its motion in the magnetic field of the earth can be split up into two motions as already noted by Störmer[5]: a motion in a meridian plane, and a motion of rotation of the meridian plane about the magnetic axis.

The inclination θ of the particle's path with respect to the meridian plane is given by[5]

$$\sin\theta=(r\cos\lambda/v)(d\varphi/dt), \quad (5)$$

[7] The introduction of the real magnetic field of the earth as determined empirically from measurements on the earth's surface, by the method of Gauss, can be made by using the results of A. Schmidt, Zeits. f. Geophys. **2**, 38 (1926).

where v is the velocity, and from the conservation of kinetic energy we have,

$$\dot{r}^2+r^2\dot{\lambda}^2+r^2\dot{\varphi}^2\cos^2\lambda=v^2. \tag{6}$$

From (2) and (4) we find, eliminating $\dot{\varphi}$,

$$\tfrac{1}{2}d(r\dot{r})/dt-v^2=(e^2M^2/m^2)(1/r^4)(2\gamma r+\cos^2\lambda) \tag{7}$$

and therefore, by integration,

$$r^2\dot{r}^2-v^2r^2=\frac{e^2M^2}{m^2}\left[-\frac{4\gamma}{r}+2\int\frac{\cos^2\lambda dr}{r^3}-C\right]. \tag{8}$$

From (2), (4), (6) and (8) we have, eliminating $\dot{\varphi}$ and $\dot{r}$, and dividing by (8),

$$\frac{r^2d\lambda^2}{dr^2}=\frac{-4\gamma^2/\cos^2\lambda-\cos^2\lambda/r^2+C+2\int\cos^2\lambda dr/r^3}{v^2m^2r^2/e^2M^2-4\gamma/r-C+2\int\cos^2\lambda dr/r^3}, \tag{9}$$

which when integrated by parts gives

$$\frac{r^2d\lambda^2}{dr^2}=\frac{-4\gamma^2/\cos^2\lambda+C+\int\sin 2\lambda d\lambda/r^2}{v^2m^2r^2/e^2M^2-4\gamma/r-C-\int\sin 2\lambda d\lambda/r^2-\cos^2\lambda/r^2}. \tag{10}$$

These equations may be more readily discussed by using a normalized coordinate

$$x=(mv/\pm eM)^{1/2}r, \tag{11}$$

where the sign in the denominator is to be taken either plus or minus according to whether we are dealing with positive or negative particles. In terms of the kinetic energy of the particle measured in electron-volts (11) may be written

$$x=r(V/300McZ)^{1/2}(1+600m_0c^2/\epsilon V)^{1/4}, \tag{12}$$

where V is the potential measured in volts, c the speed of light in vacuum, $Z\epsilon$ the absolute value of the particle charge, ϵ the electronic charge, and m_0 the rest mass of the particle. Placing for r the radius of the earth (6370 km) we obtain a value x_0 which fixes the scale of our normalized coordinate with respect to the earth and is a measure of the energy of the rays.

Likewise it is convenient to use instead of our γ a new γ_1 defined by

$$\gamma_1=-(\pm eM/mv)^{1/2}\gamma, \tag{13}$$

so that (5) now becomes,

$$\mp\sin\theta=-2\gamma_1/x\cos\lambda+\cos\lambda/x^2, \tag{14}$$

where the minus sign refers to positive particles and the plus sign to negative.

IV.

We now examine the three possibilities mentioned above, and investigate first, for any given kinetic energy, those points on the earth where no particles can arrive. For the sake of concreteness we consider the case of electrons, the discussion being similar for protons or other particles. For any given $x_0<1$ and $\gamma_1>1$ no rays coming from infinity can reach the earth because the domain of admissible values of $\sin\theta$ forms a closed region without any connection with infinity. The limiting value λ_1 of λ is therefore given by Eq. (14) with $\sin\theta=1$ and $\gamma_1=1$, and the region where the rays do not come extends from $\lambda=0$ to $\lambda=\lambda_1$. For $x_0>2^{1/2}-1$ there is no region where the rays are completely excluded.

For latitudes greater than λ_1 and values of $x_0<1$ there are trajectories coming from infinity but they have a limit, and this limiting trajectory must be asymptotic to a periodic orbit. Störmer[8] gave the estimate that no periodic orbit exists for $\gamma_1<0.5$. This estimate can be improved by using Eq. (10). A good approximation of the mean value of x for a periodic orbit can be found directly from (9) in agreement with the numerical computations of Störmer. By neglecting the integral and using a mean value for λ, the constant C in the denominator of Eq. (9) must be so chosen that this denominator has a double root. We adopt as the mean value of $\sin^2\lambda$

$$\sin^2\lambda = \sin^2\lambda_m/2, \tag{15}$$

where λ_m is the maximum value of λ, and estimate λ_m as the inclination of the tangent drawn from the origin to the curve $\sin\theta=1$, in Eq. (14). This condition gives,

$$\cos^3\lambda_m=\gamma_1^2. \tag{16}$$

The fourth degree equation fixing the value of x for the periodic orbit is found to be

$$x^4-2\gamma_1x+1-\sin^2\lambda_m/2=0 \tag{17}$$

and C is then given by,

$$C-1=[2(2\gamma_1x_P-1)(1-\gamma_1x_P)+\sin^2\lambda_m/2]/4\gamma_1x_P^2, \tag{18}$$

[8] Reference 5, p. 248.

TABLE I. *Calculated values.*

λ_m	γ_1	x_P	C
0°	1	1	0
10°	0.978	0.984	1.023
20°	0.911	0.934	1.092
30°	0.806	0.810	1.198
31° 40′	0.783	0.736	1.200

where x_P is the root of (17). Table I gives the collected values of λ_m, γ_1, x_P and C.

As the approximation (16) is in good agreement with Störmer's values for $\gamma_1=0.97$ and $\gamma_1=0.8$ we may use it to determine the value of γ_1 at which it becomes impossible to have a double root. Thus we find the value $\gamma_1=0.783$, which must be very close to the limiting value of γ_1 for which periodic orbits disappear. Therefore for values of $\gamma_1<0.783$ there is no limiting trajectory and the earth is reached by rays from all directions. Just as we have proceeded for the first domain ($\gamma_1=1$) we can now determine, using Eq. (14) with $\sin\theta=-1$ and $\gamma_1=0.783$ a limiting value λ_2 such that for values of λ greater than λ_2 the rays of energy corresponding to x_0 will reach the earth from all directions. This applies only to values of x_0 inside of the periodic orbit. We have carried out the numerical integration of Eq. (10) for $\gamma_1=0.911$, corresponding to $\lambda_m=20°$, beginning at $\log x=-0.04$ and $\lambda=14.14°$ and decreasing values of λ. From the result of this integration we have made estimates of the inclination η with the radius vector of the asymptotic family of trajectories passing through points of coordinates $x=0.5$, 0.6, 0.7, 0.8 and 0.9, both for $\lambda=0°$ (equator) and 10°.

TABLE II. $\lambda=0°$.

x_0	θ_1	θ_2	θ_3	η	$I(\%)$
0.5	0°	21°	61°	62°	65
0.6	−34	−15	10	57	32
0.7	−55	−34	−11	71	17
0.8	−70	−46	−26	83	3
0.9	−82	−52	−46	90	1

TABLE III. $\lambda=10°$.

x_0	θ_1	θ_2	θ_3	η'	η''	$I(\%)$
0.5	− 7°	13°	50°	44°	69°	60
0.6	−41	−20	5	42	59	29
0.7	−65	−39	−15	58	68	16
0.8	−90	−51	−26	79	84	—
0.9	−90	−57	−46	90	90	—

For each pair of values of x_0 and λ we know three points of the cone, i.e., the value of θ_1 for $\gamma_1=1$ for which the angle $\eta=0$, θ_2 for $\gamma_1=0.911$ for which we have the estimated values of η as described above, and θ_3 for $\gamma_1=0.783$ for which $\eta=90°$. From these data we have made a graphical integration of the total solid angle of the cone. For $x_0=0.9$ and 0.8, which are greater than 0.736, i.e., the value of x_0 for the limiting periodic orbit we have to replace θ_3 by the values corresponding to $\gamma_1=0.88$ and $\gamma_1=0.80$, respectively, for which the periodic orbit has the value $x_P=0.9$ and $x_P=0.8$, obtained by interpolation from Table I. For $x_0>1$, x_0 is greater than the corresponding value for any periodic orbit and therefore the rays come at every point from every direction.

Our collected results are given in Tables II to V. The last column labelled I gives the percentage intensity and the columns marked η', η'' refer to the north and south, respectively. It is seen

TABLE IV. $\lambda=20°$ *and* 30°.

	x_0	θ_1	θ_3	$I(\%)$
$\lambda=20°$	0.5	−30°	25°	44
	0.6	−69	−10	15
$\lambda=30°$	0.5	−90	− 9	14

TABLE V. *The latitudes at which the cosmic-ray intensity would become zero* (λ_1) *and would reach maximum value* (λ_2) *for various equivalent energies.*

x_0	λ_1	λ_2
0.1	64.4°	66.7°
0.2	49.0	57.3
0.3	34.8	50.1
0.4	12.0	44.3
0.5	—	39.9
0.6	—	36.3
0.7	—	33.9

TABLE VI. *Equivalent electron voltages corresponding to various values of* x_0.

x_0	Electrons (10^{10} volts)	Protons (10^{10} volts)	α-particles (10^{10} volts)
0.1	0.0596	0.01722	0.01842
0.2	0.238	0.1618	0.2308
0.3	0.536	0.449	0.760
0.4	0.954	0.861	1.564
0.5	1.490	1.397	2.625
0.6	2.145	2.050	3.928
0.7	2.920	2.823	5.46
0.8	3.821	3.719	7.25
0.9	4.830	4.729	9.27
1.0	5.96	5.85	11.52

that there is a slight predominance of the rays coming from the south. Table VI gives the energy measured in electron-volts equivalent to $x_0=0.1$ to $x_0=1.0$ for electrons, protons and alpha-particles.[9] The curves of variation of intensity with magnetic latitude are plotted from these tables in Fig. 1.

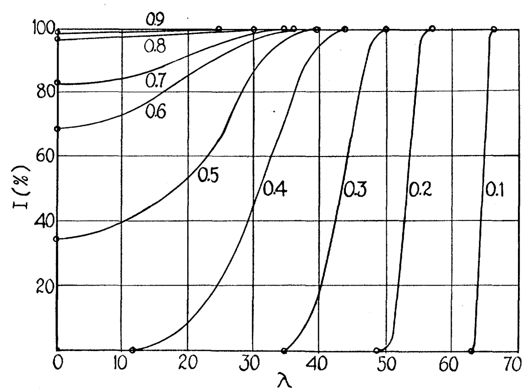

FIG. 1. Dependence of cosmic-ray intensity on magnetic latitude.

V.

From an examination of the curves in Fig. 1 we conclude, first, that the cosmic radiation must contain charged particles of energy between that corresponding to about $x_0=0.3$ and $x_0=0.7$ (see Table VI for equivalent voltages). It seems possible to represent the experimental curve by a suitable mixture of rays of these energies, but it is not at all excluded that a part of the radiation may consist of photons or neutrons.

The sign of the charged particles of the cosmic radiation, or the sign of the predominant part of the rays if they are a mixture of positive and negative particles, is within the reach of experimental detection. For negative particles there must be in the region of rapidly varying intensity a predominant amount of rays coming from the east, and conversely for positive rays. If a large part of the radiation is uncharged this effect may be missed if observations are made too near the magnetic equator.

Because of the fact that in regions near the magnetic equator there is a predominance of rays coming nearly horizontally, the absorption by the atmosphere may be increased. The small southern effect mentioned elsewhere seems to be of the second order and could only be computed by a more refined calculation.

Finally, the fact that Compton's result definitely shows that the cosmic rays contain charged particles gives some experimental support to the theory of super-radioactive origin of the cosmic radiation. In presenting this theory one of the present authors[3] wrote: "I think that a possible test of the theory is that, if I am right, cosmic rays cannot be formed uniquely of photons, but must contain, like the radioactive rays, fast beta-rays and alpha-particles, and even new rays of greater masses and charges. I have shown that the momenta of such rays must be reduced by the expansion (of the universe) in about the same ratio as that of photons."

[9] For the magnetic moment of the earth we take the value 8.04×10^{25} e.m.u. See Handb. d. Physik **15**, 288, chapter by G. Angenheister.