The Effective Neutron Collision Radius

The accurate measurements of Dunning and Pegram[1] of the transmission coefficients for neutrons of ten elements from carbon to lead show only a slow variation of total scattering per atom with atomic weight. One must therefore conclude (a) that the nucleus is the chief agent in scattering and (b) that the interaction function is not chiefly that of a polarized or polarizable particle in the field of a charge Ze. It seems more likely that the interaction is negligible until the approach is close and then rises very sharply. Such collisions are approximated by collisions between rigid spheres. Since the wave-length of the incident neutrons is of the order of magnitude of the radius of the nucleus (10^{-12} cm) one must use wave mechanics. In fact a classical calculation would yield less than one-third of the scattering for a given collision radius in this region. The solution[2] for the scattered wave is given by

$$\psi = -iC\sum_{n=0}^{\infty}(2n+1)\frac{F_n(kR)}{G_n(kR)}P_n(\cos\theta)\frac{e^{-ikr}}{kr}, \qquad (1)$$

where $k=2\pi mV/h=2\pi/\lambda$, R = nuclear radius + neutron radius, $F_n(kR)=(\pi kR/2)^{\frac{1}{2}}J_{n+\frac{1}{2}}(kR)$, $G_n(kR)=(\pi/2kR)^{\frac{1}{2}}\times H^{(2)}{}_{n+\frac{1}{2}}(kR)$, $J_{n+\frac{1}{2}}$ is a Bessel function and $H^{(2)}{}_{n+\frac{1}{2}}$ is a Hankel function of the second kind, $P_n(\cos\theta)$ is a Lagrange polynomial. This solution was first published by Mizushima.[2] The identical result was obtained by the late Dr. Gronwall of Columbia University and the values of $|\psi|^2k^2r^2/C^2$ for values of kR from 0.1 to 10 in intervals of 5° were tabulated by Mr. Taffel about two years ago. It is these tables that are used in this calculation.

The cross section per atom for scattering into a solid angle $d\omega$ is,

$$d\Phi = |\psi|^2 r^2 d\omega/C^2. \qquad (2)$$

Since the scattering experiments were performed after the neutrons had been filtered through 4 cm of lead it was assumed that all had the same velocity 35×10^8 cm/sec.[3] Φ is not very sensitive to small changes in velocity. The nuclear radius was assumed proportional to the cube root of the atomic weight (close packing) and for lead we use Gamow's value of 7.8×10^{-13} for the idealized core of the nucleus. The experimental value of the effective collision cross-section area is given by $1/\lambda N$ where N is the number of nuclei per cc of the scatterer and λ is the mean free path obtained from $p=e^{-d/\lambda}$ where p is the fraction transmitted and d is the thickness of the scatterer. Because of the finite size of the scatterer and the ionization chamber, this expression must be corrected since a considerable fraction of the singly scattered neutrons can enter the ionization chamber. The correction is made from Eq. (2) and varies from element to element. It amounts to 27 percent in lead and 9 percent in carbon. From the experimental value of the effective collision area we obtain R from Eq. (2) and by subtraction of the radius of the nucleus we obtain a neutron radius. The results are given in the table.

Element:	C	Al	S	Fe	Cu	Zn	Sn	I	Hg	Pb
Neutron radius ($\times10^{13}$):	1.45	1.37	1.24	1.15	1.33	1.15	1.54	1.08	1.33	1.43.

The average value is 1.31×10^{-13} and the experimental error amounts to ±0.2. It will be observed from the table that there is no progressive change of neutron radius with atomic weight, the variations are haphazard and within the limit of error. The scattering function $d\Phi$ seems to be in agreement with the preliminary results of Dunning and Pegram on large angle scattering in the case of carbon. The others could not be calculated from the data available.

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[1] Dunning and Pegram, Phys. Rev. **43**, 497 (1933).

[2] Mizushima, Phys. Zeits. **32**, 798 (1931).

[3] Curie, Joliot and Savel, Comptes Rendus **194**, 2208 (1932). Feather, Proc. Roy. Soc. **A136**, 709 (1932).