A Determination of e/m for an Electron by a New Deflection Method

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To fill the need of a new deflection determination and to provide a more accurate value, a new measurement of e/m is being made. The method used was conceived by Professor E. O. Lawrence. The essential points of the method are: acceleration of electrons to a continuous range of velocities by a radiofrequency electrostatic field, choice of a particular velocity by magnetic field resolution, measurement of this velocity through radiofrequency fields applied to a pair of accelerating slits and a pair of decelerating slits. A most important advantage of this method is that no acceleration voltage need be measured. This, combined with other properties of the method, practically eliminates errors due to contact potentials. Another advantage is the very high observational precision the method makes possible. The present results, although not considered final, are of an accuracy comparable with published values. The value obtained from two groups of observations made at different electron velocities (the corresponding electron voltages being about 1420 and 844 volts) is

$$e/m_0=(1.7571\pm0.0015)\times10^7 \text{ e.m.u.}$$

The uncertainty stated is a conservatively calculated probable error. A comparison of six recent and probably most reliable e/m determinations is made. The spectroscopic and free electron averages are now in good agreement. The weighted average of these six determinations is

$$e/m_0=(1.7598\pm0.0005)\times10^7 \text{ e.m.u.}$$

INTRODUCTION

INTEREST in the correct value of the specific charge of an electron has been considerably increased in the past year through the work of Bond[1] and Birge.[2] Bond in the articles referred to presents a new method for the evaluation of e and h which uses all of the existing data from experiments designed to measure h. The method is of considerable importance because it offers a more reliable method of evaluating e than has been attained by direct determination and of even greater importance it provides a means of obtaining an accurate set of values of the basic physical constants which are *self-consistent.* Birge has corrected and improved the work of Bond, giving solutions based on the more recent and reliable data. In the conclusion of his article Birge states (p. 260) that "The most probable values of e, h, and $1/\alpha$ depend primarily on the value adopted for e/m." Thus it is doubly important to obtain an accurate value of e/m, not only in order that the uncertainty as regards the discrepancy[3] of the deflection and spectroscopic values may be definitely settled, but also for the increased accuracy that will be possible in the values of e, h and α.

As is well known, the various determinations of e/m do not agree, the differences amounting in some instances to many times the stated probable error. The situation at the present time as regards the value of e/m may be briefly put as follows. The most probable value based on all of the work previous to 1929 was given by Birge[3] as

Deflection, $e/m_0=(1.769\pm0.002)\times10^7$ e.m.u.,

Spectroscopic, $e/m_0=(1.761\pm0.001)\times10^7$ e.m.u.,

two values being given because the two types of experiments gave results differing much more than their probable errors. Since 1929 no deflection measurements have been published. Two free electron measurements, however, have been made, one by Perry and Chaffee[4] in 1930 and one by Kirchner[5] in 1931–32. Essentially the same method was used by both: electrons were

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[1] W. N. Bond, Phil. Mag. **10**, 994 (1930) and **12**, 632 (1931).

[2] R. T. Birge, Phys. Rev. **40**, 228 (1932).

[3] R. T. Birge, Phys. Rev. Sup. **1**, 1 (1929).

[4] C. T. Perry and E. L. Chaffee, Phys. Rev. **36**, 904 (1930).

[5] F. Kirchner, Ann. d. Physik [5] **8**, 975 (1931) and [5] **12**, 503 (1932).

allowed to fall through a known potential difference and their velocity then determined by the time required to travel between two points having a known separation. The third determination of e/m made since 1929 was that by Campbell and Houston[6] in 1931. This was a spectroscopic measurement based on the Zeeman patterns of certain cadmium and zinc lines. The values obtained in these three measurements are of interest:

Free electrons:	Perry and Chaffee:	$e/m_0=(1.761\pm 0.001)\times 10^7$ e.m.u.,
	Kirchner:	$e/m_0=(1.7585\pm 0.0012)\times 10^7$ e.m.u.,
		$(1.7590\pm 0.0015)\times 10^7$ e.m.u.,
Bound electrons: (Spectroscopic)	Campbell and Houston:	$e/m_0=(1.7579\pm 0.0025)\times 10^7$ e.m.u.

Thus for the first time measurements on free electrons have given "low" values in agreement with the 1929 spectroscopic value, Kirchner's value even falling so low as to be more than twice the probable error lower. Campbell and Houston's spectroscopic value is even lower but in this measurement the uncertainty is twice as large. Recent evidence thus strongly indicates a low value. On the other hand, there still remains the fact that no deflection method has given a low value. The most recent deflection determination carefully performed by Wolf[7] in 1927 gave $e/m_0=(1.7689\pm 0.0018)\times 10^7$ e.m.u. Thus it is still most desirable to obtain a very accurate deflection value of e/m.

The present determination is being made in view of the above need. A new deflection method conceived by Professor E. O. Lawrence and most kindly offered to the author has been used because it seemed capable of giving a higher precision. The method has been found successful, and the results have reached a degree of precision comparable with the other recent determinations, hence they are being published at this stage. This report, however, is not to be considered final, as it is hoped to carry the measurements to a still higher precision.

Method and Apparatus

(A) Method and e/m measuring tube

The new method used for the determination of the value of e/m is a deflection method in which the electrons after being given an initial velocity are deflected in a circular path by a magnetic field. The most important difference between this method and others is that the energy of the electrons need not be known, that is, *the electron accelerating voltage is not measured.* This feature is of major importance because it very largely eliminates the errors and uncertainties due to contact potentials. A second feature of the method is that the determination of the electron velocity is accomplished through the measurement of the frequency of a radiofrequency oscillator. This is done by adjusting the electron velocity (more properly, the speed) so that the time to travel a given length of arc is exactly equal to one period of the oscillator. In this respect the method may suggest that used by Perry and Chaffee[4] and Kirchner[5] but the manner of use of the radiofrequency fields is quite different as will be seen presently. For the determination of e/m by this method three quantities must be measured: the frequency of the oscillator ν, the angle θ subtended by the circumferential path used in timing the electrons and the deflecting magnetic field H_0.

e/m measuring tube. The general scheme of the measuring tube is shown schematically to approximate scale in Fig. 1 and the more important parts in greater detail in a cross-sectional view in Fig. 2. A heavy brass box B about 31 cm square and 12 cm deep is evacuated through a vacuum line at V. The box contains a series of adjustable slits A_1, A_2, S_1, S_2, D_1, D_2, which are arranged on the circumference of a circle of radius r. The filament cylinder FC (see

[6] J. S. Campbell and W. V. Houston, Phys. Rev. **39**, 601 (1932).

[7] F. Wolf, Ann. d. Physik **83**, 849 (1927).

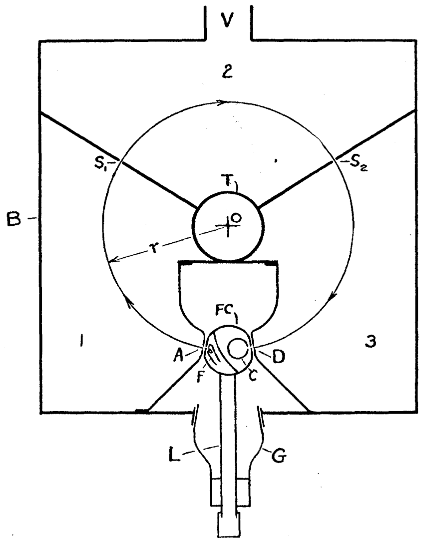

FIG. 1. Schematic diagram of measuring tube.

especially Fig. 2), made of a 3 cm length of 4.1 cm diameter brass tubing, is divided into two parts by the partition P (all parts are of brass unless otherwise stated). The ends of the left half of this tube are closed, thus making an enclosed semi-cylinderical box in which is placed the filament F, the source of the electrons. Two faces are machined on the outside of the cylinder to hold the adjustable slits A_1 and D_2. The cylinder is mounted at right angles on the long lead-in tubing L, which is supported from the box by the glass insulator G. Inside the tubing and in line with D_2 is another slit D_3 which is

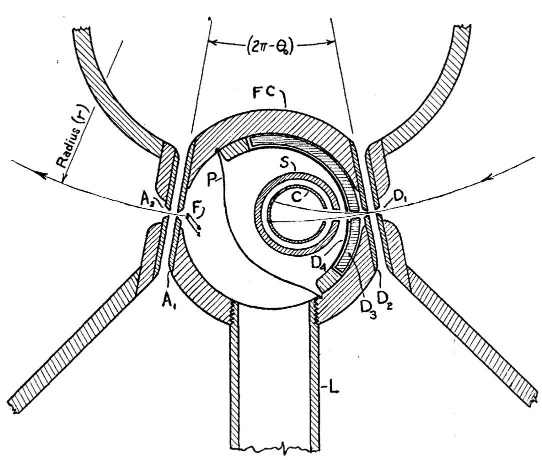

FIG. 2. Cross section of tube in vicinity of filament and collector.

completely enclosed by a shield extending from the tubing and having a slit D_4 in it. D_3 is insulated with mica. The purpose of enclosing the slit D_3 and the filament F is simply to shield them from radiofrequency fields. Coming up from the bottom of the box B and extending into the right half of the filament cylinder but not touching it is a shield S which encloses the collector C, the latter being connected to an electrometer.

The output of a radiofrequency oscillator is connected to the lead L and to the box B, the latter being grounded. Thus radiofrequency fields exist between the pairs of slits A_1-A_2, D_1-D_2, and D_4-S. The regions of the box made up of compartments (1), (2) and (3) are field free. Suitable baffles between compartments allow free circulation for evacuation. In order that the controls of the circuits associated with the filament and with the slit D_3 shall be at ground potential, their leads are brought out *through* the lead-in L and the coil I (made of tubing). Since the end of the coil connected to the box is at ground potential, the filament and D_3 leads can be brought out at this point. However, all circuits associated with the measuring tube were completely shielded with copper to protect against radiofrequency fields.

The magnetic field is produced by two large coils forming a Helmholtz system. The coils are symmetrically placed above and below the box on an axis through O. The magnetic field is thus perpendicular to the section of the tube as shown in Fig. 1 and is uniform around the circumference of any circle about O.

Simplified description of action in tube. A somewhat simplified description of the action will be given first. Electrons from the filament are accelerated across the slits A_1, A_2, during the half of each cycle of the impressed voltage in which the box is positive. There are therefore electrons passing through A_2 with velocities corresponding to all voltages from zero to the peak voltage of the oscillator. These electrons will be bent in circles of various radii by the magnetic field. For any magnetic field H (within a limit determined by the peak voltage of the oscillator) there will be electrons having a related voltage V that will be bent in a circle of radius r and that therefore will pass through the defining slits S_1 S_2 and arrive at D_1. In passing from D_1 to D_2 these electrons will experience an acceleration or deceleration depending on their time of arrival relative to the voltage cycle. With the frequency remaining constant the experimental procedure is to find the value of the field H_0 so that the time required for the electrons to travel from the slits A_1 A_2 to D_1 D_2 is equal to one period of the radiofrequency oscillator. When this is the case the electrons in passing from D_1 to D_2 experience a deceleration equal to the acceleration gained in passing from A_1 to A_2 and hence do not reach the collector C. Reference to Fig. 3 shows that the electrons accelerated at the times t_1 and t_2 and which therefore have a voltage V_0 (corresponding to the magnetic field H_0) are completely decelerated at the times t_1' and t_2', respectively. For any value of the field other than H_0, the deceleration is less than the acceleration for half of the electrons. These electrons passing on through D_2 are again accelerated in the gap between D_2 and S (disregard D_3 and D_4 for the present) and thus reach the collector C. For example if the field is smaller, corresponding to the voltage V_1, elec-

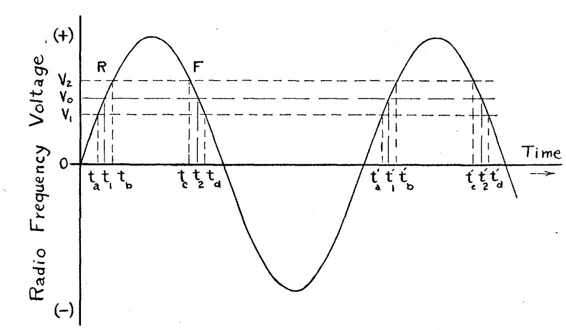

FIG. 3. Voltage-time relations for electron acceleration and deceleration.

trons accelerated at the time t_a will arrive at the decelerating slits later than t_a' and be completely stopped, while those leaving at t_d will likewise arrive later than t_d' and, experiencing only a partial deceleration, will pass on to the collector. For fields in the neighborhood of H_0, the current to the collector i_c should vary with the field in the manner shown by the solid curve (1) of Fig. 4. The resonance value of the field H_0 is indicated by zero (or minimum) current. The

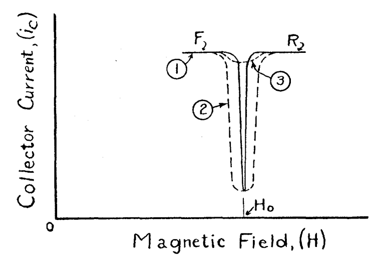

FIG. 4. Variation of collector current near resonance.

branch F is made up of electrons starting on the falling part of the cycle (as at t_d, Fig. 3) and branch R of those starting on the rising part (as at t_b, Fig. 3).

Derivation of equation for e/m. The equation giving e/m in terms of the experimentally observed quantities and the geometry of the apparatus is very simply obtained. For any magnetic field H, electrons having a velocity v given by the radial force equation

$$mv^2/r = Hev \text{ (e.m.u.)} \tag{1}$$

will be bent around through the defining slits at a radius r. The electron velocity necessary to travel from A_1 to D_2 (see Fig. 2) in one cycle is

$$v_0 = r\theta/T, \tag{2}$$

where θ is the angle in radians subtended by the path and T is the period of the oscillator. Since the oscillator frequency $\nu = 1/T$, the velocity can be written also as

$$v_0 = r\theta\nu. \tag{2'}$$

Elimination of the velocity by combining Eqs. (1) and (2′) gives the desired result:

$$e/m = \theta\nu/H_0 \text{ (e.m.u.)}, \tag{3}$$

where θ is the angle in radians subtended by the electron path, the path being that which the electron would travel with constant velocity v_0 in one period, ν is the oscillator frequency in cycles per second and H_0 is the magnetic field in gauss.

Description of action in tube. The above description of the method must be slightly modified to correspond to the actual method of operation. In order to draw electrons from the filament through the slit A_1, it is necessary to apply an accelerating potential V_A between the filament and the filament cylinder FC. In order to balance this an equal or slightly greater retarding potential V_D is applied between D_2 (i.e., the filament cylinder) and D_3. The effect of this initial velocity is simply to shift the time (relative to the radiofrequency voltage cycle) at which those electrons which pass on through the defining slits cross the gap A_1-A_2. For example if we let $V_0=V_A+V_1$, then with a magnetic field H_0, electrons crossing the gap A_1-A_2 at a time t_a (see Fig. 3) will experience a voltage acceleration V_1 and hence will leave A_2 with a velocity corresponding to the resonant voltage V_0. These electrons will then be decelerated in the gap D_1-D_2 by a voltage V_1 and further decelerated to zero velocity in the gap D_2-D_3 by a voltage V_D and hence will not reach the collector. With values of the magnetic field other than the resonance value half the electrons, as explained before, will be able to pass through D_3. They will be accelerated in the gaps D_3-D_4 and D_4-S and will reach the collector.

Contact potentials and retarding voltage V_D. From the foregoing discussion it is seen that with a magnetic field H_0 the existence of an initial electron velocity or of a continuous distribution of initial electron velocities does not in any way affect the equality of the acceleration in the A_1-A_2 slits with the deceleration in the D_1-D_2 slits. It is for the same reason that *the existence of contact potentials at any point other than* in the regions 1, 2 and 3, Fig. 1, *is immaterial.* Such contact potentials are taken care of in the adjustment of the retarding voltage V_D. This retarding voltage is adjusted experimentally to the value that gives the sharpest current minimum (see curve 1, Fig. 4). Too large a value gives a minimum with a rather flat bottom (curve 2) due to the appreciable interval of the magnetic field over which the electrons are stopped. Too small a value, instead of giving no minimum at all as might at first be expected, gives a shallow minimum, as curve 3. This is due to the fact that for conditions near resonance the electrons passing D_3 must travel a path of about 2 mm length with average velocities corresponding to a few volts before they can reach the strong accelerating field between D_4 and S and be shot into the collector. In travelling at these slow velocities the paths of the electrons are bent inward from the original circle and some are intercepted by the edges of D_3 and D_4. Thus no matter what the value of V_D (within reasonable limits of course) there is always at least a shallow minimum to be found. No variation of the observed values of e/m has been found with small variations of V_D. This is most important.

Bending of the electron path. Two things have been done to allow for the bending of the electron path mentioned above. First, the slits D_3, D_4, S and C have been widened relative to the other slits and in increasing amounts in the order given (see Fig. 2). These slits are aligned with their outer edges on the same circle as all the preceding slits. This allows electrons to reach the collector that have had their paths changed quite appreciably as well as those having paths practically unchanged. The result of this change is a narrower current minimum. Second, in order to determine if small changes in the radial location of the collector and shield relative to D_4 influence the results, the collector and its shield have been mounted eccentrically on a tapered plug in such a manner that they can be moved past the slit D_4 while measurements are being made. No changes in the observed e/m have been found with such displacement.

Effective angle subtended by the electron path. The angle θ appearing in Eq. (3) is a function of the accelerating voltage V_A. The variation, however, is quite small being of the order of one part in a thousand for the values of V_A used in the present work. As before stated the angle θ is determined by the distance (call it D) which the electrons would travel in one period *if* they maintained constant throughout this period the velocity v_0 that they have between the slits A_2 and D_1 (see Fig. 2). The electron acceleration between the slits A_1-A_2 and the deceleration between D_1-D_2 both require a finite though small time interval, the interval being of the order of 1/300 of the period T. This time interval is, of course, a function of the initial accelerating voltage V_A, being less the larger V_A. Less time taken in getting up to maximum velocity means a larger fraction of the period left to travel at that maximum velocity. Thus, increasing the accelerating voltage V_A decreases slightly the maximum velocity v_0, and hence decreases slightly the distance D that could be travelled in a whole period.

The quantitative relation between the accelerating voltage and the angle can easily be obtained if the following approximations are made. Since the time interval for acceleration, or deceleration, is about 1/300 of a period, the voltage can, to a high approximation, be assumed to vary linearly with time over such an interval. Further, since the change in accelerating voltage over such a time interval is only of the order of 1.5 percent and the change in θ that is being calculated is only of the order of 0.1 percent, the acceleration can also be assumed constant for present precision. The result is:

$$\theta=\theta_0-(2d_1/r)[1/((V_0/V_A)^{\frac{1}{2}}+1)], \qquad (4)$$

where θ_0 is the angle in radians subtended by the electron path from A_1 around to D_2 (see Fig. 2), d_1 is the spacing between the slits A_1 and A_2 and between D_1 and D_2, r is the electron radius, V_A is the initial accelerating potential and V_0 is the potential corresponding to the electron velocity between A_2 and D_1. The magnitude of the variation in θ can be visualized by noting that the limits of the path corresponding to θ are for $V_A=0$ at the slits A_1 and D_2 and for $V_A=V_m$ at the midpoints between A_1-A_2 and between D_1-D_2. To evaluate Eq. (4) it is necessary to know *approximately* the voltages V_A and V_0. V_A (of the order of 100 to 200 volts) is measured by a voltmeter, allowance being made for the filament drop in potential. V_0 (of the order of 1000 volts) is easily computed from the electron radius r, the magnetic field H_0 and any reasonable assumption for the value of e/m, being given by

$$V_0=0.08795H_0^2r^2. \qquad (5)$$

It should be emphasized that V_A and V_0 need not be known accurately. For example, an error in V_A and one volt (due, say, to a contact potential) would cause an error in the computed value of e/m of only about six parts in a million.

Method of measurement of factors determining e/m. The three factors entering directly into the determination of e/m (see Eq. (3)) were determined as follows: The angle θ was obtained through the angle $(2\pi-\theta_0)$ indicated on Fig. 2. This smaller angle was found by measuring the chord from the slit A_1 to A_2. The chord was measured for both the inner and outer slits, the mean of which in combination with the radius gave the angle. The method of measuring the radius is given in the next section. The frequency ν was obtained by harmonics from the carrier waves of various broadcast stations. The magnetic field H_0 is made up of two parts: that produced by the current i_H in the Helmholtz coils and that by the earth's field. The current i_H was measured in the usual manner by using a standard resistance immersed in oil with a potentiometer and a standard cell. Since the Helmholtz coils of No. 20 enamel wire were carefully layer-wound on accurately turned aluminum wheels, it was considered sufficiently accurate for the present precision to compute the Helmholtz constant k_r from the micrometered dimensions. This was done by formulae which take into account the finite cross section of the coils. The determination of the earth's field is explained in the next section.

(B) Auxiliary apparatus

Amplifier for magnetic field measurement and adjustment. Since the magnetic fields used in this work were of the order of 10 to 12 gauss, the earth's magnetic field of the order of 0.4 gauss could not be neglected. An audio amplifier terminated with a vacuum tube voltmeter aided in the two phases of this problem. The amplifier was of the usual resistance coupled type but was designed to respond to frequencies as low as 5 cycles per second. The over-all amplification was 300,000 and imput levels as low as 3 microvolts could be used. The imput to the amplifier came from one or the other of two 5000 turn coils placed in the brass tube T passing through the box B, Fig. 1. The coils could be rotated at any desired speed by a small motor located some distance away. The axis of one coil was coincident with the axis through O, that of the other was perpendicular. The center of the latter coil was located accurately in the plane of the electron orbit and on the axis through O. In order that there be no radial component of the earth's field in the measuring tube, the axis through O, Fig. 1, was made parallel to the earth's field. This was accomplished through the use of the parallel axis coil, a null reading on the output voltmeter indicating the desired condition. The adjustment was made by three levelling screws supporting the measuring tube. Secondly, the magnitude of the earth's field was measured through the use of the perpendicular axis coil. The field from the Helmholtz coils was adjusted to equality with the earth's field at the point O by opposing the two and again obtaining a null reading on the voltmeter. The Helmholtz current i_e so obtained, when multiplied by the Helmholtz constant at the point O, gives the earth's field. In either case the field could be adjusted to within 4×10^{-4} gauss. Since fields of the order of 10 gauss were used, this represents an accuracy of about 4 parts in 10^5.

Apparatus for aligning slits and measuring radius. The alignment of all the slits on the same radius and with the same slit width must be done very accurately. This was accomplished by having end caps on the tubing T, Fig. 1, with accurately located centers. Two dead centers supported on a U-shaped framework fit into these centers, so that the whole measuring tube (with the cover and Helmholtz coils removed) can be swung freely on the axis through O. A microscope with two cross-hairs is mounted with axis parallel but at a distance equal to the desired radius r. The slits have all been built so that they project about 0.4 mm from the face of the material mounted in, hence they can be seen from the side as viewed in the microscope. The two cross-hairs are adjusted to give the desired slit width. The pairs of slits are then adjusted consecutively to coincide with the cross-hairs.

The radius r (needed for calculation of the magnetic field and the angle) can be measured by the above arrangement with the addition of a large micrometer and an accurately turned and centered rod which is placed between the dead centers after the slits have been aligned and the measuring tube removed. The inner and outer slit radii can then be accurately measured by adjusting one edge of the bar to coincidence with the proper cross-hair, the bar being held by a framework with the other edge against the rod. The mean of the two is taken as r.

Radiofrequency source and control. The source of radiofrequency applied between the lead-in L, in Fig. 1, and the box was a short wave power-oscillator of the usual tuned-grid tuned-plate type. The circuit is given in Fig. 5. The capacity C_2 shown across the plate inductance L_2 is simply the capacity between the slits in the e/m tube and between the lead-in L and the return lead L_R which is large concentric cylinder

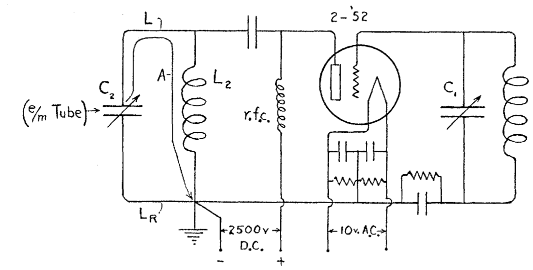

FIG. 5. Oscillator for supplying the radiofrequency voltage.

surrounding L. There is no possibility in the present method of any appreciable phase displacement of the radiofrequency voltage between the accelerating and decelerating slits. This follows from the very short path between these slits and the complete symmetry of the construction even to the symmetrically placed return lead L_R so that the capacity currents to both sets of slits have the same path lengths. The part of the tank circuit through which the filament and slit D_3 leads are led before being brought out at ground potential is indicated by the arrow A. Although only one tube is shown, actually two 75 watt tubes were connected in parallel. A well-filtered source of d.c. for the plate is necessary to avoid modulation which would alter the time between equal voltage ordinates of successive cycles.[8] The wave-length range used was from 7.5 to 10.5 meters. The peak voltage delivered to the e/m tube could be varied from zero to 3000 volts.[9]

The frequency was controlled and measured in the present preliminary work by a heterodyne method schematically shown in Fig. 6. As mentioned before, the carrier waves of various broadcast stations were used as frequency standards. Oscillator 1 is tuned to the carrier

[8] The average time would, of course, remain constant.

[9] The e/m measuring tube can be used as a high-frequency peak voltmeter by finding the maximum value of the magnetic field at which electrons can be sent around.

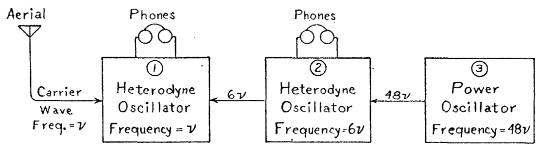

FIG. 6. Scheme for frequency control.

frequency by adjusting for zero beat note. Oscillator 2 is then turned on and its frequency adjusted so that its fundamental gives zero beat with the sixth harmonic of oscillator 1. Finally the power oscillator is adjusted to zero beat with the eighth harmonic of 2. The adjustments were monitored continuously during the taking of measurements. These zero beat adjustments were made with small vernier condensers of such size that a beat note of about 30 cycles occurred at least 7 to 10 dial divisions either side of resonance. Hence an adjustment to ± 4 cycles was easily possible.

RESULTS

General collector current variation

The hope that this new method would have a very sharp current minimum at the resonant adjustment has been realized. A typical collector current curve obtained by varying the magnetic field, the frequency being held constant, is shown by curve 1, Fig. 7. The resonant minimum at B is much narrower than the width of the printed

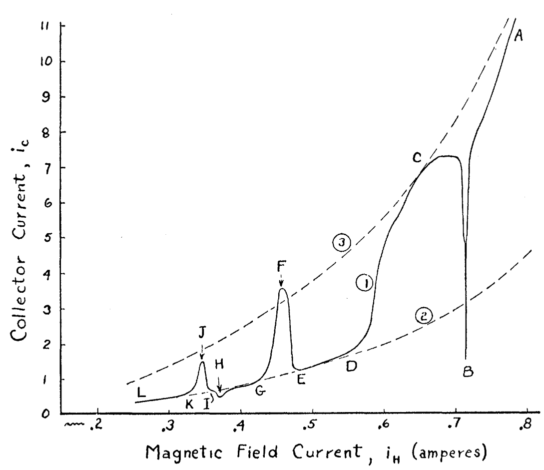

FIG. 7. Variation of collector current with magnetic field at a frequency of 3.7920×10^7 cycles and a peak voltage of 2400 volts.

line. At H there is a trace of a shallow second order resonance, that is, the adjustment in which the electrons take two periods instead of one to travel the required path. The value of the magnetic current at H is not exactly half of that at B due to the earth's field, but the net fields are exactly in the ratio of 1 : 2.

The two peaks at F and J respectively are due to special electron velocity *versus* voltage phase relations, existing only in those localized regions, and of such a nature that both R and F groups of electrons (see discussion with Fig. 4) get through the collector. The collector current is made up of these two groups in the following manner: A to B, and G to I of the R group alone, B to E and K to L of the F group alone, E to G and I to K of both groups.

Slope of collector current curve

In Fig. 4 the collector current was indicated as being constant on either side of the resonant point. That this simplified picture is not to be expected can be seen from the following. A finite slit width Δr means a finite range of electron voltage Δv which can satisfy the resonance condition. For a given slit width this voltage range is a constant and is given by

$$\Delta V/V = 2(\Delta r/r). \qquad (6)$$

The magnitude of ΔV therefore is proportional to the electron voltage V. Considering a sine wave (see Fig. 3), the time interval over which a given voltage range exists depends not only on the voltage range but also on the ratio of the electron voltage to the peak voltage V_P. It is given by

$$\Delta t/T = [1/\pi[(V_P/V)^2 - 1]^{\frac{1}{2}}](\Delta r/r), \qquad (7)$$

where T is the oscillator period. The quantity of current travelling to the collector in a pulse is, of course, proportional to Δt, hence the collector current i_c is proportional to Δt. By Eq. (5), the electron voltage is proportional to the square of the total magnetic field and therefore it is proportional to the square of the Helmholtz current i equivalent to that field. Eq. (7) may then be written in the working form

$$i_c = K/[(V_P/2593i^2)^2 - 1]^{\frac{1}{2}}, \qquad (8)$$

where K is the proportionality factor and the numerical constant comes from the electron radius and the Helmholtz constant of the present apparatus.

Curve 2 of Fig. 7 has been computed from Eq. (8), correcting i for the current equivalent to the earth's field and adjusting the constant K to give current equality at $i_H = 0.5$ ampere. V_P was 2400 volts. It is seen that the general rise of the collector current is in perfect agreement up to about $i_H = 0.55$ ampere. If instead of making the fit at $i_H = 0.5$ ampere it is made at 0.65 ampere, curve 3 is obtained which fits fairly well above $i_H = 0.65$ ampere. There is thus a threefold current increase between $i_H = 0.55$ and 0.65 ampere, before and after which the collector current varies in the manner to be expected. The cause of the threefold increase is unknown. The effect of the general positive slope in the neighborhood of the minimum is to shift the minimum to a smaller value of magnetic current which in turn gives a value of e/m too high. Fortunately, however, with good experimental conditions the minimum is so sharp that the shift is very small, being of the order of 2 parts in 100,000.

The current minimum

To show the sharpness of the current minimum the region in the vicinity of the resonant point B (Fig. 7) has been plotted in Fig. 8 with an abscissa scale 40 times that of Fig. 7. The sharpness of the minimum easily allows single readings to one part in 10,000. The two side minimum at B and C are due to geometry of the slit system since in one particular adjustment of the apparatus they disappeared (see dotted lines) only to reappear on subsequent adjustments. The heights of B and C are influenced by the magnitude of the accelerating potential V_A and the decelerating potential V_D, *but any reasonable adjustment does not affect the location of the minimum A.*

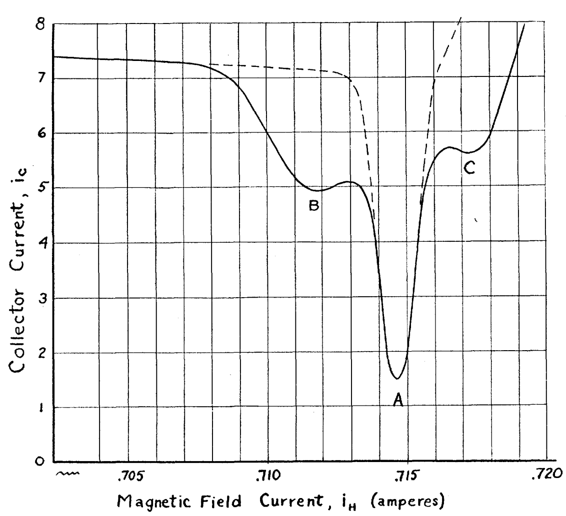

FIG. 8. The current minimum of Fig. 7 with abscissa scale magnified 40 times.

Summary of results

A summary of the results is given in Table I. The data were taken at two frequencies (see column 1) and for various accelerating potentials V_A (column 2), each run consisting of the number of observations of the current minimum given in column 4. The corresponding electron voltages were respectively about 1420 and 844 volts. The average Helmholtz current for each run appears in column 5. This has been corrected for the change with temperature of the standard resistance. These average currents have in turn been averaged by least squares in groups depending on the accelerating potential (column 6). The very high consistency of the readings of any group is evident from the observational probable errors given below each average, this error being less than 2 parts in 100,000 for the best experimental conditions. The values of e/m corresponding to these group averages are given in column 6.[10] These have been computed on the basis of θ_0 (i.e., without allowing for the change in angle with V_A as given in Eq. (4)) in order that a comparison between the computed and experimentally observed change with V_A could be made. This comparison appears in Table II. The observed changes are simply differences of values in column 7 of Table I, and the calculated changes are differences in the values of column 8, the latter computed by Eq. (4). The observed changes are greater than the calculated, that for the higher frequency

[10] The numerical values used in Eqs. (3) and (4) were as follows: angle $\theta = 5.9348$ radians, electron radius $r = 9.907$ cm, Helmholtz constant at this radius $k = 17.3616$, mean value of the earth's magnetic field $= 0.43291$ gauss, and slit spacing $d_1 = 0.128$ cm. The electron voltages V_0 are given above. The slit width was 0.066 cm.

TABLE I. *Summary of e/m data and results.*

1 Frequency and station	2 Run No.	3 V_A (volts)	4 No. observations	5 Average i_H (amp.)	6 Group average i_H (amp.)	7 Corresponding e/m uncorrected	8 $(\Delta e/m)_\theta$ Angle correction	9 e/m_0	10 Frequency average e/m_0
3.7920×10^7 (KGO_{48})	1	71	13	0.71424	0.71426	1.75354	−0.00140	1.75701	1.75699
	2	71	10	0.71424	± 1	± 3		± 3	± 2
	3	71	12	0.71424					(R_e/R_i =0.80)
	4	71	20	0.71430					
	5	121	10	0.71415	0.71415	1.75381	−0.00173	1.75695	
	6	121	10	0.71414	± 2	± 4		± 4	
	7	121	10	0.71416					
	8	171	10	0.71403	0.71397	1.75423	−0.00197	1.75713	
	9	171	10	0.71391	± 5	± 12		± 12	
2.9280×10^7 ($KFRC_{48}$)	10	81	30	0.54518	0.54518	1.75555	−0.00183	1.75661	1.75679
					± 3	± 9		± 9	± 12
	11	121	20	0.54497	0.54498	1.75617	−0.00210	1.75696	(R_e/R_i =1.86)
	12	121	20	0.54499	± 3	± 9		± 9	
	13	121	12	0.54496					

being within the experimental error but that for the lower frequency being considerably outside. The reason for this discrepancy is apparent when the effect is considered of the width of the current minimum on its location. The minimum becomes broader as the accelerating voltage is increased. This should make the values of e/m obtained at higher accelerating voltages too large and such is the case (see column 7 and "observed" values Table II). The reason that

TABLE II. *Calculated and observed angle corrections.*

Frequency $\times10^{-7}$	Change in V_A (volts)	Change in Uncorrected e/m ($\times10^5$) Computed	Observed
3.7920	71 to 171	57	69±15
2.9280	81 to 121	27	62±18

at the lower frequency the observed change is so much larger than the computed is because the current minimum even for the best conditions is much broader. Hence the effect is increased at the lower frequency.

The values of e/m_0 containing the angle as well as the relativity correction are given in column 9 for each accelerating voltage. The relativity corrections in e/m for the higher and lower frequency data are respectively +0.00487 and +0.00289.

The least squares average for each frequency with the probable error is tabulated in column 10. In all cases in this paper the probable error given is the larger of the two errors computed by external and internal consistency. The consistency of the various e/m_0 values for a given frequency is shown by the ratio of the external to the internal probable errors[11] as given below each average. For the higher frequency the ratio is less than unity. This indicates that the error due to broadening and consequent shifting of the minimum as V_A is raised to 171 volts has no appreciable effect on the result, that is, runs 8 and 9 have little weight. This is due to their relatively large probable error. For the lower frequency the ratio of two errors is 1.86. The chance of the error varying by this fraction due purely to statistical fluctuation is only 1 to 3.8. In this case the error due to the shifting of the minimum with increasing V_A is noticeable. This results from two things: the magnitude of the shift is greater due to a broader minimum and second, the number of observations taken at the two accelerating potentials happened to be such that the two resulting probable errors are equal. The agreement, however, between the two frequency values is good since they differ by only 1 part in 10,000. Changing the frequency changes the energy of the electrons being measured. The

[11] R. T. Birge, Phys. Rev. 40, 207 (1932).

TABLE III. *A comparison of the constancy of e/m_0 with change in electron energy.*

Experimenter	Percent electron energy change	$\Delta e/m \times 10^4$	Relative change in e/m for same percent energy change
Dunnington	40.9	2	1.00
Kirchner	5.87	5	17.4
Perry and Chaffee	41.6	11	5.4

constancy of the present results with change in energy is compared with that obtained in other e/m determinations in Table III. The data for Perry and Chaffee were taken from the two frequency groups whose percentage difference in electron energy is nearest to that of the author.

The final value of e/m_0 as computed by least squares from the two frequency averages is $e/m_0=(1.75699\pm0.00002)\times10^7$ e.m.u. The correction of the magnetic field from international to absolute units[5] requires an increase of 0.5 parts in 10^4. The result then becomes:

$$e/m_0=(1.75708\pm0.00002)\times10^7 \text{ e.m.u.}$$

The uncertainty given is the observational probable error determined simply by observations on the minimum. The ratio of the external to internal probable errors for this average is 1.12 and the chance of the error varying by this fraction solely because of statistical fluctuations is 1 to 1.25. This shows that the frequency variation does not appreciably affect the magnitude of the result or its probable error. This is because of the much larger probable error at the lower frequency. In other words, the result largely depends on runs 1 through 7. This mathematically attained result is in agreement with the author's estimate of the relative reliability of the data. The previously mentioned effect of the shifting of the minimum with its increased breadth would lead one to expect a slightly higher value at the lower frequency. Since the results obtained were lower for the lower frequency, there is apparently some other effect that is a function of the frequency and that varies in a sense opposing the first.[12]

[12] The effect may at least partially be due to one or both of the broadcast stations consistently maintaining an average frequency above or below their assigned frequency. In this case the uncertainty can be eliminated by using various harmonics of the same carrier wave.

ERRORS

A summary of the various probable errors and their sources is given in Table IV. The formula for e/m (Eq. (3)) has three factors, θ, ν and H_0 but actually the determination of the field H_0 involves two things: the Helmholtz constant k and the current i equivalent to the total field. These four factors are listed at the left of the table. The quantities that enter into the determination of these factors are listed in the next column with their estimated (or calculated) uncertainties in the third column. The probable errors calculated by least squares appear in the last column. The probable error in e/m_0, also calculated by least squares is given below the table.

It should be noted that in the case of the angle θ_0, the probable error computed from the uncertainties of the radius and chord is that for the small angle $(2\pi-\theta_0)$, shown in Fig. 2. Since θ_0 is about 17 times greater, its proportional uncertainty is only 1/17th as large.

Since the constant of the Helmholtz coils was calculated and not checked by any experimental method, an allowance has been made of 4×10^{-4} for unknown errors (i.e., other than from uncertainties in the dimensions). This should allow for such things as an appreciable permeability of the brass and aluminum used in and about the tube, etc. A generous allowance has also been made in the field current i for the possible existence of any factors disturbing the location of the minimum. It should be noted that the final probable error in e/m_0 (given below the table) depends almost entirely on these two allowances, all other errors being small in comparison.

In computing the frequency error, the computed error in the setting of the heterodyne oscillators was based on an assumed uncertainty of each individual setting of 10 cycles. This latter is twice the precision of a single setting.

The estimated uncertainties given in the third column are not probable errors, that is, they are not the errors which are as likely to be exceeded as fallen short of. Rather they are the errors for which the chance is small that the true error will exceed them, probably about one chance in five. This means that they are twice

TABLE IV. *Summary of probable errors.*
Probable error in e/m_0: 6.77 parts in 10,000 or $\Delta e/m_0 = \pm 0.00119$.

Factors	Quantities entering into factors	Estimated uncertainty Parts in 10^4	Probable error Parts in 10^4
Angle θ_0	Radius Chord (of $2\pi-\theta$)	5 14	0.90
Field constant k	Effect of uncertainty in radius of electron orbit Effect of uncertainty in radius of Helmholtz coils Allowance for unknown errors	0.75 1 4	4.19
Field current i	Observational uncertainty (including frequency correlation) as given by twice the obs. p. e. Voltage of standard cell (by Bur. Standards) Resistance of standard resistance (by Bur. Standards) Precision of Wolf potentiometer Allowance for factors disturbing location of minimum	0.25 1.0 0.5 0.2 4	5.23
Frequency ν	Standards, limit of error $\left\{\begin{array}{l}\text{KGO} \ -0.63\times 10^{-4}\\ \text{KFRC}-0.82\times 10^{-4}\end{array}\right.$ Errors in setting heterodyne oscillators	0.30 0.13	0.33

the probable errors. However, to be conservative, these estimated uncertainties will be taken as the probable errors.

On the foregoing assumption, the probable error in e/m_0 is ± 0.00119. Retaining only four decimals and allowing still more for additional uncertainties because of the preliminary nature of the work, the value that will be adopted for the present work is

adopted probable error in

$$e/m_0 = (\pm 0.0015)\times 10^7 \text{ e.m.u.}$$

In view of the above manner of determination, this is considered a conservative estimate.

CONCLUSION

Author's value of e/m_0

The value of e/m_0 and its probable error found in the present investigation is[12a]

$$e/m_0 = (1.7571 \pm 0.0015)\times 10^7 \text{ e.m.u.}$$

The present probable value of e/m_0

For the purpose of comparing this result with that of preceding experimenters and of obtaining the present most probable value of e/m, the data of Table V is offered. The table contains the results of all important determinations of e/m made in the last ten years.[13] The only exception is that of Wolf[7] whose result now appears to be in conflict with those of all other methods.

The table may be divided into two parts, the first consisting of three spectroscopic determinations and the second consisting of three free electron determinations. The value of e/m_0 given for Houston is the result as corrected by Birge[3] and the value given for Kirchner is the weighted mean of his two frequency determinations with the probable error based on internal consistency. The probable error quoted for Babcock is that based on his statement that he believed his result good to one part in a thousand. His observational probable error was 0.0012. The weights as computed by least squares have been listed in column 6. These were used in computing the averages of columns 7 and 8 and the probable errors given under each average. A separate average for the spectroscopic and free electron data has been computed to emphasize the present agreement between the two classes of data.

A final average of e/m_0 based on all the data is given in column 8. This value is

[12a] This result differs from that published in a letter, Phys. Rev. **42**, 734 (L) (1932), because of a change in the value used for the standard resistance (the certificate arriving from the Bureau of Standards after the letter was published) and because of a new and more accurate calculation of the magnetic field constant k of the Helmholtz coils.

[13] A comparison has recently been made by Birge using three previous determinations and the author's uncorrected value,[12a] Phys. Rev. **42**, 736 (1932).

TABLE V. *Probable value of e/m based on six recent determinations.*

Experimenter	Date	Method	e/m_0 $\times 10^{-7}$	Probable Error (r) $\times 10^4$	Weight $=10^{-6}/r^2$	e/m_0 $\times 10^{-7}$	e/m_0 $\times 10^{-7}$
Houston[14]	1927	Fine structure	1.7608	8	1.563	1.7605 ± 7 ($R_e/R_i=0.526$)	1.7598 ± 5 ($R_e/R_i=0.88$)
Babcock[15]	1929	Zeeman effect	1.7606	18	0.309		
Campbell and Houston[6]	1932	" "	1.7579	25	0.160		
Perry and Chaffee[4]	1930	Linear acceleration	1.761	10	1.000	1.7593 ± 7 ($R_e/R_i=1.12$)	
Kirchner[5]	1932	" "	1.7587	9	1.235		
Dunnington	1932	Magnetic deflection	1.7571	15	0.444		

probable value based on six determinations

$$e/m_0=(1.7598\pm0.0005)\times10^7 \text{ e.m.u.}$$

It is important to note that in all three averages the ratio of the error based on external consistency to that based on internal consistency (R_e/R_i) is near to or less than unity. This shows that the errors assumed by each investigator have been quite conservative. The agreement of the six determinations made with such widely varying methods indicates the reliability of their average.

Discussion

The author's value of e/m_0 is seen to be lower than that of the other recent determinations. The improvement, however, of the present method over preceding ones is indicated by the constancy of the value of e/m_0 with change in frequency, that is, with change in electron energy, the energy correlation being 5 to 17 times better than in the two most recent free electron determinations. The improvement is also indicated by the high observational precision and by the greatly reduced effect of contact potentials.

However, it is to be emphasized that the measurements have all been made with one alignment of the slit system and are not considered by the author to be of a final nature. The apparatus has now been moved to the California Institute of Technology and the measurements will be continued at that institution with the aim of confirming or disproving the present results and of increasing the accuracy as much further as is reasonably possible and desirable.

It is also desirable to emphasize the importance of the human equation in accurate measurements such as these. It is easier than is generally realized to unconsciously work toward a certain value. One cannot, of course, alter or change natural phenomena (for example, the location of the current minimum in the present experiment), but one can, for instance, seek for those corrections and refinements which shift the results in the desired direction. Every effort has been made to avoid such tendencies in the present work.

The author desires to thank Professor E. O. Lawrence for the use of the method which made possible the present results and for suggestions during the course of the work. He also wishes to thank Professor R. T. Birge for his interest and for his criticism of the paper and David H. Sloan for suggestions on the radiofrequency apparatus.

[14] W. V. Houston, Phys. Rev. **30**, 608 (1927).
[15] H. D. Babcock, Astrophys. J. **69**, 43 (1929).