Photoelectric Currents in Gases between Parallel Plate Electrodes

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The passage of photoelectrons between parallel plate electrodes through a gas has been investigated theoretically in its dependence on accelerating field strength, pressure, and nature of the gas. It is found that the physical phenomenon of importance in such an experiment is the Ramsauer scattering in the immediate vicinity of the emitter. A theoretical expression for the ratio of observed current to saturation current is in excellent agreement with experiments on H_2 and N_2 by N. E. Bradbury. The theory allows the calculation of Ramsauer cross sections of the gas from the observed currents.

In general, problems of conduction of electricity through gases are of such a complicated nature that an exact theoretical discussion is impossible. However, attempts employing diffusion concepts and gas kinetic ideas have in many instances led to very useful results. Perhaps a better approach in many cases would be to abandon the use of the differential equations of diffusion and to consider from the outset the elementary physical processes as fundamental, and then arrive at the desired results by applying statistical methods to the tremendous number of processes involved. This is in many instances difficult to do for we usually do not have problems of *equilibrium*, but rather of *steady flow*. It has occurred to the authors that the problem of flow of electrons between two parallel plate electrodes in a gas (the electrons being emitted photoelectrically from one of the plates) should be amenable to such a treatment.

Consider, then, two parallel plate electrodes a distance L apart in a gas at such a pressure that there are N gas atoms or molecules per cubic centimeter. Let light shine on one of these plates of such an intensity that n_0 photoelectrons are liberated per second. Further let a potential $V=XL$ be applied between the electrodes in such a direction that these electrons are drawn away from the emitter and a current flows. This current will vary with the field strength X, N, and the nature of the gas. As the gas is pumped out of the tube containing the plates, the current will increase to a saturation value corresponding to all electrons liberated at the source being collected. When the gas pressure is not zero, the electrons will undergo scattering collisions with the gas atoms and a part will be returned to the emitting plate.

Thus the elementary process in such an experiment is elastic (Ramsauer) scattering of the photoelectrons. Let us consider the scattering of electrons of a fairly definite energy E_0 so that we can assign a Ramsauer cross section $\sigma(E_0)$ to the molecules. Suppose that at each collision an electron has a probability ω of being removed from the current stream. Then the number of electrons left after travelling a distance ξ is $n=n_0e^{-\sigma N\omega\xi}$.

In the experiment proposed the process responsible for removal of electrons is scattering backward at such an angle that the electrons can reach the emitter. For the present we shall assume that the return or non-return of the electrons is determined by their first collision. We may now write $x=\xi\ \overline{\cos\phi}$ where x is the component in the direction of the field of the distance traversed and $\overline{\cos\phi}$ is the mean value of the angle between the actual path and the direction of the field which is taken as normal to the emitting plane. This mean value in the following will be taken over all the electrons having their first return scattering at a distance x from the emitter.

The fraction ω depends on the initial energy of the electrons, the field strength X, and the distance x from the emitter at which the scattering takes place. In order that an electron return to the emitter, the kinetic energy associated with its velocity towards the plate u must be such that $\frac{1}{2}mu^2>eXx$. But its total kinetic energy at collision is $\frac{1}{2}mV^2=E_0+eXx$. Therefore the condition for return is that

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$$u/V=\cos\theta>[eXx/(E_0+eXx)]^{1/2}$$

or $\theta<\cos^{-1}[eXx/(E_0+eXx)]^{1/2}$. Thus if an electron is to return to the plate it must be scattered within a cone of angle $\cos^{-1}[x/(x+E_0/eX)]^{1/2}$.

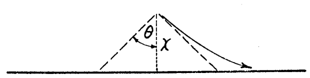

FIG. 1. Cone of return for an electron scattered backward at x.

Such a cone subtends a solid angle, (Fig. 1),

$$\Omega_0=2\pi[1-(x/(x+E_0/eX))^{1/2}].$$

Therefore the fraction of those electrons scattered that reach the plate is

$$\omega(x)=\tfrac{1}{2}[1-(x/(x+E_0/eX))^{1/2}].$$

We may now write

$$n=n_0\exp\left(-(\sigma N/\overline{\cos\phi})\int_0^x\omega(x)dx\right) \quad (1)$$

for the number of electrons reaching x without removal from the current stream. The current arriving at the collecting electrode may be written

$$i=n_0e-R \quad (2)$$

where R, the return current is given by

$$R=n_0e\int_0^L\exp\left(-(\sigma N/\overline{\cos\phi})\int_0^x\omega(x)dx\right)\times(\sigma N/\overline{\cos\phi})\omega(x)dx. \quad (3)$$

Eqs. (1) and (2) are, of course, exactly equivalent, but for practical purposes Eq. (2) is in a more convenient form. This is so because we wish to carry out in detail the example of quite high pressures where most of the return takes place very close to the emitter. Under these conditions we may make several approximations. First, the field does not have a chance to deflect the electrons appreciably before their first collisions so we may write $\overline{\cos\phi}=\frac{1}{2}$, the value corresponding to electrons emitted with a uniform distribution in angle. Second, for very small x, $\omega(x)\sim\frac{1}{2}$. We may therefore approximate (2) for this case by

$$i=n_0e-n_0e\int_0^\infty e^{-\sigma Nx}\sigma N[1-(x/(x+E_0/eX))^{1/2}]dx. \quad (4)$$

We have now extended our integral from 0 to ∞ instead of from 0 to L. This is allowable when $L\gg(1/\sigma N)((1/\sigma N)$ corresponds to the mean free path). This extension of the limits of integration corresponds to the fact that for such pressures and constant field strength, the current is independent of the electrode distance.

Eq. (4) may be rewritten

$$i=n_0e\int_0^\infty e^{-\sigma Nx}[x/(x+E_0/eX)]^{1/2}\sigma N dx$$

or the ratio of the current i to the saturation current n_0e is

$$i/i_0=\int_0^\infty e^{-y}[y/(y+E_0\sigma N/eX)]^{1/2}dy.$$

Denoting $ex/E_0\sigma N$ by a^2 we have

$$i/i_0=\int_0^\infty e^{-y}[y/(y+1/a^2)]^{1/2}dy. \quad (5)$$

This integral has been evaluated in the form of a series for small a.[1] We find

$$i/i_0=\Gamma(3/2)a-\tfrac{1}{2}\Gamma(5/2)a^3+(1\cdot3/2\cdot4)\Gamma(7/2)a^5\cdots. \quad (6)$$

This series is divergent but usable in calculation since the error made in summing any number of terms is of the same sign and less than the first term neglected. For large a we find

$$i/i_0\sim1-(1/a^2)\log a. \quad (7)$$

This approximation is usable for $a>4$. For $a<0.2$ we may approximate i/i_0 by

$$i/i_0\sim\tfrac{1}{2}\pi^{1/2}a=\tfrac{1}{2}\pi^{1/2}(eX/E_0\sigma N)^{1/2}. \quad (8)$$

The upper limit of the error of this approximation at $a=0.2$ is 3 percent. In the intermediate region $0.2<a<4$ no satisfactory analytic expression for i/i_0 has been found, but Eq. (5) has been evaluated by numerical integration in this range. In the following table i/i_0 is tabulated for a set

a	0.200	0.707	1.000	1.414	2.000	3.000	4.000
i/i_0	0.172	0.492	0.603	0.708	0.791	0.867	0.902

of values of a. These values are exhibited graphically in Fig. 2.

[1] Malmsten, Handl. Stockh., 1841.

Before applying these results we must justify the fact that multiple collisions have not been considered. In the first place, most of the return current is produced by scattering in the very first layers of gas molecules and the nearer to the emitter a return electron is produced the smaller its chance of suffering *any* collision before capture. Also, if a second collision takes place, its *chance* of being a return collision is greater than at the point of first collision. Moreover, if this

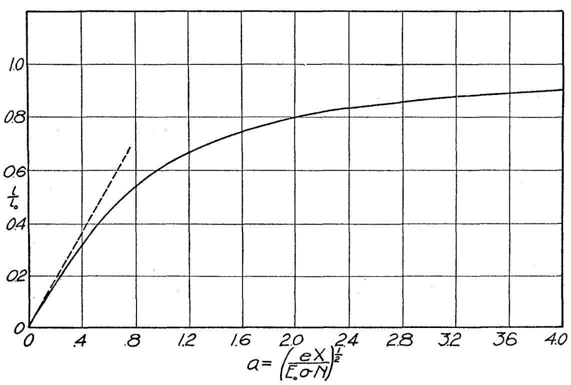

FIG. 2. Curve showing variation of i/i_0 with a obtained from the evaluation of Eq. (5). For small values of a the slope is essentially constant and equal to $\frac{1}{2}\pi^{1/2}$.

second collision is not of the return type it is clear that it produces an electron practically equivalent to one just starting from the emitter so that its chance of subsequent return is of the order of $1-i/i_0$. In the experiments about to be discussed $1-i/i_0$ is in general greater than 0.8.

One of us (N.E.B.) has studied experimentally the passage of photoelectrons through H_2 and N_2 at pressures ranging from a few centimeters of mercury to pressures near atmospheric.[2] Since, in these experiments, i/i_0 was usually less than 0.2, it is possible to compare these results with the simple theoretical expression Eq. (8). Remembering that $a=(eX/E_0\sigma N)^{1/2}$. Eq. (8) predicts a linear relation between the observed i/i_0 and $(X/p)^{1/2}$ where p is the gas pressure (for experiments performed at one temperature). In Figs. 3 and 4 the observed i/i_0 is plotted against $(X/p)^{1/2}$ (X in volts per centimeter and p in mm Hg). The predicted linearity is realized within the accuracy of the experiments.

[2] N. E. Bradbury, Phys. Rev. **40**, 980 (1932).

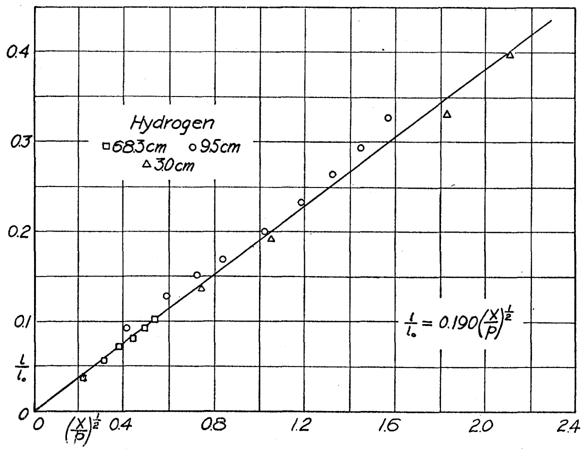

FIG. 3. Experimental values of i/i_0 plotted as a function of $(X/p)^{1/2}$.

Eq. (8) also predicts the slope of this line in terms of the cross section σ. As a further test of the validity of the theory we may make use of the observed slopes to calculate the scattering cross section. Doing so we find

$$H_2 \qquad \sigma=0.85\times10^{-15}\ \mathrm{cm}^2$$
$$N_2 \qquad \sigma=1.07\times10^{-15}\ \mathrm{cm}^2.$$

In making these calculations E_0 was taken to be 0.78 electron-volts. This was presumably near the maximum of the energy distribution curve of the electrons emitted from the plate in these experiments. Thus the above values of σ are for electrons of approximately this energy. These results may be compared with Ramsauer cross section obtained by other methods. For example Normand[3] gives $\sigma=1.31\times10^{-15}$ for H_2 and

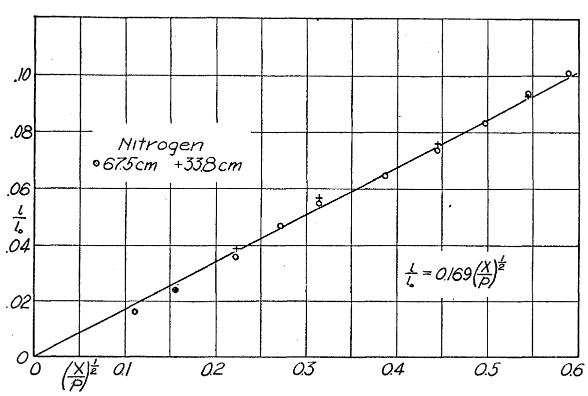

FIG. 4. Experimental values of i/i_0 plotted as a function of $(X/p)^{1/2}$.

[3] C. E. Normand, Phys. Rev. **35**, 1217 (1930).

$\sigma = 1.03 \times 10^{-15}$ for N_2. However, in the region corresponding to electrons of this energy Normand's curves are varying so rapidly with electron energy that exact comparison is impossible.

We have here introduced the cross section as the only property of the gas of importance in these experiments. This is natural, as we have seen, since the return of electrons to the emitter is determined by their first elastic collisions. The slight loss of energy at each collision (dependent on the ratio of the mass of electron to that of the gas molecule) does not enter. By the time the electron has lost an appreciable fraction of its initial energy in this way its distance from the emitter makes its chance of return negligible.

J. J. Thomson has given an expression for i/i_0 in experiments of this type in terms of the mobility of the carrier. This expression may be equated to our expression for i/i_0 and the mobility solved for. In this way we arrive at a simple analytic expression for the mobility which is in general agreement with the theoretical equation derived by Compton giving the proper variation of k_0 with X/p and λ. The expression so obtained, however, contains a small term involving E_0. This is not surprising and arises in part from the assumptions made in evaluating Eq. (2) and in part from the fact that the electrons leaving the emitter do not at once attain their terminal energy.

Another interesting result of this investigation is that one would have to go to field strengths of the order of 100 times those employed by Bradbury to reach 3/4 saturation. This is in agreement with results of Sanders[4] who found that in the region immediately before breakdown of the gas by cumulative ionization, the current had not yet reached its saturation value.

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[4] F. H. Sanders, Phys. Rev. **41**, 667 (1932).