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A Mechanism of Acquirement of Cosmic-Ray Energies by Electrons

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It is shown that the growth of stellar magnetic fields such as occur in sunspots can give rise according to Faraday's law of electromagnetic induction to electric fields capable of giving electrons energies corresponding to 10^{10} volts. Moreover, the rate of acquirement of energy can reasonably be greater than the rate of loss by collisions with atoms. It would be difficult to realize energies as high as 10^{10} volts from the magnetic fields of spots such as occur on the sun. Energies corresponding to 10^9 volts are, however, within the range of possibility, and it is suggested that electrons projected from such spots may play a part in auroral phenomena. For cosmic rays one must, however, probably look to the stars for the necessary conditions. Because of the repulsion of the current responsible for the magnetic field, the electron is hurled away from the spot as it acquires its energy.

IT is usually considered that the assumption of the existence of electronic energies of the order 10^{10} volts is attended with considerable theoretical difficulties. The present note is an elaboration of one of the suggestions recently made by the writer.[1] It concerns the possibility of the production of cosmic-ray energies in stellar spots, analogous to sunspots on our sun. It is known that the sunspots possess considerable magnetic fields which grow in comparatively short intervals of time. A consideration of the existing magnitudes will show that line integrals of the electric field of the order 10^{10} volts may readily arise through the Faraday law as a result of the growth of such magnetic fields. However, the detailed working out of the matter involves certain considerations which require careful attention.

We shall consider the problem in cylindrical polar coordinates r, θ, z, in which the stellar spot is in the plane of r, θ, and the magnetic field H is symmetrical with θ and, of course, more or less perpendicular to the plane of the spot.

The problem must be treated from the standpoint of relativistic electrodynamics. It may be conveniently treated by the equations of Lagrange, or even more conveniently by the Hamiltonian equations.

The Lagrangian function for an electron in a field specified by a vector potential U and a scalar potential φ is, as is well known

$$L=-mc^2(1-\beta^2)^{\frac{1}{2}}+(e/c)(\mathbf{U}\cdot\mathbf{u})-e\varphi,$$

where m is the rest mass, u is the velocity, and $\beta^2=u^2/c^2$.

φ is zero in our problem, so that transforming to polar coordinates, we have

$$L=-mc^2\left(1-\frac{r^2\dot{\theta}^2}{c^2}-\frac{\dot{r}^2}{c^2}-\frac{\dot{z}^2}{c^2}\right)^{\frac{1}{2}}+\frac{e}{c}(\dot{r}U_r+r\dot{\theta}U_\theta+\dot{z}U_z).$$

[1] In a paper under the above title presented at the Chicago Meeting of the American Physical Society, November, 1932.

The momenta P_r, P_θ, P_z, are given by

$$P_r = m\dot{r}(1-\beta^2)^{-\frac{1}{2}} + eU_r/c, \quad (1)$$

$$P_\theta = mr^2\dot{\theta}(1-\beta^2)^{-\frac{1}{2}} + erU_\theta/c, \quad (2)$$

$$P_z = m\dot{z}(1-\beta^2)^{-\frac{1}{2}} + eU_z/c. \quad (3)$$

The Hamiltonan function $\mathcal{H}$ is given by

$$\mathcal{H} = \dot{r}P_r + \dot{\theta}P_\theta + \dot{z}P_z - L \quad (4)$$

and is easily shown to be $\mathcal{H} = mc^2(1-\beta^2)^{-\frac{1}{2}}$. Solving (1), (2) and (3) for $\dot{r}$, $\dot{\theta}$, $\dot{z}$, and substituting in (4) we readily find

$$\mathcal{H} = mc^2\left\{1 + \frac{1}{m^2c^2}\left[\left(P_r - \frac{e}{c}U_r\right)^2 + \left(\frac{P_\theta}{r} - \frac{e}{c}U_\theta\right)^2 + \left(P_z - \frac{e}{c}U_z\right)^2\right]\right\}^{\frac{1}{2}}. \quad (5)$$

Now in our problem $U_r = U_z = 0$, and U_θ is independent of θ for a magnetic field symmetrical with regard to θ. In fact, the electron currents responsible for U are all of a circular type symmetrical about the z-axis. Thus P_θ is constant, i.e., from (2)

$$mr^2\dot{\theta}(1-\beta^2)^{-\frac{1}{2}} + erU_\theta/c = er_0U_0/c, \quad (6)$$

where r_0 and U_0 refer to the values of r and U_θ at the initial instant when $\dot{\theta}=0$. Thus

$$\mathcal{H} = mc^2\left\{1 + \frac{1}{m^2c^2}\left[P_r^2 + P_z^2 + \frac{e}{c}\left(\frac{r_0U_0}{r} - U_\theta\right)^2\right]\right\}^{\frac{1}{2}}. \quad (7)$$

Now $\mathcal{H}$ is explicitly a function of t through U_θ. Hence, as follows directly from the Hamiltonian equations

$$d\mathcal{H}/dt = \partial\mathcal{H}/\partial t = -(e^2/\mathcal{H})(r_0U_0/r - U_\theta)(\partial U_\theta/\partial t)$$

$$\tfrac{1}{2}d\mathcal{H}^2/dt = e^2(U_\theta - r_0U_0/r)(\partial U_\theta/\partial t). \quad (8)$$

Since $\mathcal{H} = mc^2(1-\beta^2)^{-\frac{1}{2}} = T + mc^2$, where T is the kinetic energy,

$$\tfrac{1}{2}(d/dt)(T+mc^2)^2 = e^2(U_\theta - r_0U_0/r)(\partial U_\theta/\partial t). \quad (9)$$

Hence T increases with t so long as $rU_\theta \geqq r_0U_0$. This is certainly satisfied at the initial instant in view of the definitions of r_0 and U_0. It will be satisfied for all subsequent instants if $d(rU_\theta)/dt \geqq 0$; i.e., if $\partial U_\theta/\partial t + (\dot{r}/r)(\partial/\partial r)(rU_\theta) \geqq 0$; i.e., if

$$-(E_\theta - \dot{r}H_z/c) \geqq 0. \quad (10)$$

Bearing in mind that E_θ is in the negative direction of θ for an increase of U_θ in the positive direction of θ, we see that condition (10) is simply the condition that the force due to the electric field accelerating the electron in the negative direction of θ must be always greater than the force due to the magnetic field which acts in the opposite sense when $\dot{r}$ is positive. If (10) is satisfied always, the electron will never loop around in its path since to so loop $-(E_\theta - \dot{r}H_z/c)$ would have to be less than zero in order to balance the centrifugal acceleration at the instant when $\dot{\theta}$ was zero. Since $\dot{r}$ can never exceed c, the condition for continual increase of energy will always be satisfied if $|E| > |H_z|$. If there is anything like a uniform growth of magnetic flux with time, E will certainly be greater than H initially, and we shall find that there is time for high electronic energy to be acquired before H has grown to a magnitude sufficiently large to prohibit further increase.

If the electron starts from rest, at $t=0$ and if the magnetic field is zero then we have $U_0=0$, and

$$\tfrac{1}{2}(d/dt)(T+mc^2)^2 = e^2U_\theta(\partial U_\theta/\partial t). \quad (11)$$

So far we have made no assumption as to the way in which the magnetic field H depends upon r. Since $H = \text{curl } U$, Stokes theorem applied to an annulus $2\pi rdr$ perpendicular to z gives

$$2\pi rH_z = \partial(2\pi rU_\theta)/\partial r. \quad (12)$$

If, for purposes of illustration we take a case where H_z is inversely proportional to r, we shall enjoy the advantage of having U_θ independent of r as well as of θ. We shall consequently write

$$H_z = H_0Rt/r\tau \quad (13)$$

where τ is the time taken for it to acquire a field intensity H_0 at some assigned radius R comparable with the radius of the spot.

With U_θ independent of r and $U_r=0$, $\mathcal{H}$ is independent of r, so that P_r is constant with time, which, through (1) shows that $\dot{r}$ is zero for all time since it is zero initially. Thus the electron describes a circular path. As a matter of fact an examination of the equations shows, as might be expected, that, under these conditions, the centrifugal acceleration of the electron and the magnetic deflecting force increase together with the time in such a manner as to keep each other balanced.

Thus, with the assumption involved in (13), (12) gives $U_\theta = H_0 Rt/\tau$ and (11) gives

$$(d/dt)(T+mc^2)^2 = 2e^2 H_0^2 R^2 t/\tau^2 \qquad (14)$$

and $(T+mc^2)^2 = e^2 H_0^2 R^2 t^2/\tau^2 + m^2c^4$. Since T is zero when t is zero.

For the energies in which we are interested, energies of the order 10^{10} volts, mc^2 is negligible, so that $T = eH_0Rt/\tau$ or, expressed in volts, $V = 300H_0Rt/\tau$. If $R = 3\times10^{10}$, $H_0 = 2000$, $\tau = 10^6$, $t = 1$ second, we find $V = 2\times10^{10}$ volts.

The conclusion is then to the effect that in one second during the initial formation of a stellar spot about 50 times the earth's diameter and of such character as to give rise to a magnetic field of 2000 gauss in 12 days, an electron can acquire a velocity comparable with 2×10^{10} volts. The electric field E is given by

$$\int_0^R \frac{H_0 R}{r\tau} 2\pi r dr = 2\pi RcE,$$

so that $E = H_0R/c\tau$ and the magnetic field at the end of a time t is $H_0 t/\tau$. Hence in the present instance, the magnetic field would only just have attained a value equal to the electric field by the time the electron had received 2×10^{10} volts velocity. In the present case, the fact that $\dot{r}$ is zero insures that the electron would go on acquiring energy continually, even after the magnetic field had become equal to or greater than the electric field; but, this feature is special to our particular assumption as to the dependence of the magnetic field upon r. Naturally our particular assumption regarding H is mainly relevant only to the mathematical simplification of the problem and not to its essential features. As a matter of fact the magnetic field we have assumed would not have zero curl, and would require the existence of circular electron currents in the regions where H has the property specified. Difficulties regarding the infinite value of H at $r=0$ are irrelevant since we only require the variation of H with r postulated to exist over a very thin range of r. If we had a uniform magnetic field instead of one of the type assumed, the electron would not describe a circular orbit but would diminish its distance from the origin in its path. This complicates the details of our calculation but not the order of magnitude of the results.

We have given no attention to the motion of the electron parallel to z. It is obvious that since the electronic motion takes place in the opposite sense to that of the increasing electronic currents which are producing the magnetic field, the electron will be repelled from them, and so will shoot out into space, which is a desirable consummation from the point of view of our theory.

An examination of the foregoing calculation shows that it would be difficult to secure energies of 10^{10} volts from spots of a size such as occur on our sun. However, energies one-tenth of this amount are within the range of possibility. This fact is particularly interesting in suggesting that while we may have to look to stars other than our sun for sources of cosmic rays, the spots on the sun may be able to furnish electrons such as could play a part in auroral phenomena, since the position of the auroral zone suggests energies of the order of 10^9 volts for the electrons producing auroras. It is moreover a significant fact in this connection that correlations between sunspot activity and auroral and terrestrial magnetic phenomena have been observed. Another matter of significance arises from the fact that although, for purposes of visualization of the situation, we have imagined a spot which grows 2000 gauss in 10^6 seconds, as a matter of fact the electronic energy of 2×10^{10} is acquired before the spot has been growing for more than one second, or before the field has attained a value of more than 2×10^{-3} gauss in our example. Naturally it is not intended to imply that a situation exists in nature in which the problem we have cited is duplicated in all its details. All that it is intended to show is that with magnetic fields extending over large areas in stellar spots, relatively small rates of changes of these fields existing for brief periods can give rise to large electronic energies. Our problem has been designed to illustrate the feasibility of the idea in one particular case.

Finally, we must consider the possibility of loss of energy by collision. The work done on the electron over its mean free path λ is $E\lambda e$. In our present example $E = RH_0/\tau c = 2\times 10^{-3}$ e.s.u. Hence the work estimated in volts is $10^{-3}\times 600\lambda = 0.6\lambda$ volt. Hence if λ were greater than 50 cm the electron would acquire more energy over its mean free path than it would lose at collision, and the energy would continue to increase with time. The pressure above the surface of a star falls off very rapidly with distance and is probably far below that corresponding to a mean free path of 50 cm in regions which are sufficienty near to the surface to experience the magnetic fields of the spots. It is probable, therefore, that the efficiency of the process of accumulation of energy is but little reduced by collisions.