Momentum Transfer to Cathode Surfaces by Impinging Positive Ions in a Helium Arc

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The momentum transfer to an auxiliary cathode has been studied in the positive column of a low-voltage helium arc. The auxiliary cathode was a flat molybdenum plate insulated on one side by glass and suspended in such a way that its deflection gave a measure of the pressure against it. The measured pressure on the cathode is believed to be due to two phenomena. The first is the recoil of those ions which retain some of their kinetic energy after neutralization. The second is a radiometer effect due to the heating of the cathode by positive ion bombardment. On the basis of these assumptions it was possible to calculate from the experimental data an accommodation coefficient for helium positive ions and the fraction of the measured current carried by electrons. Although the accuracy of the experiment is not high, the values of the accommodation coefficient are in qualitative agreement with those obtained by Compton and Van Voorhis. The fraction of the current carried by electrons agrees fairly well with the results of Harrington.

Introduction

In 1929 Tanberg[1] measured the pressure on the cathode of a copper arc in vacuum. On the assumption that this pressure was the recoil resulting from the evaporation of copper atoms, it was possible to compute a temperature of about 500,000 degrees for the cathode spot. Compton[2] showed that if one assumed a reasonably small departure from unity in the accommodation coefficient for the positive ions such that they retained after neutralization a small fraction of the kinetic energy gained in the cathode drop, one could account for the observed pressure. Although the impact of a charged ion against the cathode contributes nothing to the pressure against it (on account of the counterbalancing pull during its attraction to the cathode), nevertheless if the neutralized ion leaves the cathode with any momentum there is imparted to the cathode an equal opposite momentum. Evidence for the existence of an accommodation coefficient for positive ions was found by Compton and Van Voorhis[3] from measurements of the heating of an auxiliary cathode by positive ion bombardment in a low-voltage arc.

At the suggestion of Dr. Compton, the work reported here was undertaken to test the above hypothesis with a nonvolatilizing cathode and to look for further evidence for the existence of an accommodation coefficient for positive ions. With a nonvolatilizing cathode, there is no possibility of a pressure as a result of a stream of metal vapor leaving the cathode. Preliminary work along this line, carried on at Princeton University, was reported in a letter to the Editor.[4] As a result of this preliminary experience, the apparatus has been redesigned and the work carried through, during the past year, to much more reliable conclusions, which are here reported.

The accommodation coefficient was originally introduced and defined by Knudsen[5] in terms of temperatures by $\alpha=(T_i-T_r)/(T_i-T_s)$, where T_i, T_r and T_s are the temperatures of the incident gas molecules, the reflected molecules and the reflecting surface, respectively. Langmuir and Blodgett[6] redefined the coefficient in the more general terms of energies, thus avoiding restrictions as to Maxwellian distribution of velocities among the gas molecules. In the present paper,

[1] Tanberg, Phys. Rev. **35**, 293 (1930); Phys. Rev. **35**, 1080 (1930).

[2] K. T. Compton, Phys. Rev. **36**, 706 (1930).

[3] C. C. Van Voorhis and K. T. Compton, Phys. Rev. **35**, 1438 (1930); Phys. Rev. **37**, 1596 (1931).

[4] Lamar, Phys. Rev. **37**, 842 (1931).

[5] Knudsen, Ann. d. Physik **34**, 593 (1911).

[6] Blodgett and Langmuir, Phys. Rev. **40**, 78 (1932).

this more general definition is used and the very high energy of the molecules in comparison with the average energy of the surface atoms makes the latter term so insignificant as to justify its neglect.

Ideally Simple Theory

Imagine a plane collecting electrode immersed in a uniformly ionized gas and maintained at a potential V negative with respect to the surrounding space. If its current density I consists entirely of the positive ions which are drawn into it through the surrounding space charge sheath, the pressure P against it is due to the rebound of these ions following their neutralization at its surface. This rebound energy V_r, in electron-volts, is expressible in terms of the accommodation coefficient $\alpha=(V-V_r)/V$. Under these conditions the pressure is

$$P=(I_+/e)(2MeV_r)^{1/2}=(I_+/e)(2MeV(1-\alpha))^{1/2}.$$

Here the rebound is assumed to be normal to the surface. If, as is more probable, the rebounding neutralized ions are scattered in random directions, a factor 2 should appear in the denominator. This comes about in the integration of the component of the momentum normal to the collector surface. The integration is carried out over a hemisphere cut by the plane of the collector surface from a sphere whose center lies in an element dS of the collector surface. In the apparatus to be described, the plane collector was the bob of a pendulum, whose deflection gave a direct measure of the pressure against the collector.

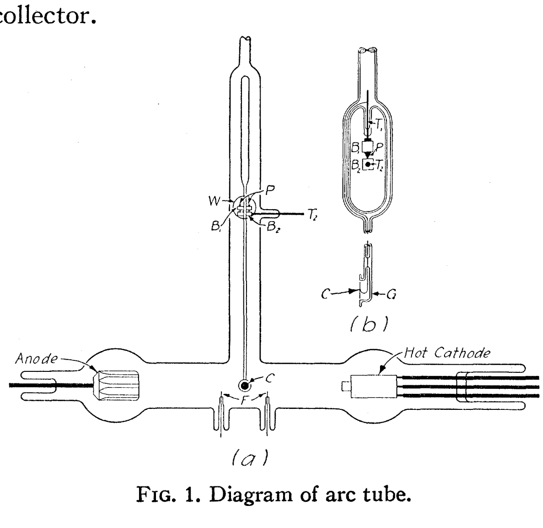

Fig. 1. Diagram of arc tube.

Actually the experiment was not so simple, because of the existence of three complicating factors: (1) lack of homogeneity of ionization, so that the conditions around the collector changed as the pendulum deflected; (2) radiometric force due to heating of the collector surface by the energy there liberated; (3) presence of secondary electrons emitted from the collector and contributing appreciably to its current. These complications are taken into account in the actual conduct of the experiment and analysis of results.

Apparatus

A diagram of the arc tube is shown in Fig. 1. The arc cathode which is of the thermionic type and the arc anode were obtained from the General Electric Company. A detailed sketch of the auxiliary cathode C is shown in Fig. 1b. It consists of a molybdenum plate insulated on one side by the glass G. A small clearance was left between the glass and the plate to prevent any sputtered metallic films deposited on the glass from making electrical contact with the auxiliary cathode. The auxiliary cathode assembly was part of a glass pendulum suspended in a side tube as shown. In order to support the pendulum a tungsten rod T_1 was sealed through the glass of the pendulum. A steel block B_1 was attached to the rod T_1 by means of a set screw. To the block were fastened two steel points P. These points set in polished indentations in the steel block B_2 which was supported by a tungsten rod T_2 sealed through the glass of the arc side tube. The current carrying leading-in wire from the auxiliary cathode C went up the inside of the hollow glass pendulum and was attached, on the inside, to the tungsten rod T_1. In order to prevent the carrying of current by the sharp steel points, a two mil copper wire was shunted between the two steel blocks B_1 and B_2. Metallic contact could then be made from the outside of the arc tube to the auxiliary cathode C by making connection with the tungsten rod T_2.

A small mirror (not shown in the figure) was fastened to the steel block B_1. A light beam was introduced through the lens-shaped window W on to the mirror and focussed on a scale 1.5 meters away. A perfect vacuum joint was made between the lens and the ground surface of its supporting side tube by means of Glyptal lacquer.

A small cylindrical probe wire (not shown in the figure) was sealed through the main arc tube immediately behind the auxiliary cathode *C*. On each side of the auxiliary cathode are the two test filaments.

The thermionic equipotential cathode was heated by alternating current from a transformer. A steady e.m.f. for the arc was supplied by a 120 volt bank of storage batteries. The resistance in series with the arc was placed on the hot cathode side of the batteries, so that it was possible to obtain, from a potential divider across the main batteries, 120 volts negative with respect to the anode for application to the auxiliary cathode. An additional 200 volts for the auxiliary cathode were obtainable from storage B batteries. The voltage of the auxiliary cathode was controlled chiefly by a high series resistance so as to eliminate all possibility of striking an arc to this cathode.

The helium was introduced into a two-liter glass reservoir bulb through a chabizite trap immersed in liquid air. The trap had previously been baked out at 500°C. In the reservoir bulb was an iron anode and a misch-metal cathode between which an arc was maintained for further purification of the helium.

The pendulum was set up and calibrated dynamically before the tube was assembled. The necessary data were the weight of the pendulum, its period for small oscillations and its period with an additional known load at a known distance from the axis of oscillation. From these, the distance of the center of gravity from the axis and thus its sensitivity could be easily calculated. It was feared that the current carrying leading-in wire might introduce an additional torque; so after the tube had been assembled, the period of the pendulum was again measured. As its value was within a percent of the previously determined value, it was evident that the leading-in wire introduced no appreciable torque.

After filling the tube to the desired pressure of helium and allowing the hot cathode to reach its normal temperature, the arc was started by means of a high-frequency leak tester.

Experimental Procedure and Calculation of Results

The ideal conditions described in the simple theory above were not entirely realized in the actual discharge tube. It was found necessary, therefore, to take certain data by way of correction as follows. It was found that even with the auxiliary cathode floating (i.e., with electron and positive ion currents to it equal) the reading of the scale (and thus the pressure acting on the cathode assembly) changed as the total arc current changed. With increasing arc current, the motion of the cathode was always away from the center of the arc tube. A reasonable assumption is that this pressure on the cathode and surrounding glass was due in part to the rebound of the neutralized positive ions, and in part to a radiometric pressure resulting from heating of the cathode assembly and from radial temperature gradients in the arc. One would expect all of these effects to be approximately proportional to the arc current for the following reasons. Langmuir and Mott-Smith[7] and others have found that at least in the case of mercury the mean kinetic energies of electrons and ions are approximately independent of arc current for constant gas pressure, and that the concentrations of electrons and ions are approximately proportional to the arc current. In this work, measurements taken with the probe electrode immediately behind the auxiliary cathode showed that the electron mean kinetic energy was approximately independent of arc current. Further test of the above assumptions was not so conclusive due possibly to the shielding action of the auxiliary cathode as it assumed different positions in the arc tube. However, for the present purposes, the assumptions are considered sufficiently well justified. The pressure due to the rebounding neutralized ions should therefore be proportional to the arc current. The heating of the cathode and surrounding glass is probably due to bombardment by electrons and positive ions and thus should also be proportional to the arc current. The radial temperature gradients existing in the tube should be proportional to the power input and (since the arc voltage was

[7] Langmuir and Mott-Smith, G. E. Rev. **27**, 449, 538, 616, 762, 810 (1924).

approximately constant) proportional to the arc current. Since the cathode moved from one position in the tube to another as the pressure acting on it changed, it was necessary to find the pressure to be expected from the above causes for different positions of the cathode in the discharge tube and to subtract these pressures as a correction from the pressures observed during bombardment under voltage. This information was obtained in the following way. Readings of the scale were recorded for different arc currents and a sample plot is shown in Fig. 2a. Subtraction of the reading at zero current from any other reading gave the deflection or pressure acting. Since these pressures were proportional to arc current, the ratio of pressure to arc current gave the pressure per unit arc current to be expected from the above complicating causes for the particular position of the cathode in the discharge tube. Since the scale reading gave the position of the cathode in the arc, these pressures per unit arc current were plotted against the scale reading as shown by a sample in Fig. 2b. This curve with slight alterations to be described later served as a correction curve for data taken during bombardment under voltage.

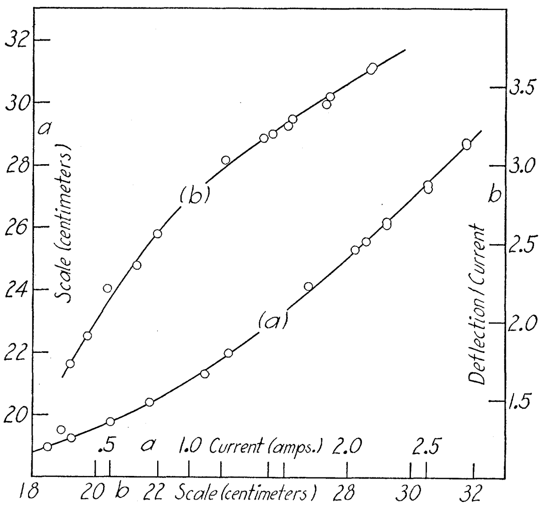

Fig. 2. Curve (a) shows scale readings as a function of arc current when the auxiliary cathode is floating. Curve (b) shows deflection per unit arc current as a function of scale reading.

For any constant arc current the potential of the space surrounding the auxiliary cathode was determined in the following way. It was feared that the large electron currents, to be expected with the auxiliary cathode above the potential of the surrounding space, would burn out the leading-in wire shunting the supports of the pendulum. Thus the current voltage characteristic curves for the auxiliary cathode were not continued above space potential. However, current voltage curves were taken with the small cylindrical probe wire immediately behind the auxiliary cathode in the manner described by Langmuir and Mott-Smith.[7] From a plot of the logarithm of the electron current *vs.* the negative potential of the probe wire with respect to the anode, space potential was determined in the usual way. The potential difference between the probe wire and the auxiliary cathode was measured with both floating. The potential of the space surrounding the auxiliary cathode was then assumed to be the potential of the space surrounding the probe wire plus the above-mentioned difference. As the difference amounted to only a few volts, these values of space potential were considered to be sufficiently accurate for the present investigation.

In order to obtain the pressure contributed by the positive ions striking the auxiliary cathode, the deflections of the auxiliary cathode were measured for different negative voltages with the arc current constant. One hundred volts negative with respect to the anode was taken as a reference voltage (a reading of the scale at this voltage being taken after each other reading). A sample plot of deflection *vs.* negative voltage with respect to the surrounding space is shown in Fig. 3a. For convenience the scale readings are also shown. At the low-voltage end of the curve there is a kink which is attributed to that part of the radiometric pressure which results from electron heating of the auxiliary cathode. Since no electrons can reach the auxiliary cathode at the higher negative voltages, this part of the radiometric pressure should not be included in any corrections made to the curve of Fig. 3a. Thus the amount of this pressure at floating potential represented by AB was reduced to unit arc current and subtracted from the correction curve (Fig. 2b) described above at the scale reading corresponding to AB. The remainder of the curve of Fig. 2b was reduced in the same ratio. The correction AB was taken at floating potential since it applied to the curve of Fig. 2b taken for convenience at this same potential. The deflections due to the positive ions striking the auxiliary cathode were then obtained by subtracting from each observed deflection (Fig. 3a), a correction amounting to the deflection to be expected in the absence of large negative voltage. These expected deflections were, of course, the differences in ordinates of the reduced correction curve (Fig. 2b), corresponding to the scale readings before and after voltage changes (Fig. 3a), multiplied by the arc current. The resulting curve, which presumably represents the pressures due to the positive ions striking the cathode, is shown in Fig. 4. Fig. 3b shows the measured currents to the auxiliary cathode plotted against the negative voltage with respect to the surrounding space. Use will be made of this curve later.

Since the pressure contributed by the positive ions resulted not only from their rebound after neutralization but from radiometric effects due to their heating of the cathode, a method was devised for studying this radiometric effect independently of other effects. When the auxiliary cathode was only slightly negative with respect to space an electron current was able to reach it. Such electron currents were obtained in the usual way by subtracting algebraically the extra-

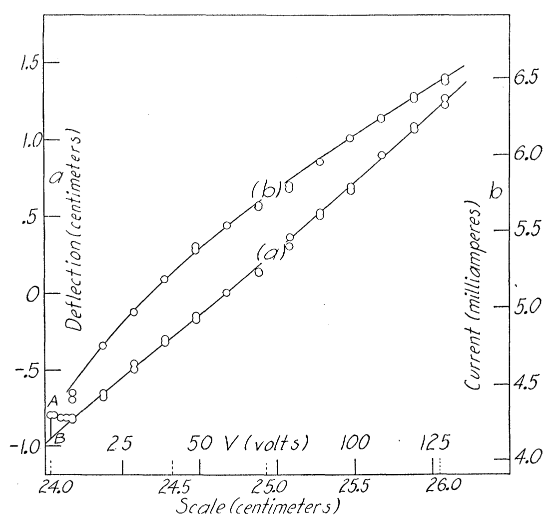

FIG. 3. Deflections and measured currents to the cathode as functions of the negative voltage of the cathode with respect to the surrounding space and of scale readings.

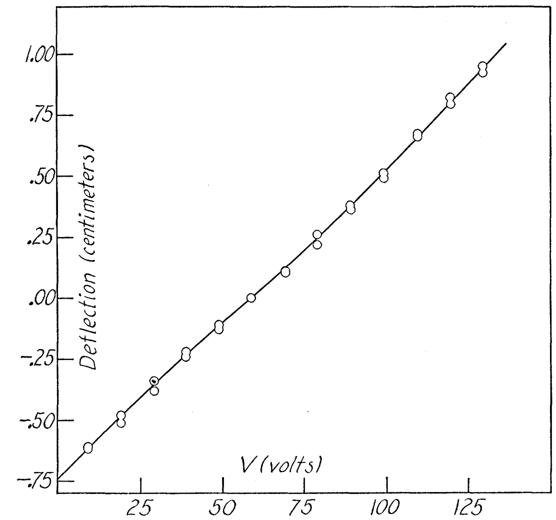

FIG. 4. Deflections resulting from bombardment of the cathode by positive ions as a function of the negative voltage of the cathode with respect to the surrounding space.

polated values of the apparent positive ion currents from the measured currents. Since, near space potential, these electron currents increase rapidly with decreasing negative voltages, the heating and thus the radiometric pressures due to these currents should change considerably with negligible change in the pressure due to the positive ions. The effective voltage of the incoming electrons is the sum of the mean kinetic energy and the work function of the cathode, thus the electron power input is the product of this quantity and the current. From a plot of the logarithm of the electron current *vs.* the negative potential with respect to the anode of the stationary probe wire mentioned above, the mean kinetic energy was determined in the usual way as described by Langmuir and Mott-Smith.[7] The deflections or pressures due to electron heating were corrected for other effects from a curve similar to that of Fig. 2b in the manner described above. A plot of these deflections *vs.* electron power input is shown in Fig. 5. As was expected, the deflections were proportional to electron power input. The slope of this curve which we will call $F(p)$, therefore, represents the radiometric pressure per unit power input when the gas pressure in the tube has the value p. The slopes $F(p)$ of a series of such curves are shown in Fig. 6 plotted against the pressure.

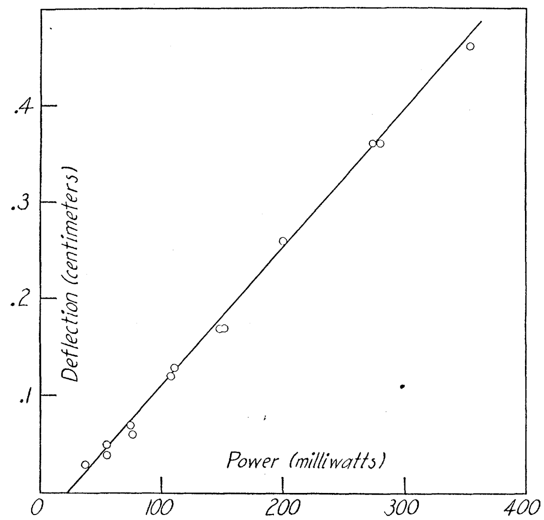

FIG. 5. Deflections resulting from bombardment of the cathode by electrons as a function of electron power input.

In order of magnitude and in shape at the higher pressures, this curve agrees surprisingly well with that given by Knudsen.[8] The deviation at the lower pressures is not surprising under the conditions of the experiment since the temperature in the arc was no doubt higher at low pressures for the same total power input.

The pressure resulting from the impact of the positive ions on the cathode can be expressed as the sum of two pressures $P=P_1+P_2$. We shall call P_1 the pressure resulting from the rebound of the positive ions after neutralization. As has been shown above,

$$P_1=(i_+/2e)(2MeV(1-\alpha))^{1/2}$$
$$=1.44(10^{-2})i_+(V(1-\alpha))^{1/2} \text{ dynes cm}^{-2},$$

where i_+ is the positive ion current density in milliamperes, V is the negative potential of the cathode with respect to the surrounding space in volts and α is the accommodation coefficient. Multiplication by the sensitivity of the pendulum, which was 2.43 cm dyne^{-1}, gives

$$P_1=3.50(10^{-2})i_+(V(1-\alpha))^{1/2},$$

where P_1 is measured in centimeters on the scale. P_2 represents the radiometric pressure resulting from heating of the auxiliary cathode by positive ion bombardment. As was shown by the use of electron heating, the radiometric force is proportional to the power input with $F(p)$ as the constant of proportionality when the gas pressure has the value p. The power input is $i_+\alpha V$ if the work function for positive ions is neglected. If it is included, V is increased by the mount φ_+ of this work function. The radiometric pressure is therefore given by $P_2=i_+\alpha VF(p)$. The total pressure P is then given by

$$P=P_1+P_2=3.50(10^{-2})i_+(V(1-\alpha))^{1/2}+i_+\alpha VF(p).$$

If f is the fraction of the measured current density i which is carried by electrons, then $i_+=i(1-f)$. By substituting, $P=i(1-f)[3.50\times(10^{-2})(V(1-\alpha))^{1/2}+F(p)\alpha V]$.

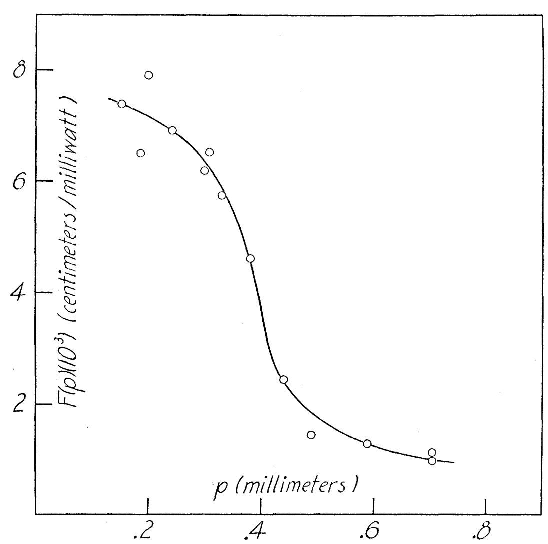

FIG. 6. Deflection per unit power input $F(p)$ as a function of gas pressure p.

TABLE I.

V=negative potential of cathode with respect to space; α=accommodation coefficient; f=fraction of current carried by electrons; φ_+=work function of cathode for positive ions; V_i=ionization potential of the gas; φ_-=work function of cathode for electrons.

	α		f	
V	$\varphi_+=0$	$\varphi_+=V_i-\varphi_-$	$\varphi_+=0$	$\varphi_+=V_i-\varphi_-$
35	0.681	0.463	0.588	0.615
65	.570	.480	.573	.612
95	.512	.449	.536	.563
125	.410	.365	.516	.532

$\alpha_{ave.}=0.491$; $f_{ave.}=0.567$.

[8] Knudsen, Ann. d. Physik 6, 129 (1930).

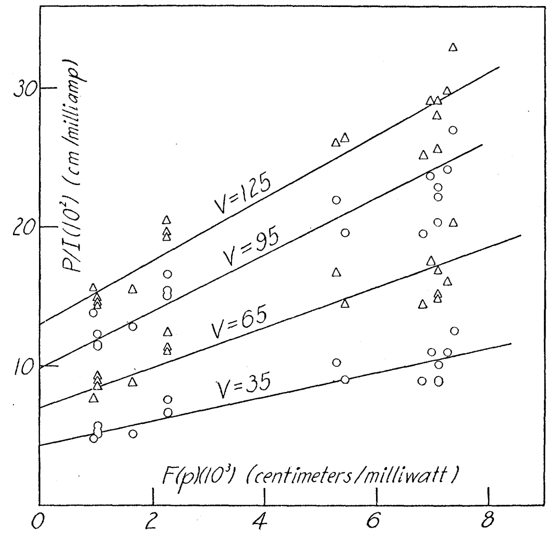

FIG. 7. Deflection per unit current to the cathode P/i as a function of deflection per unit power input $F(p)$ for different negative voltages.

If f is independent of the gas pressure p, then P/i should be a linear function of $F(p)$ by keeping V constant. Fig. 7 shows P/i vs. $F(p)$ for a series of different voltages. The slopes and intercepts were obtained by the method of least squares laying equal responsibility for error on abscissa and ordinate. From these slopes and intercepts, values of α and f were computed. An assembly of the results is given in Table I.

Discussion of Results

The results in Table I were computed on the basis of two limiting values for φ_+ in estimating the radiometric effect, namely, $\varphi_+=0$ and $\varphi_+=V_i-\varphi_-$, where V_i is the ionization potential of the gas and φ_- the electron work function of the cathode. φ_+ has not been included in P_1 since its inclusion gives values of α intermediate between the other two. From the work of Oliphant[9] on the production of metastable atoms from positive ions reflected from a metal surface, it seems improbable that φ_+ could be as great as $V_i-\varphi_-$.

The calculations of f are subject to an uncertainty inherent in the apparatus which will now be discussed. It was possible for some ions to reach the back of the cathode through the clearance between the cathode and surrounding glass. Since these ions were trapped, they contributed nothing to the pressure P_1 and, since they probably lost most of their energy in collisions before reaching the cathode, contributed nothing to P_2. They were recorded as current, however, and therefore appeared as an increase in f. The cathode was slightly elliptical in shape, having major and minor diameters 1.30 and 1.23 centimeters. The clearance between cathode and glass was about 0.2 millimeter or about 6 percent of the total area. Since ions at the edge could reach the cathode in directions other than normal, the fraction of the positive ion current which reached the back of the cathode was probably somewhat greater than 6 percent (10 percent being a reasonable estimate). The values of f given in Table I were computed on the assumption that 10 percent of the positive ions reached the back of the cathode.

The average value of f shown at the bottom of the column is in fair agreement with the high values obtained by Harrington[10] is a continuation of work by Uyterhoeven and Harrington[11] on the fraction of current carried by electrons.

The values of f given in the table represent average values taken over different gas pressures and arc currents. Recent work reported by Found and Langmuir[12] would indicate that in neon, f is not constant as the arc current is varied, so that our procedure in reducing observations to a function of P/i should not be accurately justifiable. In the work reported here observations were made at constant gas pressure over as wide a range of arc currents as the apparatus would permit (a factor of never more than three). Within our limits of accuracy no change in P/i was observed, so that our procedure would seem to be justified for our purposes. However, a possible effect to be described below might have compensated to some extent for such a variation, if present. The apparatus did not permit a direct test of the constancy of f as gas pressure varied.

The second uncertainty in the results is as

[9] Oliphant, Proc. Roy. Soc. **A124**, 228 (1929).

[10] Harrington, Phys. Rev. **38**, 1312 (1931).

[11] Uyterhoeven and Harrington, Phys. Rev. **35**, 124 (1930).

[12] Found and Langmuir, Phys. Rev. **36**, 604 (1930); Phys. Rev. **39**, 237 (1932).

follows. If there are collisions within the sheath so that the energy gained by a positive ion in falling through the sheath is carried to the cathode by more than one particle, although the energy reaching the cathode is probably unaltered, the momentum for a given energy is increased. The pressure P_2 resulting from the heating by the energy delivered is therefore probably unaltered. The pressure P_1, however, which results from the momentum delivered should be increased. Thus when the sheath thickness (measured in mean free paths for the positive ions) is large, P_1 should be large. This condition should be realized for low currents, high voltages and high pressures. The effect of such a phenomenon would be to cause an apparent decrease in α for an increase in voltage. The results show such an effect. The sheath thicknesses were in some cases as much as five kinetic theory mean free paths thick and thus this effect should be expected, although Dempster[17] and others have found surprisingly long mean free paths for positive ions.

TABLE II. *Accommodation coefficients of neutral helium atoms.*

Metal	t°C	α	Reference
Pt	−100	0.49	(13)
	+200	0.37	(13)
	20	0.34	(14)
W	20	0.06	(15) (clean)
	50	0.53	(16) (dirty)
	50	0.17	(16) (clean)

The values of the accommodation coefficient found by Compton and Van Voorhis for helium ions on molybdenum ranged from 0.35 at 36 volts to 0.55 at 126 volts. Since the ions are probably neutralized just before reaching the surface, ions and neutral atoms should behave alike when they strike a surface. Table II shows values of the accommodation coefficient for neutral atoms on various metal surfaces. Since the energies involved in the work of Compton and Van Voorhis and in the present work are much higher than those involved in any given in Table II, there is no reason to expect the values to be exactly the same. However, it is interesting to note the agreement. As one would expect, Table II shows an extreme sensitiveness of the accommodation coefficient to surface cleanness. In the present work it is believed that the continual bombardment of the surface by positive ions insured a reasonably clean surface. An interesting treatment of accommodation coefficients by K. T. Compton is soon to appear in the Proceedings of the National Academy of Sciences.

The results of this investigation, although subject to errors, seem definitely to prove that the rebounding neutralized ions exert a pressure on the cathode, give an estimate of the value of the accommodation coefficient and an estimate of the fraction of the current which is carried by electrons. These estimated values are of the same order of magnitude as values obtained by quite different methods. Compton's[2] theory that rebounding neutralized ions exert pressures of the order of those observed in various experiments is therefore verified directly (in the present work) and indirectly (in the work of Van Voorhis[3] and Compton).

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[13] Soddy and Berry, Proc. Roy. Soc. **A84**, 576 (1911).
[14] Knudsen, Ann. d. Physik **46**, 641 (1915).
[15] Roberts, Proc. Roy. Soc. **A135**, 192 (1932).
[16] Michels, Phys. Rev. **40**, 472 (1932).
[17] Dempster, Proc. Nat. Acad. Sci. **11**, 552 (1925).