From the characteristics of collector No. 2 which are not affected by the distorting action of high-speed primary electrons the reflection factors for ultimate electrons can be obtained. Thus, for example with a nickel collector, for a temperature of electron distribution  $T_e = 15,000^{\circ}$ K, the reflection factor  $\alpha$  for ultimate electrons was found to be 0.28.

It is interesting to note, that the positive ion sheath of collector No. 1 was always thinner on the cathode side, than on the anode side.<sup>2</sup> Under certain conditions we

## How Far Do Cosmic Rays Travel?

Two entirely different suggestions have been advanced in the literature as to where the cosmic rays originate. The first suggestion is that cosmic rays are of local origin (upper earth atmosphere, our own planetary system, etc.). The other suggestion is that cosmic rays are produced or have been produced throughout the universe, or even more specifically, throughout interstellar or intergalactic spaces. This latter view has especially been advanced by R. A. Millikan.

The purpose of this paper is to examine these hypotheses somewhat more closely and to establish a relation between them and the red shift of extragalactic-nebulae.

Suppose that on the basis of the second suggestion mentioned above, the generation of cosmic rays is given as  $\epsilon$  erg/cm<sup>3</sup> sec., where  $\epsilon = \epsilon(r)$  is only a function of the distance r from the observer. Then the radiation intensity  $\sigma$ from a half sphere of radius R is given by

$$\sigma = \frac{1}{4} \int_0^R \epsilon(r) dr \text{ in ergs/cm}^2 \text{ sec.}$$
(1)

Provided that  $\epsilon(r) = \epsilon_0 = \text{constant}$ , this gives

$$\sigma = \epsilon_0 R/4. \tag{2}$$

We know, however, that, because of the red shift

$$\boldsymbol{\epsilon}(\boldsymbol{r}) = \boldsymbol{\epsilon}_0 (1 - \boldsymbol{r}/D) \tag{3}$$

where  $D \sim 2000 \times 10^6$  light years. This gives

$$\sigma = (\epsilon_0 R/4)(1 - R/2D) \tag{4}$$

or if the red shift is proportional to r all the way up to r = D the total intensity from the universe

$$\sigma_t = \epsilon_0 D/8. \tag{5}$$

In these cases no light signal could ever reach us from distances r > D. In spite of an infinite number of luminous stars,  $\sigma_t$  would be finite and one of the old arguments for the necessity of a finite space would have to be discarded.

The difficulty which arises in relation to the suggestion that cosmic rays are created throughout intergalactic space now is this. According to the observational data the ratios of the intensity due to the galaxy  $\sigma_q$  and the intensity due to the rest of the universe  $\sigma_u$  are

$$a = \sigma_q / \sigma_u \gg 1$$
 for visible light (6)

$$b = \sigma_g / \sigma_u \ll 1$$
 for the cosmic rays. (7)

could observe the typical electron sheath breakdown curves<sup>3</sup> for collector No. 1.

Physical Research Institute, Moscow-University, January 9, 1933.

<sup>2</sup> Compare L. Tonks and I. Langmuir, Phys. Rev. 34, 899 (1929).

<sup>3</sup> I. Langmuir, J. Franklin Institute 214, 275 (1932).

The ratio a/b is equal at the very least to a hundred. It is therefore impossible that the cosmic rays, if photons, come from luminous matter. Now according to the present estimates the average density of dark matter in our galaxy  $(\rho_g)$  and throughout the rest of the universe  $(\rho_u)$  are in the ratio

$$\rho_g/\rho_u > 100,000.$$
 (8)

If we assume that the cosmic rays are produced at a rate proportional to the density, then it follows that the above ratio b for the cosmic rays according to (2) can only be explained if these rays are collected from all distances up to  $10^7 \times d$  light years where d > 10,000 light years is the radius of our galaxy. This would correspond to a distance greater than 1011 light years. Now if the red shift were linear with distance all the time, no cosmic-ray photon could reach us from distances greater than  $2 \times 10^9$  light years. The discrepancy becomes still worse, as Dr. Tolman kindly informs me, if the cosmic rays consist of any particles of matter such as electrons or neutrons.

The following suggestions might be advanced in order to remove the above discrepancy.

(1) The extragalactic red shift may increase less than proportional to the distance for very great distances. The corresponding Doppler velocity at great distances however must then relatively soon approach quite closely the velocity of light in order to prevent a too great amount of visible light reaching us from distant hot stars (O, B-stars, etc.). It is also to be remembered that the simple Einsteinde Sitter theory requires the red shift to increase faster than the distance.

(2) The ratio (8) may be much smaller than assumed above. Difficulties however may arise contradicting the so far observed emptiness of extragalactic space. It is also to be remembered that cosmic rays at any rate are probably more strongly absorbed by any kind of interstellar matter than visible light.

(3) The "chemical reaction" producing the cosmic rays may be of a negative order, that is, it might be proportional to some inverse power of the density. One might picture, for instance, a set of quantum states of space which according to the exclusion principle is entirely filled up at higher densities. Free states might exist at very low densities and facilitate processes which are not possible at higher pressures.

(4) Cosmic rays may have been produced at a time when the universe was in an entirely different state than it is

S. D. GVOSDOVER

now. Cosmic rays might have travelled many times in circles. Their great absorbability, however, must be remembered. This hypothesis must be investigated in relation to the recent theories on expansion.

(5) The production of cosmic rays might be a local phenomenon, that is, either it takes place in the upper earth's atmosphere or at least in the neighborhood of the solar system. One might, for instance, suggest that relatively slow electrons penetrate nuclei and fast electrons

## The Uncertainty of the Electromagnetic Field of a Particle

Heisenberg's<sup>1</sup> uncertainty principle for the electromagnetic field,

$$\Delta E_x \Delta H_z > hc/l^4$$

refers to the mean measures of two perpendicular components  $E_x$ ,  $H_z$  of the electric and magnetic fields in a cube of side l. It must be understood as referring to time-mean values of this field during the time l/c.

If we consider measurements during a time t = Tl/c, thereby allowing for T elementary determinations, each of the uncertainties will be reduced by a factor  $T^{1/2}$  and the above statement becomes,

$$\Delta E_x \Delta H_z > hc/l^4 T = h/l^3 t.$$

As E and H are expressed in the same units we may define as the uncertainty of the field the largest uncertainty  $\Delta E$  of each of the six components. We therefore have for the uncertainty of the field,

$$\Delta E > (hc/T)^{1/2}/l^2 = (h/t)^{1/2}/l^{3/2}.$$

We must compare this uncertainty with the value E of the components of the field. The electric field is  $E = ze/r^2$ where ze is the charge and r the distance of the charge from the cube where the measurement is made. The uncertainty of r is then the side l of this cube. We have therefore,

or

$$(\Delta E/E)(l^2/r^2) = (hc)^{1/2}/ezT^{1/2}$$
  
 $(\Delta E/E)^2(l/r)^3 = (hc/e^2z^2)(r/ct).$ 

both sources: first the uncertainty  $\Delta E$  of the measurements, second the uncertainty l of r in the formulation of the law. The most favorable case comes when these two un-

certainties are such that  $\Delta E/E = 2l/r$ . We therefore find, according to whether we consider T or t independent of l

are ejected, a process which has recently been found to occur for impinging protons. The above process might be repeated several times in order to boost up the energies.

A more detailed paper on the suggestions made in this letter will appear shortly.

F. ZWICKY

California Institute of Technology, Pasadena, California, January 9, 1933.

 $\Delta E/E = (4hc/e^2z^2T)^{1/6} = (200/z^2T)^{1/6}$ 

 $\Delta E/E = (8hcr/e^2z^2t)^{1/5} = \lceil (690/z^2)(r/ct) \rceil^{1/5}.$ 

We may therefore conclude that: (1) For instantaneous determinations (T=1) the field of an electron, proton or atomic nucleus is practically undetermined. The uncertainty of the instantaneous field of a particle depends only on its charge which must be at least  $6 \times 10^7 e$  in order that the field may be determined to within one percent. (2) The uncertainty of the field of a given particle, for instance an electron or a proton (z=1) depends on the number of times ct/r that light is able to travel from the charge to the point where the field is measured. In order to have an uncertainty less than one percent we must take the determination as a mean over a time  $t = 7 \cdot 10^{12} r/c$ . It is interesting to apply these consequences of the uncertainty principle to the original Bohr atom. Bohr was right when he considered the field of the nucleus as determining the orbit of the electron, since this field is static and remains significant when averages are taken over long periods of time. He was also right in neglecting the radiation of the moving electron, because we see now from the uncertainty principle that the only determined field is the average field during a time in which the electron has made more than 1010 revolutions. If we average the field before forming the Poynting vector we of course cancel the radiation altogether.

G. LEMAITRE

University of Louvain, at present at Massachusetts Institute of Technology, January 9, 1933.

<sup>1</sup> W. Heisenberg, *The Physical Principles of the Quantum Theory*, p. 52, Chicago (1930); see also G. Lemaitre, *L'indétermination de la loi de Coulomb*, Ann. de la Soc. Sci. de Bruxelles, 51-B, p. 12, (1931).

148