experimental data for the susceptibilities of salts in solution as given by the most recent determinations by Ikenmeyer, Hocart and others. The values for A, Kr and Xe in brackets are calculated from Slater's approximate wave functions. For Cl^-, Br^- and I^- it has been found that χ_A has a larger value in univalent than in bivalent solutions. The agreement between columns (3) and (4) is remarkably good and indicates that the Kirkwood-Vinti equation is probably a very good approximation to the truth; moreover, the values of α in columns (5) and (6) derived from these values of R_∞ and χ_A should be fairly reliable. In column (7) are given the values of α obtained by J. E. and M. G. Mayer for *free* ions. The latter are smaller than the former which is in accordance with the general argument given by the Mayers.

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May 14, 1933.

On a New Type of Reasoning and Some of Its Possible Consequences

A. On a principle of flexibility of scientific truth

From a deeper scrutiny of the foundations of scientific truth it follows that every scientific statement referring to observations should possess a certain minimum degree of flexibility. In other words, no set of two-valued truths can be established with the expectation that this set ultimately will stand the test of experience. Formulations of scientific truth intrinsically must be *many-valued.*

I cannot here give a more complete justification of the above contention, except the mention, that any closed set of truths can be stated only after the adoption of a certain definite set of rules describing the type of thinking which is to be used. As *a priori* there are many such sets of rules, we cannot hope to embrace any part of nature completely by restricting ourselves to a definite set. We therefore must require of every scientific statement that it be in accord with what might be called the *principle of flexibility of scientific truth* if we wish to be in an organic relation to experience. The study of this principle must not at all be thought of as an interesting but useless pastime. On the contrary, this study suggests a new and so far unexploited type of reasoning concerning especially the fundamental concepts in physics. If properly handled our principle promises to lead the way to many new discoveries. The new type of reasoning which I propose may be compared in some respects to the systematic negation of postulates and the construction of more general sets of truths, a procedure well known in mathematics, but it goes deeper, in that the underlying "truth" necessarily is many-valued.

If then in physics we are confronted with any statement which has the appearance of a well-established absolute truth, we must question it, guess at more general possibilities in accordance with many-valued logics and consult the experiments. In particular, it may be anticipated that no set of nonstatistical truths will stand the test of experience. Passing from generalities to definite applications I give a few examples.

B. On the continuity of time and space

One statement in physics which has the appearance of an absolute truth refers to the simultaneous "in between" relation connecting the coordinates of time and space. Take for instance an electron which enters and leaves a lead plate at the times t_1, t_2 at the points P_1, P_2, respectively, without crossing the boundaries at any other points. We then say that all processes which have happened to the electron between P_1 and P_2 must also have happened at a time between t_1 and t_2. Our principle of flexibility negates this statement. This means that there must exist processes which for happenings between P_1 and P_2 produce corresponding times t which may be both inside or outside of the interval (t_1t_2), this no matter how we may define our measurements of time and space. It is of great importance

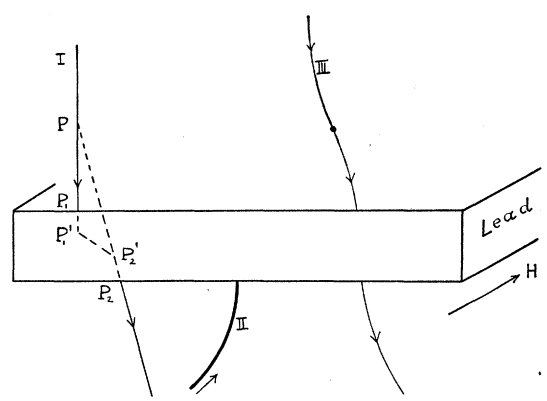

Fig. 1.

that electron tracks as shown in Fig. 1 have actually been observed[1] and that they indicate the existence of effects which according to the customary definition of time occasionally *produce* a *reversal of time.*

If one wants to explain track (I) with our ordinary notions in physics, one must assume that a double scattering at some points P_1' and P_2' has taken place. This, however, is irreconcilable with the fact that the occurrence of single scatterings through angles as shown at P_1' is not correspondingly more frequent than the double scattering. We therefore conclude that there exists a *single* process, so far unknown which produces a parallel displacement of the electron track, as well as an apparent reversal of time. It is a problem for further investigation to determine what kind of a process this is. As a very tentative suggestion the idea might be advanced that a fast electron, on a hard impact stores itself with its total energy in another elementary

[1] C. D. Anderson, Phys. Rev. **41**, 405 (1932). Fig. 23 of this paper shows an electron track of the type I.

particle and by some kind of a resonance effect (similar to the exchange of electrons) jumps sidewise and continues its path in the shown direction.

The claim that measured time can reverse means that in no given system of coordinates time is flowing continually in the same direction along the paths of all elementary particles. If the spatial length on a world line increases, time need not necessarily increase simultaneously, but may occasionally decrease, depending on the interaction with other world lines. This leads to a many-valued truth in contradistinction to a two-valued truth which answers the law of the excluded middle. This law says that an electron undergoes some process between P_1 and P_2 at a time between t_1 and t_2 *or* it undergoes a process outside of (P_1P_2) at a time outside of (t_1t_2). In our new way of thinking additional values of the truth must be introduced, namely, that the electron makes some process *between* P_1 and P_2 at a time *outside* of the interval (t_1t_2) or *vice versa*. The statistical distribution over the four values of the truth depends of course on the interaction between the electrons with other particles.

C. Degrees of freedom

Light quanta may change their number of degrees of freedom by combination with other quanta. However, elementary particles of matter are not supposed to change the number of their degrees of freedom. If this assertion could be verified experimentally with certainty we would be in possession of an absolute truth. The principle of flexibility therefore suggests that we negate the statement of the constancy of degrees of freedom for material particles and that we search for phenomena verifying this negation. One way out is, that matter might be annihilated and transformed into radiation. The only direct indication for the possibility of annihilation is the fact that the incoherent scattering of γ-rays from nuclei seems to contain two components of energies approximately equal to mc^2 and $2mc^2$ where m is the mass of the electron.

Another absolute statement is that of the constancy of the electric charge and the magnetic moment of elementary particles. This statement also implies the possibility of determining *simultaneously* the mass and the charge of a particle. From our principle of flexibility we must negate the statement and assume that the charge and the magnetic moment may occasionally change. Dr. C. Anderson has called my attention to a heavily ionized cloud chamber track (type II in Fig. 1) which might be due to an α-particle ejected by cosmic rays and moving in the direction of the arrow. However if from the curvature and the magnetic field its energy is computed it follows that the range should be smaller than the length of the observed track. One might then assume that the energy of the α-particle is high enough to explain the observed range. However in this case the applied magnetic field can produce the observed curvature only if the α-particle has suffered several collisions. One must further assume that the α-particle *accidentally* lands normal to the lead. This explanation of track II rests on two highly improbable assumptions. Therefore I think it must be discarded. I suggest instead that we are confronted with the track of a particle of about the mass of an electron emerging from the plate. In order to explain the high ionization the particle must have a charge several times that of an ordinary electron which would be a verification of our prediction.[2]

The change of sign of a charge might in some cases find an interpretation as a complete exchange of energy of a positive and a negative particle. Such a process would produce tracks of the type III in the figure. Tracks of this kind also occur in cloud chamber photographs. I think they imply that it is not only impossible to identify each of two electrons during a respective impact but that the same is true for a positive and a negative electron.

Another statement which deals with numbers only is the so-called exclusion principle in quantum mechanics. I am inclined to think that the negation of this statement might prove fruitful in the construction of an adequate theory of the nucleus.

D. The electric flux

The total flux of the electric field through a surface enclosing a finite material system has always been regarded as one of the most fundamental and indestructible quantities in physics. But according to our principle of flexibility we must even doubt this assertion and search for processes which destroy electric charges individually (not only in pairs as the recently proposed mutual annihilation of positive and negative electron). The existence of this process might prove to be a necessity in order to balance the electric budget in the universe. As a special difficulty I mention that if cosmic rays contained only negative electrons the earth would shortly charge up to potentials which would prevent the arrival of more electrons. Possible ways out are the assumption that positive and negative electrons arrive in equal numbers and equal speeds or the assumption that charges get annihilated. It will be of importance to test these questions experimentally.

E. Concluding remarks

I emphasize once more that the principle of flexibility of scientific truth refers to any type of symbolic statements of truth as truth can be secured by *measurements*. Any other type of truth does not fall within the realm of present natural science and is therefore not subject to our discussion. The special predictions which I have derived are possibilities among others and not at all unambiguous, although I feel that most of them have a fair chance to be verified experimentally in the near future. There are of course many still more fundamental concepts in physics such as reproducibility, description by arithmetical numbers, etc., which some day will no longer be adequate rules of scientific thinking. However it must be kept in mind that it is not necessarily fruitful to negate all at once all of the absolute postulates in science. Indeed a hundred years ago it would have been of no avail to negate the constancy of mass, the possibility of identification of particles, etc.,

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[2] M. Dehlbrueck, Nature **130**, 629 (1932), for entirely different reasons, has also postulated the existence of particles with a mass similar to that of the electron but of a charge greater than that of the electron.

as at that time there was probably no chance at all to verify the consequences of such negations. The principle of flexibility must be handled with great care, having always in mind the possibility of experimental verification. It seems to me that the time for an extended application of the principle is ripe now. The conceptional difficulties in quantum mechanics may be interpreted as due to the peculiar inconsistencies of this theory which in certain respects conforms with our principle of flexibility, whereas in other respects, some of which we mentioned in this paper, quantum mechanics and the relativity theory are based on very antiquated notions. It should also be clear from our discussion that the recent controversies regarding the absolute truth of uncertainty principle *versus* causality are quite futile, as scientific truth intrinsically cannot be absolute.

Finally I hope that the principle of flexibility of scientific truth is itself flexible enough so as not to annihilate itself through its own tools after the fashion of Epimenides, the Cretan.

I wish to thank Professor E. T. Bell for many discussions.

F. Zwicky

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Pasadena,
May 17, 1933.

Remarks on the Preceding Note on Many-Valued Truths

The new type of reasoning suggested in Professor Zwicky's note appears to be closely related to some of the projects which have occupied workers in the foundations of mathematics during the past two decades (since L. E. J. Brouwer's rejection of the law of excluded middle as a universally valid law of reasoning), and more particularly in the last five years. For this reason, Professor Zwicky's totally independent approach should be of interest to pure mathematicians. Conversely, some of the recent work in the foundations of mathematics may be of interest to those concerned with the foundations of theoretical physics. That Professor Zwicky arrived independently at his conclusions, gives a new interest to the mathematics and suggests further mathematical investigations. Professor Zwicky's principle of flexibility obviously has a wider scope than the new mathematics.

Many-valued logics have been created and studied in considerable detail by Tarski and Lukasiewicz, and their followers. There is a readable popular exposition of some of this work in a paper in the *Monist*, October, 1932, by Professor C. I. Lewis. The paper contains examples of such logics.

It will be noticed that one of Professor's Zwicky's suggestions challenges the universal applicability to physics of the law of identity. This law has also been scrutinized and rejected in some recent work in mathematics, and in quantum mechanics, where, however, only the identifiability of particles of the same type has been questioned. There thus remains of the Aristotelian system only the law of contradiction. This, so far as I know, has not been challenged outright, although its statement in a many-valued logic must be modified.

If many-valued logics become current, one statement of Professor Zwicky's is likely to receive prompt confirmation from the mathematical side. He predicts that description by "arithmetical numbers" will some day cease to be an adequate rule of scientific thinking. So far as those properties of integers which are independent of order relations are concerned, it may be reasonably doubted now whether "arithmetical numbers" are either necessary or sufficient in any reasoning. For (as shown by the present writer in a paper in the *Transactions of the American Mathematical Society for 1927*), common arithmetic is abstractly identical, except for order relations, with the common two-valued logic of classes. This extends to the logic of relations. By a remarkable coincidence, the problem of extending this to a many-valued logic, and hence getting a fundamentally new generalization of the concept of number, was already under way when I first heard of Professor Zwicky's similar idea. The generalization was proposed solely for its intrinsic interest as an extension of the classical theory of ideals in arithmetic, without any notion that it might have a scientific interpretation. Instead of the integers 0, 1 (for true, false, respectively), of Boolean algebra, we may have some or all of the rational numbers in the interval 0 to 1 as truth values of propositions, and a given truth value may be interpreted as a probability. The last, however, is merely one possibility of interpretation, and does not affect the abstract formulation of the generalized arithmetic.

There is another point in illustration of Professor Zwicky's principle of flexibility. If it is true that the theory of general relativity eliminates the observer (through the principle of covariance), and if it is true that the quantum theory retains the observer (through the indeterminacy principle, or whatever physical imagery is supposed to justify this principle), then a unification of relativity and quantum mechanics transcends a two-valued logic because it controverts the law of the excluded middle. To effect any unification which shall be more than superficial algebra, one or other of the theories to be unified must be radically changed, or resort must be made to a more than two-valued logic.

From the considerations adduced in Professor Zwicky's note, it appears that the time is now ripe for the adoption of the theory of many-valued truths as a working hypothesis.

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May 17, 1933.