The Effect of Tension on the Electrical Resistance of Single Bismuth Crystals

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(Received October 19, 1932)

The compensated potentiometer method of measuring minute variations in small resistances which was developed by P. W. Bridgman has been used in the study of the effect of tension, applied parallel to the direction of current flow, on the resistance of single bismuth crystals. The tension coefficient of resistance at 30°C has been found to depend on the orientations both of the principal (111) and secondary (11$\bar{1}$) cleavage planes with respect to the tension. For the limiting case of the principal cleavage plane perpendicular to the tension, the coefficient is independent of the orientation of the secondary cleavage planes; in that of the principal cleavage plane parallel to the tension, the coefficient varies very little and is very probably independent of the secondary orientation. In the case where the normal to the principal cleavage plane makes an angle of about 60° with the tension, the variation of the coefficient with the orientation of the secondary cleavage plane seems to be a maximum. This variation involves a change in sign as well as in magnitude, so that for certain orientations the coefficient becomes positive instead of remaining negative. The coefficient shows trigonal symmetry, as it must do if it is to be consistent with the known corporeal trigonal symmetry of the bismuth crystal. The paper presents only these empirical results.

THE EFFECT on the resistance of single crystals of various elements produced by hydrostatic pressure[1] has previously been studied, but not the effect of tension.[2] It is to be expected that the effect of tension will be more complicated than that of hydrostatic pressure, since in the case of the latter the symmetry of the crystal is unchanged. As a result the pressure coefficient of resistance is a function only of a single parameter, the orientation of the main cleavage plane with respect to the direction of the electric current, and, as Professor Bridgman has shown, may be expressed in terms of but two constants, one associated with the current flowing perpendicular to the normal to the principal cleavage plane and the other with a current parallel to it. In the case of tension, on the other hand, the applied force must cause a change in the symmetry of the crystal, a change which depends not only on the orientation of the principal cleavage plane with respect to the tension, but also on the orientation of the secondary cleavage planes. The problem is thus a more complicated one, since the effect may very possibly depend on more than the change of symmetry. Experimentally, however, it has been found that the coefficient may be represented within experimental error as a function of only two parameters which may be taken to

[1] P. W. Bridgman, Proc. Amer. Acad. **60**, 305 (1925); **63**, 351 (1929).

[2] Trapeznikowa (K. Akad. Amsterdam **34**, 840 (1931)) finds no effect of tension or of unidirectional pressure on the resistance of bismuth crystals. However, the sensitiveness of his apparatus was such that he could not detect changes of less than 1 percent whereas the changes to be reported in this paper are at most 0.2 percent of the initial resistance.

be the primary and secondary orientations of the crystal with respect to the tension. At the present time, it is only possible to arrange these experimental results of the effect of tension on the resistance of single bismuth crystals in a consistent set of curves without, however, giving any theoretical basis for the curves.

Experimental Arrangement

To measure the small changes in resistance involved, the compensated potentiometer method developed by Professor Bridgman,[3] was used. Fig. 1 gives the electrical connections of this method. The potential difference V_{AB} across the potential leads to the crystal AB is balanced against the potential fall V_{CD} of the supplementary compensating circuit Q, and, when the effect of the tension on the resistance is large enough, this in turn is balanced

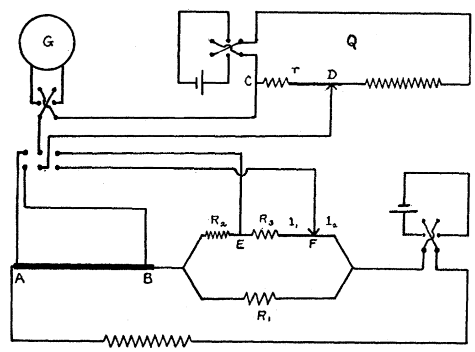


Fig. 1.

against the fall V_{EF}, so that the unknown resistance R_{AB} is calculable in terms of the known resistances R_1, R_2, R_3, l_1 and l_2. The fractional change in resistance is then

$$\Delta R/R = \Delta R_{AB}/R_{AB} = \Delta l/(l_1 + R_3) \tag{1}$$

where the slide wire lengths are reduced to ohms. As Professor Bridgman has pointed out, the advantage of this method is that the result is independent of the current flowing through the crystal provided that that remains constant during a single reading. This is cared for by using a large and heavy 6-volt storage battery as the source of the current which was usually kept at about 0.45 amp. The largest value of the fraction $\Delta R/R$ actually measured was 22×10^{-4}. Inasmuch as the bismuth resistances themselves were less than 0.01 ohm, the precision of the method is very high. When the effect was too small to be measured by this *null* method, the average change in reading of the galvanometer due to the application of the tension was determined by reading the galvanometer alternately with and without the tension, plotting the observed readings against the time scale and finding the average distance between the two curves. Ideally, both curves should be straight horizontal lines, but the zero drift of the galvanometer, small changes in the voltage

[3] P. W. Bridgman, Proc. Amer. Acad. **56,** 61 (1921).

of the compensating circuit, and slight variations in temperature which produce in addition to small changes in resistance small thermal e.m.f.'s give rise to small drifts, as in Fig. 2, which is representative of the type of curve obtained in this way. The deflections in most cases were much greater than the drifts observed. These changes in galvanometer readings with tension are proportional to the changes in potential difference and thus to the resistance changes of the crystal, the constants of proportionality[4] depending in an easily ascertainable manner on the known e.m.f.'s and resistances of the circuit. This *deflection* method was used in each determination of the tension coefficient to check the linearity of the change of resistance with tension and was checked in the case of the greatest tension against the *null* method for absolute value.

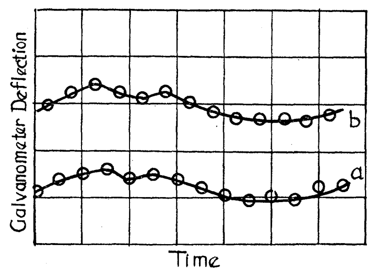


Fig. 2. *a* without tension; *b* with tension.

Since temperature fluctuations produce undesirable changes in the resistance to be measured, the crystal was mounted in a constant temperature bath of kerosene. This was thoroughly stirred and was kept at the constant temperature of approximately 30°C by means of a metal thermostat which, although it drifted slightly, prevented any large sudden change in temperature. In Fig. 3 is sketched the general arrangement of the crystal mounting and constant temperature bath. The heating coil is not shown in the figure, but is wound on a square frame larger than the brass box *mmmm* so that the heat source is not concentrated in one spot. The crystal itself AB is mounted on a brass support inside a heavy copper cylinder *pp* 5/32 inch thick, and this

[4] The voltage sensitivity of the galvanometer, which was made as great as possible by placing the scale about five meters from the galvanometer, varied with each value of the total crystal resistance. The reason for this is obvious. For, applying Kirchhoff's laws to the network, we have, where ΔE is the increase in V_{AB}, Δi_{CD} the change in current in CD produced by the lack of balance, and I_G the current through the galvanometer,

$$\Delta i_{CD} = I_G \tag{a}$$

a relation arising from the condition on current at the junction C, and

$$I_G R_G = (V_{AB})_0 + \Delta E - (V_{CD})_0 - \Delta i_{CD} r \tag{b}$$

arising from the condition on potential falls in the closed circuit $ABCD$. But $(V_{AB})_0$ and $(V_{CD})_0$ are the values of the potential falls for the balanced case and so are equal; hence

$$I_G R_G = \Delta E - I_G r. \tag{c}$$

The voltage sensitivity is consequently

$$\Delta E / I_G = R_G + r. \tag{d}$$

This method will then function with the maximum of ease, if r has approximately the value of the critical damping resistance of the galvanometer.

whole mounting is placed in air inside the brass box. The addition of the brass box seemed advisable, since in its absence it seemed impossible to stir the kerosene sufficiently inside the copper cylinder to insure constancy of temperature. With the addition of the brass box, erratic fluctuations in temperature were no longer observed.

The crystals used which were from $1\frac{1}{2}$ to 3 inches long and had a diameter of approximately $\frac{1}{8}$ inch, had all been made by Professor Bridgman from bismuth furnished by Kahlbaum, and were most generously put by him at the writer's disposal. He had previously used these same crystals in his determination of the specific resistance of bismuth crystals and of their

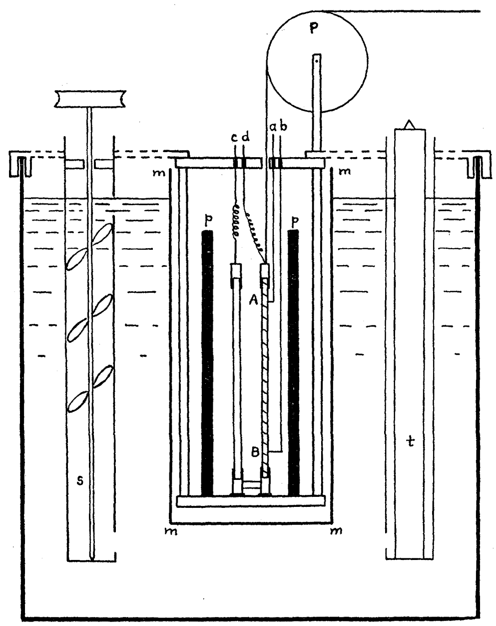


Fig. 3. *AB* crystal; *a*, *b* potential leads; *c*, *d* current leads; *pp*, copper cylinder; *mmmm* brass box; *s* stirrer; *t* metal thermostat (shown without amplifying levers); *P* pulley.

hydrostatic pressure coefficient of resistance.[1] These crystals were soft-soldered into brass holders at either end, one of which holders was fastened firmly to the large brass support which held the Bakelite pulley *P* over which the string carrying the tension passed. Fine copper wires of 0.01 inch diameter were soft-soldered to the crystal a few millimeters from each end for potential leads and a thermocouple was soldered close to one end. The other end of this thermocouple was inserted in the kerosene and so enabled one to know when the crystal temperature had come to that of the kerosene; with the air layer between the brass box and the crystal, coming to equilibrium was a slow process and at least an hour was required before equilibrium was obtained. The string carrying the tension passed over another pulley near the galvanometer telescope; weights were hung on a steel spring attached to this end. The steel spring prevented the sudden application of the whole tension to the crystal. In view of the low elastic limit (22.1 kg/cm² shearing stress across the principal cleavage plane)[5] of single bismuth crystals, 1200 grams was the greatest force applied with crystals of the given cross section, or about 15–20 kg/cm².

Results

Forty-five single bismuth crystals of different orientations were studied, the adiabatic tension coefficient of resistance β at 30°C being determined in each case. This coefficient is defined as the ratio of the change in resistance produced by a tension of 1 kg/cm² to the total resistance. It is adiabatic in that the time intervals between consecutive applications of the tension were about one minute, so that each reading was taken within half a minute of the establishment of the new state. This short interval would not allow for the establishment of thermal equilibrium. On the basis of an adiabatic process

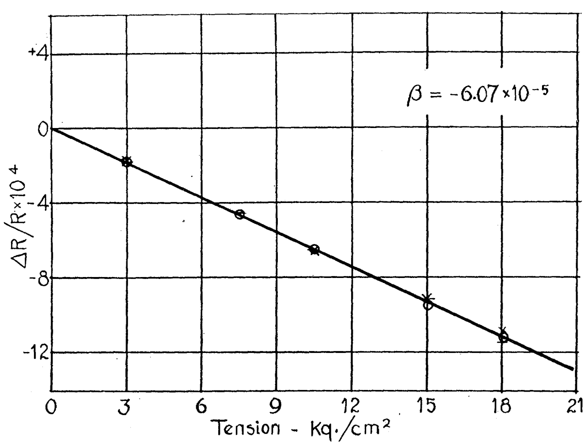


Fig. 4. Circles, plus signs and crosses signify different days' readings.

taking place, the change of temperature arising from the sudden application of the maximum tension comes out by computation to be about —0.006°C. Since the temperature coëfficient of resistance of bismuth crystals is 0.00445, this change of temperature produces a relative change in the resistance of -0.27×10^{-4} which corresponds to a change in the computed value of the coefficient β of about -0.18×10^{-5} in each case, the correction to be subtracted in passing from the adiabatic to the isothermal case. Furthermore, no correction was made for the change in resistance arising from the change in dimensions caused by the application of the tension. This varies with the orientation of the crystal, but the correction of the tension coefficient β was always less than -0.4×10^{-5}. Both these corrections were usually negligible with respect to the values of the coefficient actually found and were always of the same order of magnitude as the experimental errors to be expected.

Fig. 4 shows the value of $\Delta R/R$ for a given crystal taken on different days with different tensions and with different distances between the potential

[5] Georgieff and Schmid, Zeits. f. Physik **36**, 759 (1926).

leads. The linearity of the relation between $\Delta R/R$ and the tension applied is clearly in evidence as well as the fact that the experiments under different conditions gave entirely consistent results. It was further checked that the value of the coefficient was independent of the direction and magnitude of the current through the crystal. The specific resistance was determined roughly in every case and with a few exceptions came within 1 percent of Professor Bridgman's value for the given orientation; the dimensions of the crystals, particularly the distance between the potential leads, were not found with sufficient accuracy to warrant any better check.

The tension coefficient of resistance of a given single bismuth crystal was thus shown to be independent of the experimental variables, and so to be a constant characteristic of the given crystal; but in different crystals it varied with the orientation of the cleavage planes of the crystal with respect to the direction of the electric current. In these experiments the crystals were all cylindrical in shape with a diameter much smaller than the linear length; the direction of the electric current coincided with that of the axis of this cylinder and with the direction of the tension. The orientation of the crystal with respect to the axis of the cylinder can be defined in terms of two angles, θ and ϕ. The angle θ determines the orientation of the principal cleavage plane (111) which is perpendicular to the trigonal axis of the crystal and is defined as the angle between the normal to the principal cleavage plane and the longitudinal cylindrical axis of the crystal. The angle ϕ determines the orientation of a secondary cleavage plane with respect to the principal cleavage plane and the cylindrical axis. If the crystal is split along a principal cleavage plane, the section obtained is elliptical in shape. The major axis of this ellipse is the intersection of the plane through the normal to the principal cleavage plane and the cylindrical axis of the crystal with the principal cleavage plane. On the surface of this ellipse are found fine lines making angles of 60° with each other; these are the intersections of secondary cleavage planes with the principal cleavage plane. If the crystal be cleaved along one of these lines, the normal to the new cleavage plane will be found to make either an angle of 71° with that of the principal cleavage plane, or an angle considerably greater than 90°.[6] Those planes making angles of 71° are the ones to be considered. The angle ϕ is then to be defined as the angle between the projection on the principal cleavage plane of the normal to this secondary cleavage plane and the major axis of the elliptical section of the principal cleavage plane. Since bismuth has trigonal symmetry, these secondary planes will recur every 120°; and if the change in resistance depends on the orientation of these secondary planes, it should show a periodicity of 120°. It is to be noted that the angle ϕ as measured at one end of the crystal is, because of the known symmetry of the bismuth crystal, the negative of that measured at the other end, and hence it is impossible to differentiate between ϕ and $-\phi$, or between ϕ and $120-\phi$, since the tension is double-headed and always

[6] The writer is indebted to Professor Charles Palache of the Department of Mineralogy and Petrography of Harvard University for pointing out to her the characteristics of bismuth crystals here described.

involves two forces which are opposite in direction but equal in magnitude and act on the two ends of the crystal. Thus if the values of the tension coefficient are found between 0° and 60°, those between 60° and 120° are immediately known. With these definitions of the angle determining the orientation of the crystal with respect to the direction of the electric current, crystals having approximately the same orientations gave consistent values for β;

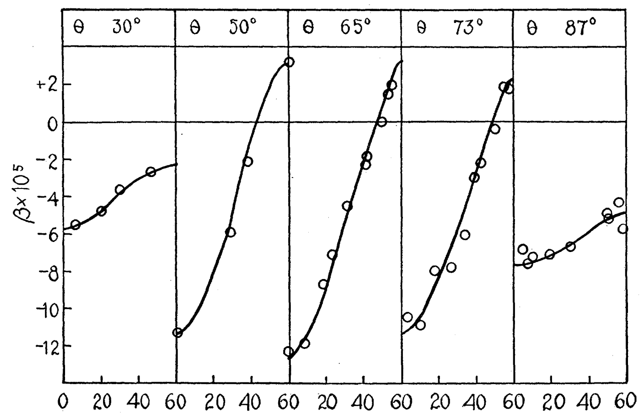


Fig. 5.

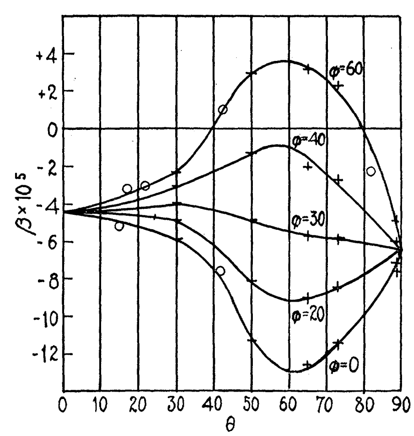


Fig. 6. Plus signs indicate points interpolated from Fig. 5; circles indicate additional points given directly by experiment.

e.g., one crystal defined by $\theta=63°$ and $\phi=9°$ gave β as -11.82×10^{-5} while another with $\theta=69°$ and $\phi=9°$ gave -11.83×10^{-5}. The angle ϕ was not determined to better than 5° and θ was uncertain to half that amount; hence this agreement is satisfactory.

The dependence of the tension coefficient β on θ and ϕ may be plotted in two ways: (1) giving β as a function of ϕ with θ constant, Fig. 5; and (2) giving β as a function of θ with ϕ constant, Fig. 6. In Fig. 5, thirty-eight of the forty-five experimental points were plotted in five groups, each group containing the points for which θ was approximately that of the group. The 50° group, for instance, was composed of four observations, for $\theta = 50°$, 54°, 50° and 50°. There was an even greater divergence in values for the other groups, which gives one good reason why the points do not lie directly on a smooth curve; the 65° group, for instance, includes orientations of 60° to 69°, although the greater number of observations are nearer the mean value. Seven observations lay too far from any one of the group values to be included in Fig. 5, but six of these, indicated by circles, fell readily into Fig. 6. Except for these points, Fig. 6 was plotted by interpolation from Fig. 5. One point for $\theta = 41°$ and $\phi = 47°$ could not be fitted into either figure, but when its value of $+0.35 \times 10^{-5}$ is considered in the light of Fig. 6 it is seen to be fairly

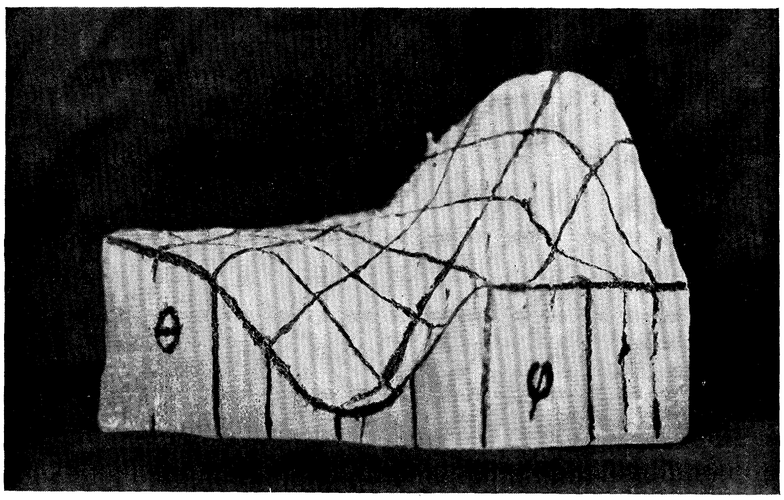


Fig. 7. Three-dimensional model of the variation of β with θ and ϕ. θ increases from left to right from 0° to 90°, intermediate lines being drawn every 15°. ϕ increases from front to back from 0° to 60°, intermediate lines being drawn every 10°. It is to be noted that the intersection of the surface with the plane $\theta = 90°$ gives a very nearly horizontal line and so leads to the conclusion that β (represented by the vertical heights of the model) is independent of ϕ for $\theta = 90°$.

consistent with the curves shown there. Since in plotting the results separately with respect to θ and ϕ, only approximate orientations could be used for each point, it was deemed desirable to plot the coefficient directly as a function of the two variables in the form of a solid model, Fig. 7. This showed nothing very different from the information given by Figs. 5 and 6, although it seemed to point more clearly to the probability that the coefficient is completely independent of the secondary orientation for $\theta = 90°$.

In studying Fig. 5, which gives the dependence on ϕ of the tension coefficient of resistance for crystals with θ constant, it is immediately seen that the coefficient β has trigonal symmetry since, with ϕ and $120 - \phi$ indistinguishable, the curves must be symmetrical about 60° and return to the 0° values at 120°. Furthermore, β depends very markedly on the secondary orientation ϕ. For instance in the case of $\theta=65°$, it varies from -12.6×10^{-5} at $\phi=0°$ to $+3.5\times10^{-5}$ at $\phi=60°$. This variation is interesting, both as regards magnitude and sign. Bismuth is abnormal in that the tension coefficient for the polycrystalline form is negative, but as shown in Fig. 5 for certain orientations of single crystals it becomes positive. It is further to be noted that when θ is close to 0° or to 90°, β varies less with the orientation of the secondary cleavage plane. At small angles of θ, this is to be expected, for in the limiting case of $\theta=0$, the elliptical cross section becomes circular so that ϕ is entirely indeterminate. Or considering it from a physical point of view, in this case the tension is applied perpendicular to the principal cleavage plane and so does not deform the elementary triangles or hexagons formed on the surface by the intersections of the secondary with the principal cleavage planes; thus the same change of symmetry will occur irrespective of the orientation of the secondary planes. Thus symmetry demands that for $\theta=0°$ β be independent of ϕ. For $\theta=90°$ it is more difficult to understand why the effect should not depend on the angle ϕ, since symmetry does not here demand any such independence of ϕ. In this case the elliptical cross section has become indefinitely elongated and consequently has the well-defined direction of the axis of the cylinder. It would appear that the tension would change the symmetry of the elementary triangles or hexagons differently depending on their position relative to the axis of the cylinder. Experimentally, however, there is found little or no variation of the coefficient with ϕ when the principal cleavage plane is parallel to the axis of the crystal. The evidence, although not conclusive, is in the direction that the coefficient is independent of ϕ at $\theta=90°$. This appears more clearly in Figs. 6 and 7. The points are not sufficiently numerous at either $\theta=0°$ or 90° to determine the limiting values with a high degree of accuracy. However, there seems to be little doubt that at $\theta=0$, β is somewhat less negative than it is at $\theta=90°$. There is some evidence from these curves that the tangent to these curves becomes horizontal at $\theta=60°$; in most cases this means a minimum or maximum, but for $\phi=30°$ it would seem to indicate a point of inflection only. The rapid variation of β with θ near 90° explains the comparatively wide scattering of the points in Fig. 5 for $\theta=87°$.

The fact that the tension coefficient varies so much with the orientation of the crystal may explain the discordance of the various values of β found for polycrystalline bismuth by different observers. Table I gives the various

TABLE I.

Williams[7]	-5.35×10^{-5} per kg/cm^2
Zavattiero[8]	-4.25 "
	-3.30 "
Bridgman[9]	-4.66 "
	-2.92 "
Rolnick[10]	-2.81 "

[7] W. Ellis Williams, Phil. Mag. **13**, 635 (1907).
[8] E. Zavattiero, Rend. Accad. Lincei **29**, (1), 48 (1920).
[9] P. W. Bridgman, Proc. Amer. Acad. **57**, 41 (1922).
[10] Harry Rolnick, Phys. Rev. **35**, 506 (1930).

values found. The discrepancies are far greater than can be accounted for by experimental error. It is reasonable to assume that the procedure of each of these observers in producing the specimen of bismuth was such as to emphasize different crystalline orientations in the polycrystalline pieces; such an assumption would be sufficient to explain the differences observed.

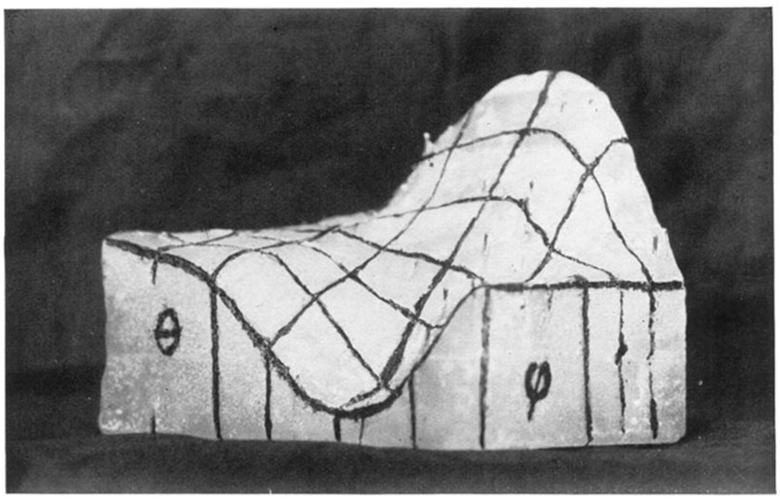
The hydrostatic pressure coefficient for bismuth single crystals[1] is for $\theta=0°+2.45\times10^{-5}$ and for $\theta=90°+7.5\times10^{-5}$; these are of the same order of magnitude as the tension coefficients found here. The sign of the pressure coefficient is positive and hence abnormal for pressure, in the same way that a negative coefficient is abnormal for a tension effect.

Conclusion

In this study of the tension coefficient of resistance in single bismuth crystals at 30°C, where the tension has been applied parallel to the axis of the cylinder in which the crystal is cast and along which the current flows, the coefficient has been found to depend both on the orientation of the principal cleavage plane and of the secondary cleavage planes with respect to the axis of this cylinder. For $\theta=0°$ and $\theta=90°$, this coefficient is apparently very little dependent on the orientation of the secondary cleavage planes, whereas for θ close to 60° its dependence on the orientation of the secondary cleavage planes becomes a maximum. This variation involves a change in sign as well as in magnitude, so that for certain orientations the coefficient becomes positive instead of remaining negative. The coefficient shows trigonal symmetry, as it must if it is to be consistent with the corporeal trigonal symmetry of the bismuth crystal.

This paper has been successful, therefore, in finding empirical relations between the tension coefficient of resistance and the orientations of the principal and secondary cleavage planes. To make the solution of the problem complete, a physical theory should be formulated to predict the relations found. However, progress would be made if a formal geometrical theory could be established which would permit the results to be expressed in terms of the parameters θ and ϕ and of three or more unknown but determinable constants. Further experimental work is planned in this field with other crystals of the same type of symmetry, of different types of symmetry, and possibly at radically different temperatures.

It is a pleasure to thank the Director of the laboratory and the authorities of Harvard University for the privilege of working in the Research Laboratory of Physics, and Professor P. W. Bridgman for the suggestion of the problem itself and also of practical details involved in carrying the work to completion.



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