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The Measurement of X-Ray Emission Wave-Lengths by Means of  
the Ruled Grating

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With a plane ruled grating, measurements have been made of the emission wave-lengths in the  $L$  series for the elements Va (23) to Zn (30) and in the  $K$  series for the elements C (6) to Si (14). With the exception of the  $K\alpha$  lines of carbon, fluorine and oxygen, the method of the ruled grating yields a longer wave-length in every case than that found for the corresponding lines by the crystal method. The Moseley diagrams representing the observed results are found to be smooth curves. The possible variation of apparent wave-length with a variation in the grazing angle of incidence has been investigated. For angles such that the diffracted lines were not too diffuse, the computed wave-lengths were independent of the incident angle.

INTRODUCTION

SEVERAL investigations have now been carried out to determine the wave-lengths of x-ray spectral lines by means of ruled gratings. In certain of these investigations measurements have been made relative to some particular emission line whose wave-length assumed as a standard has been measured by the Bragg method with a crystal. In other experiments<sup>1,2,3,4</sup> the wave-lengths have been determined directly from the measurable constants of the apparatus. The results for the wave-lengths so far obtained by this latter method are found in every case to be greater than the corresponding values found by means of the usual crystal method. Disagreement exists only in the magnitude of the discrepancy.

The apparent existence of this difference raises a very important question. If the measurements by means of crystals are correct then there is some defect in applying to the ruled grating in the case of x-rays the same theory that is found satisfactory in the visible and the ultraviolet spectral regions. If the measurements by means of the ruled grating are correct then there is something in error in calculations involving the Bragg law, other than the effect due to the ordinary refraction of x-rays in the crystal, which can be accounted for. If this error exists in the lattice constant  $d$  then it reflects upon our knowledge of certain other more fundamental constants, namely

<sup>1</sup> E. Bäcklin, *Dissertation*, Upsala, 1928.

<sup>2</sup> J. A. Bearden, Proc. Nat. Acad. Sci. **15**, 528 (1929); Phys. Rev. **37**, 1210 (1931).

<sup>3</sup> J. M. Cork, Phys. Rev. **35**, 1456 (1930).

<sup>4</sup> C. E. Howe, Phys. Rev. **35**, 717 (1930).

the number of particles per gram molecule  $N$  or the charge of the electron  $e$  from which  $N$  is derived.

The answer to this question can be given only after the accumulation of extensive data obtained with the utmost care. This must include measurements of the indices of refraction of certain crystals for various wave-lengths as well as the determination of the wave-lengths themselves by the different methods. Toward this end the present investigation was undertaken to determine as accurately as possible the wave-lengths of the emission lines in the  $K$  series for elements Si(14) to C(6) and in the  $L$  series for elements Zn(30) to Va(23).

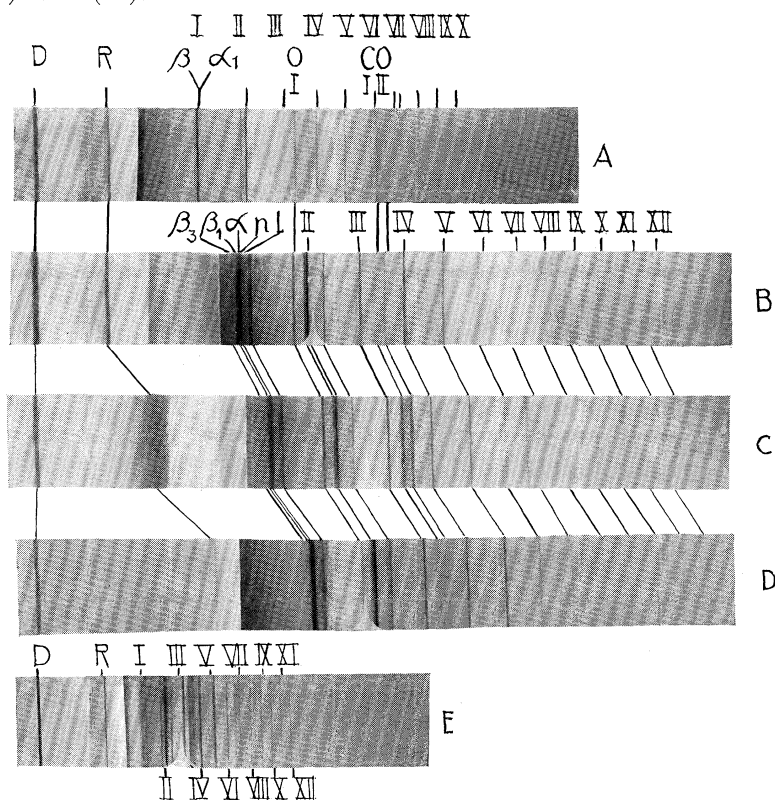


Fig. 1. Spectrograms with ruled gratings. (A)  $K$  lines of silicon,  $\theta = 30' 26''$ ; (B, C and D)  $L$  lines of copper for  $\theta = 30' 26''$ ,  $\theta = 51' 20''$  and  $\theta = 1^\circ 16' 5''$ , respectively; (E) same as (D) with photographic plate close to grating.

#### APPARATUS

The apparatus employed is similar to that used previously by one of us<sup>3</sup> in the determination of the wave-lengths of the  $L$  series lines of molybdenum. It is believed that this arrangement which makes use of outside spectral orders and employs a grating which when once properly adjusted is left absolutely fixed during a series of exposures is least subject to errors of manipulation.

The following changes in the apparatus were made. The glass grating employed in this investigation was ruled in the laboratory of Professor M. Siegbahn and obtained through his kindness. The active surface was 3 mm wide and the lines were 2 cm long. Nominally having 600 lines per mm a calibration with known wave-lengths in the arc spectrum of mercury gave for the distance  $d$  between lines ( $d = 0.00166309$  mm). The glass surface between the rulings and the outside of the plate was etched down on one side so that the direct beam going to the photographic plate was limited on one side by the edge of the ruled surface. The collimating system consisted of parallel slits 0.05 mm wide spaced 20 cm apart. A water-cooled x-ray tube of metal was used in place of the glass tube. A very fine knife edge was placed parallel and close to the rulings of the grating at the center. This reduced materially the stray radiation and general blackening on the photographic plate.

The procedure in taking exposures with the photographic plate at different distances from the grating was followed as previously described. For any particular substance on the target at least five exposures were taken with the photographic plate in a position  $a$  close to the grating and then in the position  $d$  at a maximum distance from the grating. Reproductions of some typical spectrograms are given in Fig. 1. At least six and sometimes as many as twelve measurable orders were obtained on each photogram.

## RESULTS

A summary of the results obtained for the wave-lengths in the  $K$  series for the elements investigated is given in Table I. Similarly in Table II are given the values obtained for the wave-lengths in the  $L$  series. Along with the results of this investigation are shown the corresponding values from experiments using the crystal method as well as those of other experimenters

TABLE I. Summary of wave-lengths in the  $K$  series.

Element		Crystal	Söderman <sup>5</sup>	Thibaud <sup>6</sup>	Witmer and Cork
Si	$\beta$	6.754	—	—	6.764
	$\alpha$	7.110	—	—	7.121
Al	$\beta$	7.965	—	—	7.967
	$\alpha$	8.320	—	—	8.334
Mg	$\beta$	9.539	—	—	9.558
	$\alpha$	9.869	—	—	9.898
Na	$\alpha$	11.885	11.88	—	11.910
F	$\alpha$	18.37	18.275	—	18.354
O	$\alpha$	23.73	23.567	23.80	23.644
C	$\alpha$	45.50	44.54	44.90	44.448

with the ruled grating. In the  $L$  series for the elements mentioned here, measurements by the crystal method have been carried out by Siegbahn and Thoriaeus<sup>7,8</sup> and by Karlsson.<sup>9</sup> The values from these investigations are

<sup>5</sup> M. Söderman, *Phil. Mag.* **10**, 600 (1930).

<sup>6</sup> J. Thibaud, *J. Opt. Soc. Am.* **17**, 145 (1928).

<sup>7</sup> M. Siegbahn and R. Thoriaeus, *Ark. Mat. Astr. O. Fys.* **18**, 24 (1924).

<sup>8</sup> R. Thoriaeus, *Phil. Mag.* **1**, 312 (1926); **2**, 1007 (1926).

TABLE II. Summary of wave-lengths of emission lines in the *L* series.

Element	$\beta_{3,4}$	$\beta_1$	$\alpha_{1,2}$	$\eta$	$l$	Investigation
Zn 30	11.163	11.96	12.23	13.61	13.97	Ka Crystal
	11.16	11.96	12.23	13.61	13.95	S Crystal
	—	11.96	12.25	—	14.02	Ho
	11.202	11.974	12.252	13.670	14.038	Present
Cu 29	12.07	13.027	13.306	14.87	15.26	Ka Crystal
	12.10	13.03	13.316	14.83	15.19	S Crystal
	—	—	13.60	—	—	Hu
	—	—	13.32	—	15.26	K
	—	—	13.37	14.95	15.33	Ho
	12.091	13.048	13.339	14.910	15.266	Present
Ni 28	13.14	14.25	14.53	16.28	16.66	Ka Crystal
	13.12	14.24	14.53	16.17	16.55	S Crystal
	—	—	14.51	—	—	K
	—	14.28	14.62	16.36	16.73	Ho
	—	14.255	14.556	16.271	16.660	Present
Co 27	14.24	15.63	15.94	17.86	18.25	Ka Crystal
	—	15.62	15.94	17.77	18.20	S Crystal
	—	—	15.94	—	18.28	K
	—	15.64	15.99	—	18.34	Ho
	—	15.690	15.990	—	18.321	Present
Fe 26	15.71	17.23	17.58	19.76	20.09	Ka Crystal
	15.61	17.22	17.58	19.65	20.12	S Crystal
	—	—	17.70	19.60	20.10	Th
	—	—	18.00	—	—	Hu
	—	—	17.61	—	—	B
	—	—	17.54	—	20.15	K
	—	17.29	17.66	—	20.25	Ho
	—	17.281	17.602	—	20.161	Present
Mn 25	—	19.04	19.40	—	—	Ka Crystal
	—	19.04	19.39	—	—	S Crystal
	—	19.17	19.55	—	22.34	Ho
	—	19.136	19.454	—	22.296	Present
Cr 24	—	21.19	21.53	23.28	23.84	Ka Crystal
	—	21.19	21.53	—	—	S Crystal
	—	—	21.74	—	24.73	K
	—	—	21.73	—	—	Ho
	—	21.279	21.670	—	24.788	Present
Va 23	—	—	24.20	—	—	Ka Crystal
	—	—	24.31	—	27.70	Ho
	—	—	24.215	—	27.78	Present

Ka, Karlsson, reference 9; S, Siegbahn and Thoraues, references 7 and 8; Ho, Howe, reference 4; Hu, E. L. Hunt, Phys. Rev. 30, 227 (1927); K, G. Kellstrom, Zeits. f. Physik 58, 511 (1929); Th, Thibaud, reference 6.

recorded in Table II. For the wave-lengths of certain  $\eta$  and  $l$  lines the agreement between the two crystal determinations is rather unsatisfactory. It is, however, apparent that in every case with the exception of the  $K\alpha$  lines for fluorine, oxygen and carbon the value of a particular wave-length by the grating method is greater than the corresponding value found by the crystal method. The discrepancy for the lines in the *L* series is with few exceptions found to be not as great as was observed by Howe.<sup>4</sup>

<sup>4</sup> A. Karlsson, Ark. Mat. Ast. O. Fys. 22, 9 (1930).

The Moseley diagrams for the various spectral lines observed, in which the square root of the frequency  $(\nu/R)^{1/2}$  is plotted against the atomic number, are remarkably smooth curves. No positions are observed at which the slope of a line changes abruptly as might have been expected from the consideration of the origin of the spectral lines.

Hanawalt and Prins<sup>10</sup> have reported an apparent variation in the wave-length of a particular line with a change in the incident angle  $\theta$ . The existence of such an effect would render uncertain any measurements made by the grating method and this effect was therefore investigated with considerable care. A complete series of spectrograms (i.e., five plates in each of the  $a$  and  $d$  positions) were obtained for the copper  $L$  lines with each of the following values for the grazing angle of incidence  $\theta$ :—26'33'', 30'26'', 39'36'', 51'20'' and 1°16'15''. Reproductions of three of these spectrograms are shown in Fig. 1. For any particular wave-length the smaller the value of the grazing angle of incidence  $\theta$ , the greater is the angular dispersion. This variation is not great. One very apparent effect of using a large incident angle is a broadening of any diffracted line.

TABLE III. *The wave-length of the Cu  $K\alpha_{1,2}$  line for different incident angles.*

Grazing angle $\theta$	$\lambda$ in angstrom units
26'33''	13.339
30'26''	13.339
39'36''	13.335
51'20''	13.342
1°16' 5''	13.368

The wave-length of the  $L\alpha_{1,2}$  spectral line of copper as computed for the various values of  $\theta$  is shown in Table III. For the first four angles the agreement is well within the experimental error. It is believed that the discordant value for the maximum  $\theta$  is due to an inaccuracy in the calculation caused by the increased width of the diffracted lines.

Measurements by the crystal method of long wave-lengths are necessarily subject to inaccuracies due to the refraction of x-rays in the crystal. In the region of an absorption frequency characteristic of the atoms of the crystal there is an anomalous refractive effect which influences substantially results obtained from the Bragg law. Thus in crystals of gypsum, lauric acid or palmitic acid as used in investigating wave-lengths mentioned in this paper the calculation is complicated by the  $K$  absorption frequencies for carbon, oxygen, sulphur and calcium. Failure to consider this effect for the longer wave-length  $K$  series lines together with the uncertainty in the lattice constant of crystals of the fatty acids may account for the unsystematic discrepancy between measurements by the ruled grating and the crystal.

Influenced by calculations of wave-lengths from measurements of the index of refraction of a quartz crystal, Bearden<sup>11</sup> has indicated a lack of faith

<sup>10</sup> J. D. Hanawalt and J. A. Prins, *Physica* **12**, 1 (1932).

<sup>11</sup> J. A. Bearden, *Phys. Rev.* **39**, 1 (1932); **40**, 471 (1932).

in measurements made by the ruled grating. These calculations with the index of refraction demand a knowledge of: (a) the density of the crystal; (b) the absolute value of  $e/m$  for the electron; (c) a factor characteristic of the refracting crystal which is constant for a particular wave-length but which itself demands a knowledge of the wave-length in its determination and (d) an exact evaluation of the index of refraction exhibited by the crystal for the particular wave-length being investigated. If the evident uncertainties in (b) and (c) be overlooked it would still seem that the very short distances to be measured between the relatively diffuse lines on the photographic plates in the experimental procedure of (d) must render this method impotent in expressing a preference between the other two methods.

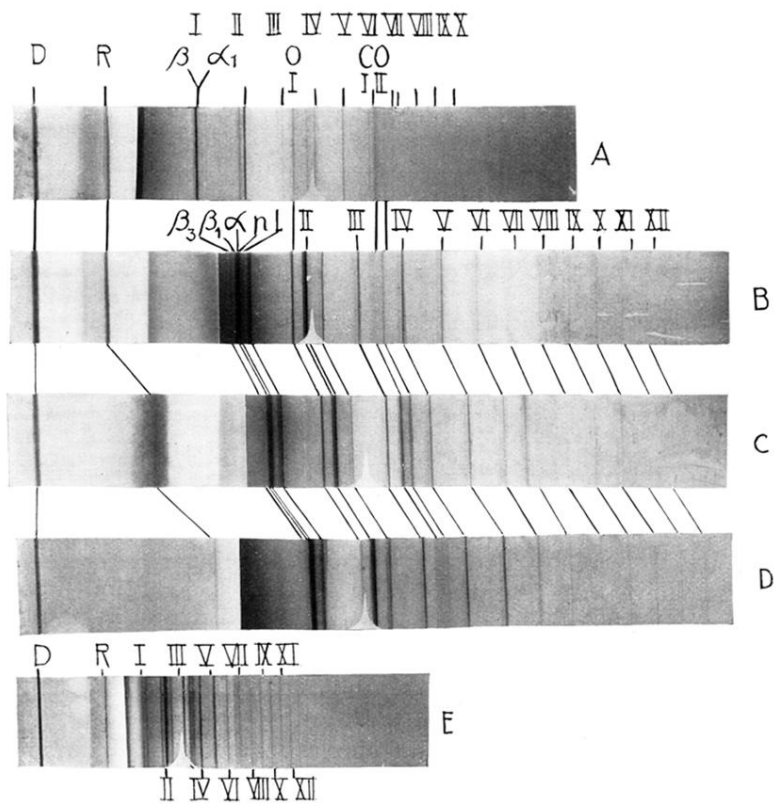


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