# Some Properties of Homogeneously Distorted Cubic Ferromagnetic Lattices

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The work of Becker and Kersten is amplified in that, starting with a convenient expression for the energy of a magnetically saturated and homogeneously distorted cubic ferromagnetic lattice, the problem is treated with especial reference to the crystallographic symmetry, rather than with the assumption of an isotropic medium, as was done by the above named authors. Application is made of the foregoing to the magnetostriction and magnetization of samples of iron, nickel, and their alloys under tension and compression. The agreement with experiment is qualitatively satisfactory. Further experimental data are needed for a quantitative check. Finally, there is a brief discussion of the effect of magnetization on elastic properties and of hysteresis, and a few important problems are listed.

In two very interesting papers by Becker,[1] and Becker and Kersten,[2] the magnetic properties of distorted lattices are discussed. These authors assume an expression for the energy of the lattice that is a linear function of the tensor components representing the distortion, and a quadratic function of the direction cosines of the direction of magnetization. In applying this expression, however, they use an approximation which eliminates the symmetry of the crystal present in the original expression for the energy. It is the purpose of this paper to carry out the various calculations without making such simplifying approximations.

## Assumptions

Although the models of Ewing and Honda have been very useful in the development of ideas about ferromagnetism, recent advances indicate that it is time to examine them critically. It seems desirable, to a certain extent, to get away from such unobservable quantities as the individual magnetic moments of the various atoms in a crystal. Indeed, since wave mechanics has taught us to treat the electric charge surrounding an atom as a continuum, it is only logical to treat the magnetization of a solid as a property of the electric density which occupies the entire space surrounding the nuclei. This indicates that we have to deal, not with an aggregate of dipoles, but with a vector field. We may expect to derive the laws governing the behavior of this vector field from general quantum-mechanical principles, but since the attempts so far made have not been entirely successful, it may be worth trying to formulate them independently, with reference to experimental results only. The first of these laws relates to saturation phenomena. It gives the energy of

[1] R. Becker, Zeits. f. Physik **62,** 253 (1930).
[2] R. Becker and M. Kersten, Zeits. f. Physik **64,** 660 (1930).

a saturated ferromagnetic lattice as a function of the direction of magnetization. This part of the problem has been attacked from an atomic point of view by Heisenberg,[3] who discusses the existence of magnetization of the type under discussion, and Bloch and Gentile[4] who take up the orientation in the crystal lattice. An attempt to establish a corrective term applicable to distorted lattices forms the body of this paper. In addition to these laws governing the orientation of the vector $\boldsymbol{I}$, we need to know how $\boldsymbol{I}$ changes in magnitude. The most generally accepted suggestion is that $\boldsymbol{I}$ does not change in magnitude at all, if we measure it in sufficiently small volumes, but is always equal to $\boldsymbol{I}_w$, the Weiss spontaneous magnetization. These small volumes must, nevertheless, contain a large number of atoms. In other words, a crystal is divided into small regions whose directions of magnetization are determined by a probability function. This model surely contains some truth, but has not been very useful in explaining the detail of demagnetization, perhaps because other phenomena, such as a periodic reversal of the direction of magnetization,[5] for instance, obscure its implications. An interesting alternative possibility is to assume that in this problem, as in the case of other vector fields, two separate treatments are called for, corresponding to geometrical optics on the one hand and wave optics on the other. In fact, in view of the general orderliness of nature on a small scale, as contrasted with the general disorder found on a large scale, one might almost suspect that all polarization and diffusion vector fields, when examined in detail, would reveal a wave structure rather than random fluctuations. However that may be, experimental evidence has recently been found, which shows that the magnetization of a single crystal is not uniform, that inhomogeneities exist, and are arranged according to well-defined geometrical patterns.[6] Therefore on purely empirical grounds we may expect a wave equation to govern the intensity of magnetization, that is, one whose solutions are some sort of oscillating functions. The existence of such an equation will then somehow have to be reconciled to the existence of spontaneous magnetization as postulated by Weiss and Heisenberg.

The above discussion has been introduced in order to emphasize two points. (1) The magnetic properties of a saturated lattice are representable by a function derivable from symmetry considerations and containing a few constants which it is the business of atomic theory to interpret. (2) Whenever other than saturated lattices are discussed, further assumptions must be made. The simplest and most convenient for our purposes are given below.

We shall assume that when the energy of a saturated lattice is less for magnetization along some one direction than for any other direction, then the lattice will actually be magnetically saturated in this one direction of minimum energy. Further, when the energy of a saturated lattice is less for magnetization in $n$ specified directions than in any other directions, then the

[3] W. Heisenberg, Zeits. f. Physik **49**, 619 (1928).

[4] F. Bloch and G. Gentile, Zeits. f. Physik **70**, 395 (1931).

[5] P. S. Epstein, Phys. Rev. **41**, 91 (1932).

[6] F. Bitter, Phys. Rev. **38**, 1903 (1931); **41**, 507 (1932).

lattice behaves as if a fraction $1/n$ of its total volume were magnetized to saturation in each of the $n$ directions. The justification for these assumptions is that they are very simple to handle, and that to a first approximation they have been found to represent certain facts very well,[7] and seem to be adequate here also. As a rule, however, they fall down when applied to problems in which two or more directions have almost equal energies, or when hysteresis is important.

We shall further assume that the energy $E_\theta'$ of a *perfect* cubic lattice insofar as it depends on the direction of magnetization may be written

$$E_\theta' = E_1' + E_2' + E_3 \tag{1}$$

$$E_1' = \text{const.} + c' \sum{}' \alpha_i^2 \alpha_j^2 \tag{2}$$

$$E_2' = \text{const.} + K_1 \sum B_{ii}\alpha_i^2 + K_2 \sum{}' B_{ij}\alpha_i\alpha_j \tag{3}$$

$$E_3 = -\boldsymbol{I}_w \cdot \boldsymbol{H}. \tag{4}$$

Here $E_1'$ refers to the undistorted cubic lattice, and results from the spin-orbit coupling.[4] The tensor components $B_{ij}$ give the position $x'$, $y'$, $z'$ of a point after distortion in terms of its coordinates before the distortion.

$$x' = (B_{ii} + 1)x + B_{ij}y + B_{ik}z, \text{ etc.} \tag{5}$$

The quantities $\alpha_i$ are the direction cosines of the magnetization along the $i$, $j$, $k$ axes which are assumed parallel to the tetragonal axes of the crystal. $E_3$ is the energy component due to the external field $H$. The summations extend over all values of $i, j, k$, the primed summation indicating $i \neq j$, etc. The constants include all terms independent of $\alpha_i$, etc.

These equations are inconvenient because they refer to an ideal cubic lattice which is not experimentally available. In order to correct this we must calculate the equilibrium configuration of the lattice under no external forces, and use this as our starting point.

## Lattice under no External Forces

Let us write $B_{ij} = A_{ij} + C_{ij}$, where the quantitites $B_{ij}$ refer to total distortions measured from the original cubic form of the lattice, $C_{ij}$ are the distortions produced in the lattice by internal forces, and $A_{ij}$ are the distortions produced by external forces alone, measured from the equilibrium configuration of the lattice. We can write

$$E_2' = \text{const.} + K_1 \sum A_{ii}\alpha_i^2 + K_2 \sum{}' A_{ij}\alpha_i\alpha_j + K_1 \sum C_{ii}\alpha_i^2 + K_2 \sum{}' C_{ij}\alpha_i\alpha_j.$$

In addition we put for the elastic energy of distortion[8]

[7] A review is contained in F. Bitter, Phys. Rev. **39**, 337, 371 (1932).

[8] This is the expression used by Becker[1] and is correct for isotropic media. In general, three elastic constants are required to describe cubic crystals. The energy is (Love, *Math. Theory of Elasticity*, page 158)

$$E_4 = (c_{11}/2)\sum C_{ii}^2 + (c_{12}/2)\sum{}' C_{ii}C_{jj} + c_{44}\sum{}' C_{ij}^2$$

which reduces to the above for $2c_{44} = c_{11} - c_{12}$. The use of this complete expression does not alter the form of Eqs. (7) or (11), but does alter the relationships 10 and 12 to

$$c = c' + K_1^2/2(c_{11} - c_{12}) - K_2^2/4c_{44};\ \chi_1 = -K_1/(c_{11} - c_{12});\ \chi_2 = -K_2/2c_{44}.$$

$$E_4 = \tfrac{1}{2}\lambda[\sum C_{ii}]^2 + G\sum C_{ij}^2,$$

$G$ being the modulus of shear, or rigidity, and $(\lambda + 2G/3)$ the modulus of compression of the (non-magnetic) cubic lattice. The quantities $C_{ij}$ are then so determined that in the absence of external forces $(A_{ij}=0)$

$$(\partial/\partial C_{ij})(E_2' + E_4) = 0.$$

Carrying this out, we obtain

$$\sum C_{ii} = \text{const.}$$
$$C_{ii} = \text{const.} - (K_1/2G)\alpha_i^2$$
$$C_{ij} = \text{const.} - (K_2/2G)\alpha_i\alpha_j$$

and, remembering that

$$\sum\alpha_i^4 = [\sum\alpha_i^2]^2 - \sum{}'\alpha_i^2\alpha_j^2$$

we obtain

$$E_4 = \text{const.} + [(K_2^2 - K_1^2)/4G]\sum{}'\alpha_i^2\alpha_j^2$$
$$E_2' = f(A_{ij}) + [(K_1^2 - K_2^2)/2G]\sum{}'\alpha_i^2\alpha_j^2.$$

Consequently, putting $E_\theta = E_\theta' + E_4$ and lumping all the terms independent of the direction of magnetization into a single constant, we obtain

$$E_\theta = E_1 + E_2 + E_3 + \text{const.} \tag{6}$$

where

$$E_1 = c\sum{}'\alpha_i^2\alpha_j^2 \tag{7}$$
$$E_2 = K_1\sum A_{ii}\alpha_i^2 + K_2\sum{}'A_{ij}\alpha_i\alpha_j \tag{8}$$
$$E_3 = \boldsymbol{I}_w\cdot\boldsymbol{H} \tag{9}$$
$$c = c' + (K_1^2 - K_2^2)/4G \tag{10}$$

where $E_1$ is the energy of the lattice including magnetostrictive strains, and where the $A_{ij}$ are measured from that configuration of the lattice in which it is in equilibrium with itself. Further, the magnetostriction given by the tensor $C_{ij}$ is more conveniently expressed by the formula for the change in length per unit length in the direction $\beta_i$, $\beta_j$, $\beta_k$

$$\delta l/l = \chi_0 + \chi_1\sum\alpha_i^2\beta_i^2 + \chi_2\sum{}'\alpha_i\alpha_j\beta_i\beta_j \tag{11}$$

which can be derived by noticing that

$$\delta l/l = \sum C_{ii}\beta_i^2 + \sum{}'C_{ij}\beta_i\beta_j + \text{const.}$$

and substituting the values of $C_{ij}$ found above. On doing this, one obtains Eq. (11) with

$$\chi_1 = -K_1/2G;\ \chi_2 = -K_2/2G. \tag{12}$$

The expression for $\delta l/l$ gives the difference in length between the final magnetically saturated state and the initial cubic state. Since this initial condition cannot be realized experimentally, we must choose some other cubic condition as our starting point and correct the above expression by choosing $\chi_0$ to fit experimental observations, instead of using for it that function of $\lambda$, $G$, etc., which results from the foregoing calculation. It is convenient to use as our reference configuration one with no applied field $H$, in which the sample is completely demagnetized. For the present we need not specify further what this demagnetized condition is. It should be noticed that formula (11) may be checked without any special assumptions regarding $\chi_0$ by measuring the difference in magnetostriction between various directions of magnetization and observation. $\chi_0$ will then drop out.

Insofar as the foregoing considerations are correct, Eqs. (6) to (12) should describe the behavior of saturated crystals. There appear five constants, $I_w$, $c'$, $K_1$, $K_2$, and $G$ which are to be interpreted by quantum theory. This has been attempted for the first two only.

## Evaluation of $E_2$

In the following we shall not be concerned with the elastic properties of crystals, but confine ourselves to the description of strains, and moreover to extensions with transverse contractions. The tensor components have been calculated for the following cases.

Extension parallel to [100] axis

$$A_{ii} = A;\ A_{jj} = A_{kk} = -\mu A;\ A_{ij} = A_{jk} \cdots = 0.$$

Extension parallel to [110] axis

$$A_{ii} = A_{jj} = [(1-\mu)/2]A;\ A_{kk} = -\mu A;$$
$$A_{ij} = A_{ji} = [(1+\mu)/2]A;\ A_{jk} = A_{ik} = 0.$$

Extension parallel to [111] axis

$$A_{ii} = A_{jj} = A_{kk} = [(1-2\mu)/3]A;\ A_{ij} = A_{jk} = \cdots = [(1+\mu)/3]A.$$

$A$ measures the extension, and $\mu$ determines the extent of the accompanying change in volume. In the ensuing formulae additive constants are neglected.

### $E_2$ for extension parallel to [100] axis

Substituting the values found above into Eq. (8) and putting $\alpha_i = \cos\theta$; $\alpha_j = \sin\theta\sin\phi$; $\alpha_k = \sin\theta\cos\phi$ one obtains

$$E_2 = [(1+\mu)/2]K_1 A\cos 2\theta, \tag{13}$$

indicating that the energy is independent of the orientation of $I_w$ in the plane normal to the extension, and is a maximum in the direction of extension if $K_1 > 0$.

### $E_2$ for extension parallel to [110] axis

Substituting the appropriate values of $A_{ij}$ into Eq. (8) and putting $\alpha_i = \sin\theta\sin\phi$; $\alpha_j = \sin\theta\cos\phi$; $\alpha_k = \cos\theta$ we obtain

$$E_2 = K_1A[(1-\mu)/2]\,[\sin^2\theta - \mu\cos^2\theta] + K_2A(1+\mu)\sin^2\theta\sin\phi\cos\phi. \quad (14)$$

Putting $\theta=\pi/2$, this gives for the (001) plane, which contains the direction of extension

$$E_2 = [(1+\mu)/2]K_2A\sin 2\phi$$

which expression has a maximum for $\phi=\pi/4$, or in the direction of extension, if $K_2>0$. However, by putting $\phi=\pi/4$, this gives for the $(1\bar{1}0)$ plane which also contains the direction of extension,

$$E_2 = -[(1+\mu)/4](K_1+K_2)A\cos 2\theta$$

or for the (110) plane perpendicular to the direction of extension, for which $\phi=-\pi/4$

$$E_2 = -[(1+\mu)/4](K_1-K_2)A\cos 2\theta$$

which shows that the energy is not independent of the orientation of $I_w$ in the plane normal to the extension unless $K_1=K_2$. Further, $E_2$ will have its maximum or minimum value in the direction of extension as long as $K_1+K_2$ has the same sign as $K_2$. But if $K_1+K_2$ has the opposite sign of $K_2$, then both maximum and minimum of $E_2$ will lie in a plane perpendicular to the extension. This latter case is of considerable importance in describing the Villari reversal in iron, as discussed further on.

**$E_2$ for extension parallel to [111] axis**

Substituting the appropriate values of $A_{ij}$ into Eq. (8), and writing for the angle $\theta$ between the direction $\alpha_i$, $\alpha_j$, $\alpha_k$ and the [111] axis whose direction cosines are $3^{-1/2}$,

$$\cos\theta = 3^{-1/2}(\alpha_i+\alpha_j+\alpha_k)$$

we obtain

$$E_2 = [(1+\mu)/2]K_2A\cos 2\theta,$$

an expression similar to that for extension along the tetragonal axes, except that $K_2$ replaces $K_1$.

## Magnetostriction

The formula given in Eq. (11) expresses a complicated relationship between the parallel and transverse components of magnetostriction in their dependence on the direction of magnetization in the crystal. Existing data are not sufficiently reliable for a satisfactory quantitative check, but are perhaps sufficient for a rough estimate of $\chi_1$ and $\chi_2$ for iron and nickel. From Eq. (11) we find the following values of $\delta l/l$. We shall evaluate these con-

Table I. *Table of theoretical magnetostrictions.*

| Case | Direction of magnetization | Direction of observation | $\delta l/l$ |
|---|---|---|---|
| *a* | [100] axis; $\alpha_i=1$, $\alpha_j=0$, $\alpha_k=0$ | $\beta_i=1$, $\beta_j=0$, $\beta_k=0$ | $\chi_0+\chi_1$ |
| *b* | " " " " | $\beta_i=0$, $\beta_j=1$, $\beta_k=0$ | $\chi_0$ |
| *c* | " " " " | $\beta_i=0$, $\beta_j=2^{-1/2}$, $\beta_j=2^{-1/2}$ | $\chi_0$ |
| *d* | [110] axis; $\alpha_i=2^{-1/2}$, $\alpha_j=2^{-1/2}$, $\alpha_k=0$ | $\beta_i=2^{-1/2}$, $\beta_j=2^{-1/2}$, $\beta_k=0$ | $\chi_0+\frac{1}{2}(\chi_1+\chi_2)$ |
| *e* | " " " " | $\beta_i=2^{-1/2}$, $\beta_j=-2^{-1/2}$, $\beta_k=0$ | $\chi_0+\frac{1}{2}(\chi_1-\chi_2)$ |
| *f* | " " " " | $\beta_i=0$, $\beta_j=0$, $\beta_k=1$ | $\chi_0$ |
| *g* | [111] axis; $\alpha_i=3^{-1/2}$, $\alpha_j=3^{-1/2}$ $\alpha_k=3^{-1/2}$ | $\beta_i=3^{-1/2}$, $\beta_j=3^{-1/2}$, $\beta_k=3^{-1/2}$ | $\chi_0+\frac{1}{3}(\chi_1+2\chi_2)$ |
| *h* | " " " " | $\beta_i=2^{-1/2}$, $\beta_j=-2^{-1/2}$, $\beta_k=0$ | $\chi_0+\frac{1}{3}(\chi_1-\chi_2)$ |

stants for iron and nickel, assuming that in the experiment the zero reading corresponds to the length for perfect demagnetization. The observations on iron are taken from Honda and Masiyama,[9] and those on nickel from Masiyama.[10] The observations are tabulated in Table II.

TABLE II. *Comparison of theoretical and experimental magnetostrictions.*

| Case | Theoretical value | Experimental value $\times 10^6$ Iron | Experimental value $\times 10^6$ Nickel |
|---|---|---|---|
| *a* | $\chi_0+\chi_1$ | {17.1<br>{15.3 | {−54.4<br>{−50.7 |
| *b* | $\chi_0$ | −15.7 | 21.1 |
| *c* | $\chi_0$ | −15.4 | 24.0 |
| *d* | $\chi_0+\frac{1}{2}(\chi_1+\chi_2)$ | {−7.2<br>{−2.7 | {−31.3<br>{−33.9 |
| *e* | $\chi_0+\frac{1}{2}(\chi_1-\chi_2)$ | 14.0 | 14.5 |
| *f* | $\chi_0$ | −9.1 | 18.3 |
| *g* | $\chi_0+\frac{1}{3}(\chi_1+2\chi_2)$ | −12.9 | −27.1 |
| *h* | $\chi_0+\frac{1}{3}(\chi_1-\chi_2)$ | 20.6 | 7.2 |

In order to fit these values we have chosen the constants shown in Table III.

TABLE III. *Magnetostriction constants* $\times 10^6$.

| | Iron | Nickel |
|---|---|---|
| $\chi_0$ | −15.5 | 21.0 |
| $\chi_1$ | 32.0 | −73.5 |
| $\chi_2$ | −12.3 | −23.5 |

These values fit the observations in Table II fairly well, except case *h* in iron, which becomes 0 instead of 20.6, and case *e* in nickel, which becomes −4 instead of 14.5. A better all-around fit might be attempted, but this is hardly worth while, since the observations were made on disks cut perpendicular to tetragonal and digonal axes in which the demagnetization is probably structurally different. The constants as evaluated in Table III are therefore only roughly reliable.

As to the dependence of the volume on the direction of magnetization, we have[11] $\delta v/v=\sum C_{ii}=\text{constant}$, a relation which holds independently of the choice of constants $K_1$ and $K_2$.

In order to calculate the change in magnetostriction produced by tension we shall make use of the assumption regarding the nature of demagnetization —the material behaves as if a fraction $1/n$ of the total volume is magnetized in each of the $n$ directions of easy magnetization. This assumption requires that the observed longitudinal magnetostriction, or the total change in length from a demagnetized state to saturation can be written:

[9] K. Honda and Y. Mashiyama, Sci. Rep., Tohoku Imp. Univ. **15**, 755 (1926).

[10] Y. Masiyama, Sci. Rep., Tohoku Imp. Univ. **17**, 945 (1928).

[11] $\Sigma C_{ii}$ is invariant to a rotation of axes, and is therefore equal to the sum of the principal axes of the tensor ellipsoid.

For iron

$$\left.\frac{\delta l}{l}\right]\begin{pmatrix}\alpha_i, \alpha_j, \alpha_k\\ \beta_i, \beta_j, \beta_k\end{pmatrix}-\frac{1}{3}\left.\frac{\delta l}{l}\right]\begin{pmatrix}1, & 0, & 0\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}-\frac{1}{3}\left.\frac{\delta l}{l}\right]\begin{pmatrix}0, & 1, & 0\\ \beta_i, & \beta_j & \beta_k\end{pmatrix}$$
$$-\frac{1}{3}\left.\frac{\partial l}{l}\right]\begin{pmatrix}0, & 0, & 1\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}.$$

For nickel

$$\left.\frac{\delta l}{l}\right]\begin{pmatrix}\alpha_i, \alpha_j, \alpha_k\\ \beta_i, \beta_j, \beta_k\end{pmatrix}-\frac{1}{4}\left.\frac{\delta l}{l}\right]\begin{pmatrix}3^{-1/2}, & 3^{-1/2}, & 3^{-1/2}\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}-\frac{1}{4}\left.\frac{\delta l}{l}\right]\begin{pmatrix}-3^{-1/3}, & 3^{-1/2}, & 3^{-1/2}\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}$$
$$-\frac{1}{4}\left.\frac{\delta l}{l}\right]\begin{pmatrix}3^{-1/2}, & -3^{-1/2}, & 3^{-1/2}\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}-\frac{1}{4}\left.\frac{\delta l}{l}\right]\begin{pmatrix}3^{-1/2}, & 3^{-1/2}, & -3^{-1/2}\\ \beta_i, & \beta_j, & \beta_k\end{pmatrix}$$

where the quantities in brackets indicate the directions of magnetization and observation, respectively. Both of these expressions reduce to

$$-\chi_1/3+\chi_1\sum\alpha_i^2\beta_i^2+\chi_2\sum{}'\alpha_i\alpha_j\beta_i\beta_j,$$

which is equivalent to our original expression (11) provided $\chi_0=-\chi_1/3$. The values in Table III are not quite consistent with this result, so that we may expect discrepancies when making use of the above assumption concerning demagnetization.

We proceed to discuss the longitudinal magnetostriction in crystals under tension and compression in the direction of magnetization. This tension or compression, whenever it is referred to in this article, means tension or compression so large that $E_1$ may be neglected in Eq. (6). The procedure is here outlined, by way of illustration, for nickel under tension along a trigonal axis. Tension along a trigonal axis in nickel makes the energy a minimum for magnetization in the plane perpendicular to the tension. Therefore the magnetostriction under tension will be that observed without tension plus the change due to the new configuration for demagnetization, which change is given by

$$-\left.\frac{\delta l}{l}\right]\begin{pmatrix}2^{-1/2}, & -2^{-1/2}, & 0\\ 3^{-1/2}, & 3^{-1/2}, & 3^{-1/2}\end{pmatrix}=-\left[\chi_0+\tfrac{1}{3}(\chi_1-\chi_2)\right]$$

or for the total magnetostriction under tension

$$\chi_0+\tfrac{1}{3}(\chi_1+2\chi_2)-\left[\chi_0+\tfrac{1}{3}(\chi_1-\chi_2)\right]=\chi_2.$$

In this case, as in all others, the magnetostriction under tension does not involve $\chi_0$, and is therefore independent of our special assumptions about demagnetization. Various other cases have been calculated and are tabulated in Table IV. Here $\chi_0$ has been put equal to $-\chi_1/3$ to show in which cases the relation

$$\left.\frac{\delta l}{l}\right]_{\text{tension}}=\frac{3}{2}\left.\frac{\delta l}{l}\right]_{\text{no tension}}$$

which was found by Becker for nickel, holds. In Table IV the values of $\delta l/l$ only are affected by the special choice of $\chi_0$.

TABLE IV. *Longitudinal magnetostriction in crystals with and without tension or compression in the direction of magnetization, assuming a special mechanism for demagnetization which requires that* $\chi_0 = -\chi_1/3$.

| Material | $K_1$ | $K_2$ | $K_1+K_2$ | Direction of magnetization | $\delta l/l$ | $\delta l/l$ with tension | $\delta l/l$ with compression |
|---|---|---|---|---|---|---|---|
| Fe | − | + | − | [100] | $2\chi_1/3$ | 0 | $\chi_1$ |
| | | | | [110] | $\frac{1}{2}(\chi_1/3+\chi_2)$ | $\chi_2$ | $\frac{1}{2}(\chi_1+\chi_2)$ |
| | | | | [111] | $2\chi_2/3$ | $\chi_2$ | 0 |
| Ni | + | + | + | [100] | $2\chi_1/3$ | $\chi_1$ | 0 |
| | | | | [110] | $\frac{1}{2}(\chi_1/3+\chi_2)$ | $\frac{1}{2}(\chi_1+\chi_2)$ | 0 |
| | | | | [111] | $2\chi_2/3$ | $\chi_2$ | 0 |
| 80% Ni, 20% Fe | ? | 0 | ? | [111] | 0 | 0 | 0 |
| 45% Ni, 55% Fe | ? | − | ? | [111] | $2\chi_2/3$ | 0 | ? |

The sign of $K_2$ for the alloys containing 80 percent Ni and 45 percent Ni is taken from data by Buckley and McKeehan.[12] Further, McKeehan and Cioffi[13] found that the magnetostriction of a wire of the 80 percent Ni alloy with and without tension is zero, while Honda and Shimizu[14] found that for a wire of the alloy containing 45 percent Ni the magnetostriction under tension is zero. In nickel and iron wires, the last named authors find the magnetostriction with and without tension as shown in Table V.

TABLE V. *Magnetostriction of wires.*

| Without tension | | With tension |
|---|---|---|
| $-4\times10^{-6}$ | in iron | $<-9\times10^{-6}$ |
| $-30\times10^{-6}$ | in nickel | $\sim-42\times10^{-6}$ |

All these results are in general agreement with the predictions of Table IV provided we assume the iron wires to be fibered with a digonal axis parallel to the wire axis, and all the other wires to be fibered with a trigonal axis parallel to the wire axis.

The calculation of magnetostriction under tension as a function of $H$ is of course quite possible in accordance with the above. We shall here be content with pointing out that both for iron and nickel wires under sufficient tension, $I$ is proportional to $H$ up to saturation, and the magnetostriction is proportional to $I^2$. This is of especial interest for iron, in that it predicts the disappearance of the change in sign in the magnetostriction which has often been observed in wires[14] and single crystals[15] magnetized in a [110] direction. The

[12] O. E. Buckley and L. W. McKeehan, Phys. Rev. **26**, 261 (1925).
[13] L. W. McKeehan and P. P. Cioffi, Phys. Rev. **28**, 146 (1926).
[14] K. Honda and S. Shimizu, Phil. Mag. **4**, 338 (1902).
[15] W. L. Webster, Proc. Roy. Soc. **A109**, 570 (1925).

change in sign, or Villari reversal, is discussed from a theoretical point of view by Heisenberg[16] and Akulov.[17] Its disappearance has been observed by Honda and Shimizu.[14]

## MAGNETIZATION[18]

The magnetization curves for crystals under tension may be calculated with the help of Eq. (6) by finding the minima of $E_\theta$. If tension produces a maximum in the direction of magnetization, the material will be more difficult to magnetize under tension, and conversely. Since $E_\theta$ has a maximum in the direction of extension for extension in nickel along the tetragonal, digonal and trigonal axes, and in iron along the trigonal axes, these conditions will give rise to properties similar to those found in nickel wires, and discussed at length by Becker and Kersten.[19] On the other hand, iron under tension along a tetragonal axis has a minimum of $E_\theta$ in the direction of extension. The four minima at right angles to the direction of extension have been bulged out. In other words, suppose an iron crystal is placed with its tetragonal axes parallel to the axes of a cartesian system of coordinates. $E_\theta$ has a minimum along the $\pm x$, $\pm y$, and $\pm z$ axes due to the fact that $C>0$ in Eq. (7). Extension along the $z$ axis changes $E_\theta$ so that it has a minimum in the $\pm z$ directions but a maximum in any direction in the $x$-$y$ plane. In order to see the effect of extension on magnetization let us assume a small field in the direction of the $+z$ axis. We shall here give up the simple assumption that this small field produces saturation, and assume instead merely that there is a slightly greater probability of finding a volume element magnetized in the $+z$ direction than in any other direction. If, now, we apply tension along the $z$ axis, we effectively dump the contents of the minima along the $\pm x$ and $\pm y$ axes into the minima along the $\pm z$ axis, which, because of the small magnetic field in the $+z$ direction, will fall more into the $+z$ than $-z$ direction, and so increase the magnetization. Iron under tension along a digonal axis is more complicated. Let us assume that extension is along the [110] axis in the model just discussed. This extension will produce maxima of $E_\theta$ along the $\pm z$ axes, and minima along the $[1\bar{1}0]$ directions, as may be seen by substituting the values for $K_1$ and $K_2$ as given by Table III and Eq. (12) into Eq. (14). A small field in the [110] direction will make the minima of $E_\theta$ in the $x$ and $y$ directions less than the minima in the $-x$, $-y$, and $\pm z$ directions. If tension is gradually applied along the [110] axis, the minima in the $\pm z$ directions are first emptied, producing an increase in magnetization. Further extension shifts the minima along the $x$ and $y$ directions into the $(1\bar{1}0)$ directions, producing a decrease in magnetization. This is in accordance with the observations of Honda and Terada[20] and others. In general, if a substance is easier

[16] W. Heisenberg, Zeits. f. Physik **69**, 287 (1931).

[17] N. Akulov, Zeits. f. Physik **69**, 78 (1931).

[18] A further discussion of magnetization with illustrations of the function $E_\theta$ will be published in a further paper.

[19] See references 1 and 2, and M. Kersten, Zeits. f. Physik **76**, 505 (1932).

[20] K. Honda and T. Terada, Jour. Col. Sci., Tokyo **21**, Art. 7 (1906).

to magnetize under tension, it is more difficult to magnetize under linear compression. This, however, is not true of iron along a [110] axis. Changing from extension to compression changes the sign of $A$ in Eqs. (13) through (15), and consequently reverses the positions of the maxima and minima of $E_\theta$. But iron under tension along a [110] axis has both maxima and minima of $E_\theta$ at right angles to the extension. Compression will reverse these, and so will not produce a minimum in the direction of compression.

## Elastic Properties

Since the energy of distortion $E_2$ as given in Eq. (9) is a linear function of the distortion, we may say that the atomic interactions responsible for $E_2$ give rise to forces that are independent of the distortion, and dependent only on the direction of magnetization. Consequently, in order to discover any change in the elastic properties of iron due to a change in magnetization from a [100] axis to a [010] axis, for instance, it would be necessary to use methods that were capable of detecting the change in elastic properties due to the application of a constant force.[21]

Finally, it follows from Eq. (12) that the modulus of shear $G$ may be written

$$G = -K_1/2\chi_1 = -K_2/2\chi_2.$$

Since $\chi_1$ or $\chi_2$ can be determined by measurements on magnetostriction, and $K_1$ or $K_2$ by measurements on the change in magnetization or magnetostriction under tension, it follows that we can determine $G$, an elastic constant, by purely magnetic measurements.[8]

## Hysteresis

Whether or not homogeneous strains are important in determining hysteresis depends entirely on the mechanism by which magnetization changes from one direction, say $A$, to another $B$. If this is essentially a rotation, so that the nature of $E_\theta$ between $A$ and $B$ actually enters into the problem, then homogeneous distortions certainly will be important. Such a condition is to be expected in rotating fields. It may very well be, on the other hand, that the values of $E_\theta$ between $A$ and $B$ are in some cases quite irrelevant. In such cases homogeneous strains would not be important.[22]

## Problems

The following problems appear to be among the most interesting and important ones confronting students of ferromagnetism today: (1) The inter-

[21] If the direction of magnetization depends on distortion the elastic behavior will be modified in a complicated way which will be discussed elsewhere.

[22] F. Bloch discusses a mechanism for such changes in direction of magnetization, Zeits. f. Physik **74**, 295 (1932).

pretation of the constants $c$, $K_1$, and $K_2$ in Eqs. (7) and (8). (2) The effect of alloying on these constants, both from an experimental and theoretical point of view. (3) The establishing of Eq. (8) for the energy of distortion and its derivative Eq. (11) for magnetostriction on a firm experimental basis. (4) A determination of the law stating how magnetization changes from one direction to another, together with the related but perhaps more difficult problem of describing demagnetization.