Second Series October 15, 1932 Vol. 42, No. 2

THE

PHYSICAL REVIEW

On the Scattering of Hard X-Rays by Solids

By S. Chylinski
Ryerson Physical Laboratory, University of Chicago

(Received August 8, 1932)

The distribution of the intensity of scattering for angles ranging from 10 to 105 degrees has been experimentally determined for various solids. For paraffin and aluminum the scattering occurred at an effective wave-length for the primary rays of 0.23A; for copper and lead the wave-length was 0.19A. These wave-lengths were determined from absorption measurements in aluminum. The scattering from paraffin and aluminum was from thin plates by the transmission method, while that from copper and lead was from thick plates by reflection. The ratios of intensities at any given angles to those at 90 degrees were corrected for the different ionization effects at those angles, on account of the change of wave-length due to the Compton effect. Comparison of the experimental results for paraffin with the predictions of the Breit-Dirac theory of scattering from free electrons shows distinct excess scattering. This part of the work has been performed with a precision of about 1 percent. In addition, the S-values (scattering per electron) were computed for the various materials from their experimental scattering functions, using the paraffin value at 90 degrees, as predicted by the Dirac theory, as the basic measure. The S-values from aluminum have a most probable error of less than 2 percent, while those from copper and lead, being based upon a not entirely satisfactory computation, are assigned less than 6 percent. Curves have been plotted of the S-values against $[\sin(\phi/2)]/\lambda$. The S-values show a large increase with the atomic number of the scatterer for the smaller values of $[\sin(\phi/2)]/\lambda$, while for the larger values they tend to come together.

I. Introduction

The unit of scattering of x-rays by matter is the electron. J. J. Thomson[1] was the first to calculate, on the basis of the electromagnetic theory, the intensity scattering function for a free electron. His classical expression is

$$I_\phi = I_0(e^4/2m^2c^4r^2)(1+\cos^2\phi)$$

where I_ϕ represents the intensity of the scattered beam at an angle ϕ with the direction of the primary beam, at a distance r from the scattering electron, the mass of which is m and charge e in e.s.u. The primary beam is assumed to be unpolarized and its intensity is I_0. c stands for the velocity of light. If we are dealing with an atom having Z orbital electrons, and if the distances between these electrons are so small as to be negligible in comparison with the wave-length of the incident x-rays, then we may suppose that all these electrons act as a unit in the scattering process. Thomson's formula shows

---

[1] J. J. Thomson, *Conduction of Electricity through Gases* p. 325, 2nd Ed.

that the scattering by the atom would then be Z^2 times the intensity due to a single free electron. If, on the other hand, the electrons in the atom are separated by distances large compared with the wave-length of the x-rays, we may suppose them to scatter independently, in which case the intensity should be only Z times that due to a single free electron. Depending, then, upon the concentration of the electrons in the atom, the intensity of the x-rays scattered from it will vary by a factor of Z. If the electrons are at distances comparable with the wave-length of the x-rays, interference, constructive as well as destructive, will take place between the x-rays scattered by the different electrons of the atom, and as a result the intensity scattered by the atom as a whole will then be between Z^2 and Z times that due to a single free electron. Compton,[2] Debye,[3] Schott[4] and others have calculated the intensity scattering function for various atomic models.

The intensity of x-rays scattered by a number of atoms depends not only upon the configuration of the electrons within each atom but also upon the configuration of the atoms in the molecules, as well as upon any special orientation of the molecules themselves. In the particular case, where the atoms are grouped in a crystal lattice, we have the so-called *Laue or Bragg Reflection* which, therefore, is only a special case of scattering. In the case of liquids or amorphous solids, the phenomenon known as "excess scattering" is probably the result of the cooperation of the three factors just mentioned. Debye[5] has shown that the last two of these factors play no roll when perfect gases are used for scattering and hence excess scattering must in that case be ascribed solely to the configuration of the electrons within the atoms. On account of this simplification of the problem great progress has been made in the theories of scattering for monatomic gases. So far as the writer is aware, only Jauncey[6] has made an attempt to solve the problem for amorphous solids.

The quantum theories predict a scattering function different from Thomson's formula. They are, however, based upon the assumption that for the limiting case of long wave-lengths, where the motion imparted to the scattering electron is negligible, the intensity of the scattered rays should approach that assigned to it by Thomson's classical formula. The older forms of the quantum theory were not successful in giving a unique solution for the scattering function. The newer forms do give such a solution and it is,

$$I_\phi = I_0 \frac{e^4}{2m^2r^2c^4} \frac{(1+\cos^2\phi)}{(1+\alpha \operatorname{vers} \phi)^3}$$

where $\alpha \equiv h\nu/mc^2$, h being Planck's constant, ν the frequency of the x-rays, and the other symbols have the same significance as before.

---

[2] A. H. Compton, Washington University Studies **8,** 99 (1921).

[3] P. Debye, Ann. d. Physik **46,** 809 (1915).

[4] G. A. Schott, Proc. Roy. Soc. Lon. **A96,** 395 (1920).

[5] P. Debye, Phys. Zeits. **28,** 135 (1920).

[6] G. E. M. Jauncey, Phys. Rev. **37,** 1193 (1931).

Breit[7] was the first to suggest this formula on empirical grounds and reasoning from the correspondence principle. Dirac[8] derived it theoretically with the help of the quantum dynamics of Heisenberg and Born, and Waller[9] as well as Gordon[10] did the same on the principles of the de Broglie-Schroedinger wave theory. It is true, that more recently, Klein and Nishina,[11] on the hypothesis of the spinning electron using Dirac's relativistic quantum dynamics, have derived a scattering function which differs somewhat from the above. However, deviations of the two formulae are of the order of $(h\nu/mc^2)^2$, while the Breit-Dirac expression differs from the classical Thomson formula by quantities of the order of $h\nu/mc^2$. In the x-ray region, where $h\nu/mc^2$ is small the deviations between the two quantum expressions are virtually negligible. Even for an $\alpha = 0.5$, which corresponds to a wave-length of 0.05A the deviation amounts to only about 10 percent.

The only experimental tests of the quantum formula for sufficiently hard rays, that have so far been published, are the indirect ones of Fricke and Glasser,[12] for effective wave-lengths of 0.18 and 0.115A, and those of Ishino,[13] and Owen, Fleming and Fage,[14] for the γ-rays from RaC. The first consisted in determining the ratios of the coefficients of photoelectric absorption to those of true absorption due to scattering in carbon, and in the latter two cases, in measuring the ratios for aluminum of the coefficients of true absorption due to scattering to the total scattering coefficients. A direct test, by comparing I_ϕ with the experimental values of scattered intensities, is attended by considerable difficulties. In the earlier technique of various authors,[15] the wave-lengths used were too long to show appreciable deviations from the predictions of the classical theory. Where short wave-lengths were used the results are difficult to interpret on account of the comparatively large corrections that would have to be made in the ionization effects at different angles. This is due to the small absorption in the ionization chamber.

The work described in this paper was undertaken with a view of testing the scattering function given by the quantum theory. For hard x-rays and scattering materials of low atomic number, we may suppose, in view of the Compton effect, that the scattered rays are more or less completely modified. In that case the intensity of scattering should most nearly approximate that predicted by the quantum theory. Paraffin was selected as the light scatterer and in addition the scattering distribution curves were determined for aluminum, copper and lead.

---

[7] G. Breit, Phys. Rev. **27**, 362 (1926).

[8] P. A. M. Dirac, Proc. Roy. Soc. Lond. **A111**, 422 (1926).

[9] Ivar Waller, Phil. Mag. **4**, 1228 (1927).

[10] W. Gordon, Zeits. f. Physik **39**, 117 (1926).

[11] Klein and Nishina, Zeits. f. Physik **52**, 852 (1928).

[12] H. Fricke and O. Glasser, Zeits. f. Physik **29**, 374 (1924).

[13] M. Ishino, Phil. Mag. **33**, 140 (1917).

[14] E. A. Owen, N. Fleming, and W. E. Fage, Proc. Phys. Soc. **36**, 355 (1924).

[15] For a summary see Compton's *X-rays and Electrons* p. 306, D. Van Nostrand Co. N.Y.C. (1926).

II. Apparatus and Experimental Procedure

A diagram of the apparatus is given in Fig. 1. A tungsten target x-ray tube was operated at 120 peak k.v. and 5 m.a. with half-wave rectification. The tube was water cooled and immersed in oil in a heavy lead box. In the front of the box was fitted a heavy lead glass plate having a thin celluloid window. This arrangement permitted the tube to be brought very near to the spectrometer with a consequent increase of intensity in spite of the absorption in the oil.

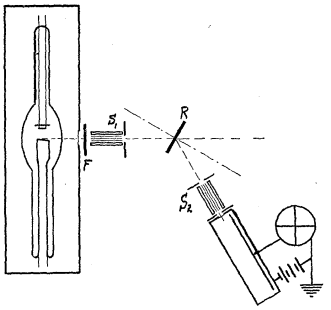

Fig. 1. Arrangement of apparatus.

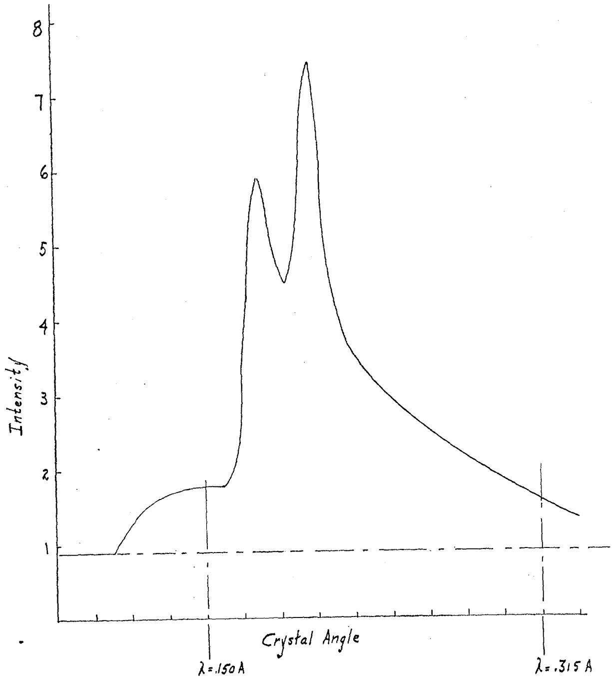

Fig. 2. Spectrum of radiation transmitted through tungsten filter.

An effort was made to work with homogeneous radiation. It was hoped, that by designing all component parts of the apparatus for maximum efficiency, it would be possible to use crystal-reflected rays. Unfortunately, however, the intensities obtained were too low for accurate measurements and the attempt had to be abandoned. Also, the balanced filter method could not be used as the two necessary elements for such filter, when using tungsten radiation, erbium and ytterbium, were not procurable anywhere in the United States. Finally, it was decided to use a tungsten filter, as shown at *F* in Fig. 1. The spectrum of the radiation transmitted through this filter, which consisted of two metallic tungsten foils each 0.065 mm thick, is given in Fig. 2. It is seen that the filtered band extends practically from about 0.15 to 0.315A.

Soller collimators were used for both the primary and scattered beams. They are shown as S_1 and S_2 in Fig. 1. They were 10 cm long and the distance between adjacent lead spacers was 0.1 cm, so that a maximum angular divergence of a little over one degree was obtained in the horizontal plane. The width of the collimators could be varied by proper slits, the height of which was 1 cm. This made the scattering angle slightly larger than recorded on the spectrometer circle, but the corrections were too small to be taken account of.

R in Fig. 1. represents the scattering substance mounted in the center of the spectrometer. In the case of paraffin and aluminum thin plates of these materials were placed so that the normal to the plate always made an angle of $\phi/2$, where ϕ is the scattering angle. This so-called "Crowther position" has certain obvious advantages over other ways of mounting the scatterer. The copper and lead plates, for reasons of greater intensities, were mounted so that the angle between the face of the plate and the primary beam was always one half the scattering angle. These plates were thick enough to absorb all of the primary radiation.

The intensities were measured with a Compton electrometer, mounted over the axis of the spectrometer, having a sensitivity of 8900 divisions per volt on a scale 150 cm away. The ionization currents could be measured with a reproducibility of about 3 percent.

At first an ionization chamber 7 cm long filled with xenon at two atmospheres pressure was used. Due to the insufficient absorption in this chamber and the large fluorescent yield of xenon certain corrections had to be made which, considering the nature of the experiment, were felt to be too large. Accordingly, another high pressure ionization chamber was constructed and filled with argon at a pressure of 1000 lbs. per sq. in. It was made of cylindrical, seamless steel tubing, 40 cm long and 4.45 cm inside diameter. The window was of 0.7 mm celluloid. The absorption in the window for the wavelengths used was negligible and hence no corrections were made for it. The absorption in the chamber of a wave-length of 0.21A is 90 percent and the fluorescent yield for argon is, according to Auger,[16] only 0.07. The corrections are discussed in more detail below. The potential across the electrodes of the ionization chamber was 135 volts, which was about twice the value needed for saturation.

All parts of the apparatus were carefully shielded. The electrometer system, including the connections to the ionization chamber, was protected by

[16] Auger, Ann. de Physique **6**, 183 (1926).

large lead baffles. Other lead baffles, suitably disposed, prevented all stray radiation from reaching the ionization chamber.

The procedure consisted in a determination of the angular distribution of the intensities scattered from paraffin, aluminum, copper and lead. In the case of the last two, care was taken to exclude the characteristic radiations from entering the ionization chamber. This was accomplished by using an aluminum filter 0.65 mm thick for copper and one 1.3 mm thick for lead. Wave-lengths of the scattered rays at the different angles were determined by measuring the absorption in several sheets of aluminum. The final step was to make an accurate measurement of the scattered intensities at 90° from the different materials under experimental conditions as nearly alike as can be realized in actual practice.

The constants of the scattering materials are as follows: paraffin, 0.276 g per sq. cm; aluminum, 0.103 g per sq. cm; copper, 5.09 g per sq. cm; lead, 3.80 g per sq. cm.

The paraffin was from a block of commercial "Parowax." The aluminum and lead were from the usual rolled sheets of these materials. The copper was of electrolytic origin. An attempt to work with finely powdered aluminum failed because no good binding material was available.

III. Experimental Results and Calculations

The ratios of the ionization currents at any angle ϕ to those at 90° are given in the second column of Tables I to IV for each of the four materials.

Table I. *Paraffin at* $\lambda 0.23A$.

Angle ϕ	Exp. ratio of ionization $i\phi/i_{90}$	Ratio from Crowther's formula	Corrected ratio $P\phi/P_{90}$	Scattering per gram s/ρ	S
10	3.23	4.46	4.9	0.0432	1.84
20	2.50	3.42	3.72	0.0328	1.46
30	2.10	2.80	3.04	0.0268	1.28
40	1.75	2.28	2.46	0.0216	1.14
50	1.53	1.92	2.01	0.0177	1.03
60	1.33	1.59	1.67	0.0147	0.99
70	1.18	1.35	1.39	0.0122	0.92
80	1.06	1.15	1.16	0.0102	0.83
90	1.0	1.0	1.0	0.0088	0.74
105	1.15	1.26	1.24	0.0109	0.85

Table II. *Aluminum at* $\lambda 0.23A$.

Angle ϕ	Exp. ratio of ionization $i\phi/i_{90}$	Ratio from Crowther's formula	Corrected ratio $P\phi/P_{90}$	Scattering per gram s/ρ	S
10	9.50	12.60	13.80	0.1290	5.70
20	4.50	6.20	6.80	0.0636	2.95
30	3.00	4.05	4.40	0.0410	2.04
40	2.20	2.90	3.13	0.0293	1.61
50	1.60	2.03	2.15	0.0201	1.22
60	1.30	1.58	1.65	0.0154	1.08
70	1.20	1.39	1.43	0.0134	1.05
80	1.05	1.13	1.14	0.0106	0.905
90	1.0	1.0	1.0	0.0093	0.81
105	1.20	1.33	1.31	0.0122	1.0

TABLE III. *Copper at λ0.19A.*

Angle ϕ	Exp. ratio of ionization	Scattering per gram	S
10	22	0.2340	11.0
20	7.95	0.0855	4.20
30	4.75	0.0510	2.69
40	3.25	0.0350	2.04
50	2.30	0.0247	1.59
60	1.70	0.0182	1.35
70	1.40	0.0150	1.19
80	1.15	0.0123	1.10
90	1.0	0.0107	0.99
105	1.07	0.0115	1.0

TABLE IV. *Lead at λ 0.19A.*

Angle ϕ	Exp. ratio of ionization	Scattering per gram	S
10	22.0	0.557	30.0
20	11.20	0.284	16.05
30	5.00	0.127	7.73
40	3.13	0.0792	5.30
50	2.15	0.0545	4.02
60	1.65	0.0418	3.56
70	1.35	0.0342	3.26
80	1.12	0.0284	2.94
90	1.0	0.0253	2.70
105	1.0	0.0253	2.53

These ratios were, of course, corrected for the leak in the electrometer system. The third column of Table I and II is derived from the second by evaluation from Crowther's formula.[17] This formula for the intensity of x-rays scattered in a direction ϕ from a thin plate of material of thickness t is,

$$I_\phi = AIts/R^2 \cos(\phi/2) \tag{1}$$

where I_ϕ is the intensity scattered in a direction ϕ, I is the intensity of the primary rays *after* penetrating the plate in the Crowther position, s is linear scattering coefficient per unit solid angle in a direction ϕ, A is the area of the ionization chamber window, and R is the distance between the window and the plate. Also, I is given by

$$I \equiv I_0 e^{-\mu t/\cos(\phi/2)}$$

where I_0 is the intensity of the primary rays at the incident face of the scattering plate, and μ is the linear absorption coefficient of the primary rays in the plate.

From Eq. (1) an expression may be set up giving the ratio of the scattering per gram per unit solid angle in any direction ϕ relative to that at 90° with the primary beam, thus cancelling out I_0.

Measurements of absorption in aluminum of the rays scattered from paraffin at 90° gave an absorption coefficient which by interpolation from the values given in Compton's table[18] corresponded to a wave-length of 0.25A. At

---

[17] J. A. Crowther, Proc. Roy. Soc. **A86**, 478 (1912).

[18] Compton's *X-rays and Electrons*, p. 184.

30° the result was 0.23A. The values for aluminum were the same within the uncertainty of the measurement. Considering the hardness of these rays, the scattered radiation from paraffin may be assumed to be completely modified at 90° and that from aluminum is virtually so. Accordingly, for these two materials, the change in wave-length at the different scattering angles was computed from the formula for the Compton effect,

$$\Delta\lambda = (h/mc)(1 - \cos\phi)$$

where $\lambda = 0.25$ at 90°. The wave-lengths so obtained were used in the correction computations given below.

When x-rays of different wave-lengths enter an ionization chamber, the ionic saturation currents which one measures, are not in general proportional to the intensities in the beams. Allison and Andrew[19] give an expression which enables one to calculate the true intensities. This expression is

$$I = cPFR \tag{2}$$

where I is the measured ionic saturation current, P is the power of the beam of x-rays entering the chamber, F is the fraction of the primary energy absorbed in the chamber, c is a constant for a given gas, which is inversely proportional to ϵ, where ϵ is the energy spent in producing a pair of ions, and R is given by

$$R = 1 - \frac{\tau_k}{\mu}\omega_k\lambda_i \sum_i \frac{f_i}{\lambda_i} e^{-\tau_i r} - \frac{\sigma_s}{\mu} e^{-\tau r} + \frac{\sigma_r}{\mu} \tag{3}$$

where τ_k is the absorption coefficient of the gas corresponding to frequency ν for the ejection of photoelectrons from the K-level, μ is the absorption coefficient of the gas for the wave-length of the primary beam, ω_k is the fluorescent yield for K-series as defined by Auger, f_i is the fractional part of the total number of quanta emitted, having the frequency ν_i of the i^{th} line of the K-series, τ_i is the fluorescent absorption coefficient of the gas for the i^{th} line of its own K-series, r is the effective path length of the radiation to the walls, σ_s is the true scattering coefficient, σ_r is the coefficient of absorption due to scattering or simply the recoil electron coefficient, and τ is the absorption coefficient for the scattered rays.

The last term of Eq. (3) has been added to Allison's original expression to take care of the Compton scattering. The σ_s is merely σ in Allison's expression, the usual scattering coefficient.

σ_s and σ_r were calculated from Compton's expressions,[20]

$$\sigma_s = \sigma_0(1 + \alpha)/(1 + 2\alpha)^2$$
$$\sigma_r = \sigma_0\,\alpha/(1 + 2\alpha)^2$$

where σ_0 is the classical linear scattering coefficient, and α has the same significance as before.

---

[19] S. K. Allison and V. J. Andrew, Phys. Rev. **38**, 1424 (1931).

[20] Compton's *X-rays and Electrons*, p. 312.

The second term of Eq. (3) is negligibly small, if we consider that the K-series of argon is around 4A and hence strongly absorbed on its way to the walls; and also, that the fluorescent yield, as stated above, is only 0.07. It may be emphasized that the last term of Eq. (3) is based on the assumption that the recoil electrons are completely absorbed in their journey to the walls. This assumption is permissible as a short calculation from Whiddington's formula for the "reach" of a photoelectron will show.

The constant c in Eq. 2 may be eliminated by taking the ratio between the ionization due to a given wave-length as compared with that due to any other. In this manner the corrected ratios given in column 4 of Tables I and II were arrived at, using the wave-lengths at the different angles as determined from the formula for the Compton effect, with the experimental determination of wave-length at 90° serving as the basis.

With a thick scattering plate, in which there is complete absorption of the primary rays, which are incident on it at a glancing angle of $\phi/2$, it can easily be shown that the ratio of the scattering per gram per unit solid angle in any direction ϕ to that at 90° with the primary beam, is given by

$$(s/\rho)_\phi/(s/\rho)_{90} = I_\phi/I_{90} \tag{4}$$

where the symbols have the same significance as above. Hence in the case of copper and lead in Tables III and IV the experimental ratios are the true ratios, if, for the moment, we leave aside possible corrections for the wave-lengths at different angles.

The absorption measurements in aluminum of the scattered rays from copper and lead gave almost identical wave-lengths. There was no difference, within experimental error, between the wave-lengths at different scattering angles. The effective wave-length, determined as above from Compton's table, was 0.19A.

It was thought desirable to place the ratios of the intensities of scattering upon an absolute basis by a comparison with paraffin, as Coven[21] had done.

For unpolarized x-rays, the Breit-Dirac formula for the scattering per gram per unit solid angle in a direction ϕ, may be written,

$$\left(\frac{s}{\rho}\right)_\phi = N\frac{Z}{W}\,\frac{e^4}{2m^2c^4}\,\frac{(1+\cos^2\phi)}{(1+\alpha\ \mathrm{vers}\ \phi)^3} \tag{5}$$

where N is Avogadro's constant, Z is the atomic number of the scatterer and W its atomic weight, and where the other symbols have the same significance as above.

This formula is, of course, strictly valid for free electrons only. But, considering the low effective atomic number of paraffin and the hardness of the primary rays in this experiment, we may assume as a first approximation that the value it gives for paraffin at 90° is the true value for paraffin. In that way we obtain a unit of measure. The average formula for paraffin is $C_{24}H_{50}$ which makes Z/W approximately equal to 0.5. Substituting this in Eq. (5),

---

[21] A. W. Coven, Phys. Rev. **38**, 1424 (1931).

we get for σ/ρ at 90° the value of 0.0088, for a wave-length of 0.23A. With this basic value for paraffin at 90° and the corrected ratios for the other angles, the column in Table I giving the scattering per gram has been calculated.

Similar calculations for the other material were made by using the separately determined ratios at 90°. These were:

paraffin/aluminum	2.43	by transmission
copper/lead	1.12	by reflection.

Also, the scattering from a 2 mm plate of aluminum, arranged for reflection, was compared with that from copper and lead. The ratios were:

aluminum/copper	2.02	by reflection
aluminum/lead	2.26	by reflection.

Since the 2 mm aluminum plate did not absorb all of the primary radiation, the latter ratios had to be divided by $(1-e^{-\mu l})=0.37$, where l is the maximum path of the x-rays in the aluminum plate.

Correcting all ratios for the different effects in the ionization chamber due to the different wave-lengths, we finally get the following relations:

$$(s/\rho)\mathrm{Al}\ 0.23\mathrm{A} = 1.06(s/\rho)\mathrm{Par.}\ 0.23\mathrm{A}$$

$$(s/\rho)\mathrm{Cu}\ 0.19\mathrm{A} = 1.22(s/\rho)\mathrm{Par.}\ 0.23\mathrm{A}$$

$$(s/\rho)\mathrm{Pb}\ 0.19\mathrm{A} = 2.88(s/\rho)\mathrm{Par.}\ 0.23\mathrm{A}.$$

The effective wave-length of the x-rays in the 2 mm aluminum plate was found to be 0.22A and in the above relations there is a small correction for the difference in the scattering power of aluminum at 0.23A and at 0.22A.

Using these relations at 90° and the corrected experimental ratios at the other scattering angles, the columns for the scattering per gram in Tables II, III and IV for aluminum, copper and lead, respectively, were calculated.

The S-columns in the tables give the scattering per electron relative to the classical scattering from a free electron. They were calculated on the assumption that the rays scattered from paraffin are completely modified, in which case we have from Eq. (5) that,

$$S = (1+\alpha \operatorname{vers} \phi)^{-3}$$

which is then the actual value for paraffin. This value at 90° is 0.74. With the help of Eq. (5) and the experimentally determined values of (s/ρ), the S-values at other angles and for the other materials were computed.

IV. Discussion of Results

In Fig. 3 the dots represent the experimental values of the ratios of scattering from paraffin at a given angle compared with that at 90°. For convenience of comparison with Dirac's theory of scattering by free electrons, three curves have been drawn, evaluated from Eq. (5) for different values of α. Curve 1 corresponds to an α of 0.05A, curve 2 to an α of 0.1 and curve 3 to an α of 0.23A. The broken curve is plotted from Thomsons classical formula for the scattering from a free electron. It is interesting to note, although the fact is probably entirely without significance, that the experimental values for angles between 30° and 90° lie almost exactly along curve 2, whereas, if the scattering from paraffin were equal to that from free electrons, we should expect these values to lie more nearly along curve 3, assuming Dirac's theory to hold for paraffin also. For angles below 30° there is a radical departure of the experimental values even from curve 2, but this circumstance can most probably be ascribed to line interferences, as, strictly speaking, none of the substances investigated can be entirely amorphous. In the backward direction also, there is excess scattering over what one would expect from the theory. Unfortunately, one could not go beyond 105°, as the lead box, containing the x-ray tube, was so close to the spectrometer, as to prevent its arm from swinging beyond that point.

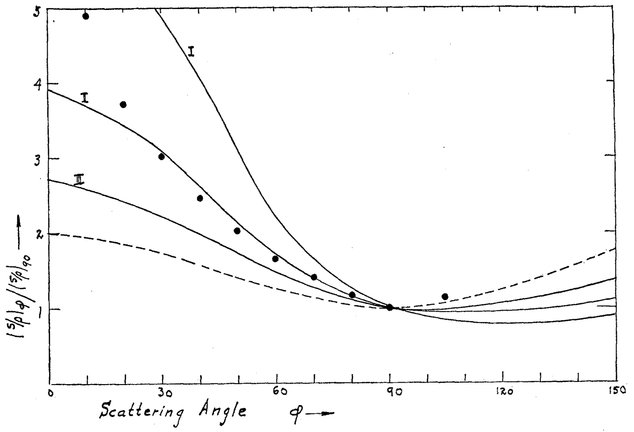

Fig. 3. Scattering from paraffin. Experimental values are represented by dots. Curves 1 2 and 3 are drawn from Dirac's formula for values of λ equal to 0.05, 0.1 and 0.23A respectively, The broken curve is from Thomson's classical formula.

But this value at 105° must be given some weight, for it, like all the other experimental values, represents the mean of the values obtained from three distribution curves for paraffin, each point on the latter being in turn the mean of four ionization measurements. It is estimated that the probable error of the dots in Fig. 3 is less than 1 percent.

There is some error introduced due to the use of Crowther's formula for the hard x-rays of this experiment. As Jauncey and Williams[22] point out, Eq. (1) was derived by Crowther when the Compton effect was still unknown. The scattered rays should be divided into coherent and incoherent rays and the simple Crowther formula should be replaced by

$$I_\phi = [AIt/R^2 \cos(\phi/2)](s_1 + s_2T) \tag{6}$$

where s_1 and s_2 are respectively the coherent and incoherent linear spatial scattering coefficients per unit solid angle in a direction ϕ, and where T is a complicated expression involving the absorption coefficients of the coherent

---

[22] G. E. M. Jauncey and P. S. Williams, Phys. Rev. **41**, 127 (1932).

and incoherent scattered rays in the plate, in the air between the plate and the window of the ionization chamber.

Since we may consider the x-rays of wave-length 0.23A, scattered from paraffin at 90°, to be almost entirely modified and hence incoherent (unless, indeed, the modified ray is not entirely incoherent), s_1 in Eq. (6) becomes zero. A calculation for this particular case shows that the value of s_2 would be less than 1 percent larger than the value of s calculated from Eq. (1). This, therefore, places an upper limit upon the error from this source, as for the smaller angles the discrepancy would be smaller. The difference between the errors for 90° and 105° is trifling.

These results for paraffin are somewhat discordant with those obtained by Coven[21] at an effective wave-length of 0.32A. He finds closer agreement with the Dirac theory for free electrons. There is, however, in his case a question as to the magnitude of the error in determining the effective wave-lengths. Also, no corrections were made for the different ionization effects at different angles. Jauncey and Harvey[23] report good agreement between the scattering from paraffin and the Dirac theory at an effective experimental wave-length of about 0.30A. They measured the ratios of the scattering at 97.5° and 120° as compared with that at 75°. Their corrections for the different ionization effects are more difficult than in this experiment. They used methyl iodide and ethyl bromide in their ionization chamber which require comparatively large corrections for the fluorescent yield.

It is difficult to see how the agreement with the theory from free electrons can be perfect, if we recall what has been said about interferences, unless, indeed, our conception of the mechanism of scattering should be modified.

In Fig. 4 are plotted the S-values against $[\sin(\phi/2)]/\lambda$ for the different scattering materials. These curves show very clearly how the scattering per electron increases with the atomic number of the scatterer. For paraffin at the larger angles, the scattering is practically that from free electrons, which condition seems to be approached also for the other materials as $[\sin(\phi/2)]/\lambda$ is increased.

The largest errors involved in measuring the scattering per electron are probably those of comparing the scattering from one material with that of another. The smallest of these is in the case of the comparison of the intensity of scattering from paraffin with that from aluminum. This is estimated to be of the order of 1 percent. Also, in the comparison of the intensity of scattering of lead to copper the error is of about the same magnitude. This is because in one case the transmission and in the other case the reflection method was used for the two materials in question. But when we come to compare the scattering from paraffin with that from copper or lead we have to use the scattering from aluminum by the reflection method as a sort of connecting link. A little thought on the calculation involved will show that it is not entirely satisfactory. This doubt is strengthened by the fact that an attempt to get the ratio of paraffin to copper, by scattering from a copper foil by the transmission method, gave a result which did not agree with the other within

---

[23] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **37**, 1203 (1931).

experimental uncertainty. There is a possibility that multiple scattering[24] may have some bearing on this, and this point will be investigated in the near future.

The possible error of method involved in determining the S-values for copper and lead is estimated, in view of the above, to be of the order of 5 percent. On account of this large error it was thought inadvisable to attempt a correction for the coherent and incoherent scattered radiations from copper and lead. Besides, this error is probably slight, as the radiation scattered from lead is almost entirely coherent and that from copper is mostly so.

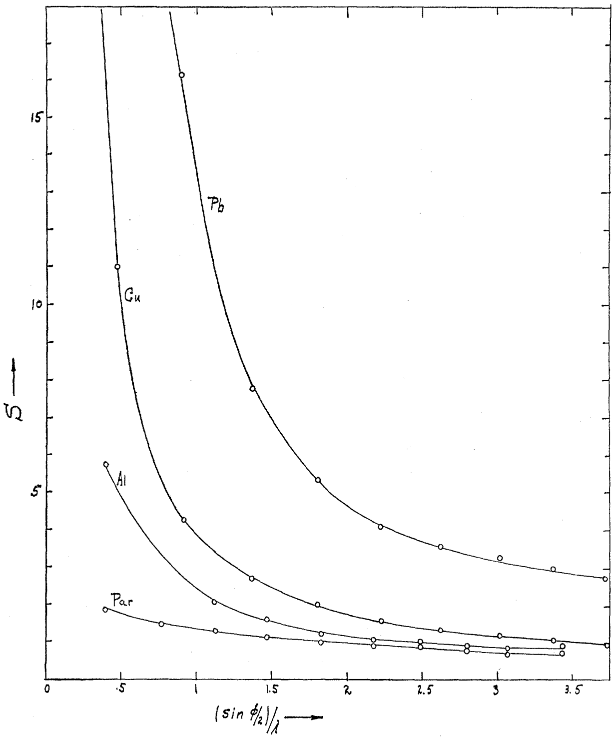

Fig. 4. Experimental values for the scattering per electron with the classical scattering from a free electron taken as a unit.

Considering all sources of error, the most probable error for the S-values is estimated to be less than 2 percent for paraffin and aluminum, and less than 6 percent for copper and lead. Included in these calculations is the error involved in the determination of the effective wave-length. It must be emphasized that in experiments of this kind where one works with more or less heterogeneous radiation, the concept of "effective" wave-length is not a closely defined one, as one learns by reading the divergent opinions expressed by different authors.[25]

---

[24] J. W. M. Dumond, Phys. Rev. **36**, 1685 (1930).

[25] See discussions under *X-Rays*, International Critical Tables, Vol. VI, 1928 Ed.

Jauncey,[6] in his theory of diffuse scattering of x-rays by solids, arrives at an expression given by

$$S = 1 - (Z - 1)(f'^2/Z^2) + (F^2/ZN)X$$

where S is the scattered intensity per electron, relative to the scattered intensity from a single electron, Z is the atomic number, F is the atomic structure factor which includes the effect of thermal agitation, f' is related to f, the true atomic structure factor, N is the total number of atoms, and X is a complicated double summation.

So far, the double summation X has been evaluated only for a simple cubic crystal consisting of atoms of one kind.[26] It has not been determined for an amorphous solid and hence no comparison can as yet be made between Jauncey's theory and the present experimental results.

It is interesting, in connection with what has been said above about the excess scattering by paraffin at 105°, to note Chao's experimental result.[27] Working with γ-rays from ThC he finds at 135° an intensity of scattering by lead approximately three times that which one would expect from the formula of Klein-Nishina. This *anomalous* scattering he attributes to a cooperation of the nuclear electrons in the process of scattering.

In concluding, I wish to express my gratitude to Professor A. H. Compton for help in the course of this work.

---

[26] G. E. M. Jauncey and G. G. Harvey, Phys. Rev. **37**, 1203 (1931).

[27] C. Y. Chao, Phys. Rev. **36**, 1519 (1930).