Second Series *September 15, 1932* *Vol. 41, No. 6*

THE

PHYSICAL REVIEW

The Faraday Effect with X-Rays

By Darol K. Froman

Physics Laboratory, Macdonald College, McGill University

(Received July 19, 1932)

Voigt's theory for the rotation of the plane of polarization of light by a magnetic field is discussed and one of the equations used by W. Kartschagin and E. Tschetwerikowa who investigated the Faraday effect with x-rays transmitted through ferromagnetic materials, is shown to be inapplicable. The theory for the effect given by Drude is modified for the case of x-ray wave-lengths giving a rotation of $\chi = -2\pi\mu eIL/mc^2$ radians. μ is the refractive index, I is the intensity of magnetization, l the thickness of the material through wich the x-rays are transmitted. The other quantities have their usual meanings. Kartschagin and Tschetwerikowa found evidence suggesting a rotation in the case of x-rays. An experiment was performed which confirmed these suggestions and which checked the equation for χ given above well within the experimental error. The experimental error was large, but a rotation of the order of 10° was shown to occur for x-rays of wave-length 0.3A transmitted through 0.05 cm of iron in a field of 300 gauss.

Introduction

The Faraday effect with x-rays has been studied by Kartschagin and Tschetwerikowa[1] using paraffin and iron in the magnetic field to produce a rotation of the plane of polarization. Their results were negative in the case of paraffin, but in the case of iron they say that their experimental results do not entirely justify the conclusion that a rotation exists but indicate a rotation of about 10°. They used iron of effective thickness about 0.01 cm in a magnetic field of 750 gauss. They compared their experimental results with Voigt's[2] theory which, under their experimental conditions, predicted a rotation of only 4′.

It is the purpose of this paper to discuss the applicability of Voigt's equation to paramagnetic materials; to modify the theory of Drude[3] for x-ray frequencies; to describe an experimental investigation on iron, indicating the differences between the method used and that of Kartschagin and Tschetwerikowa; and to compare the experimental results with the predictions of the two theories.

[1] W. Kartschagin and E. Tschetwerikowa, Zeits. f. Physik **39**, 886 (1926).

[2] W. Voigt, *Magneto- und Elektrooptik*, p. 130. Much of the fundamental work of this theory is due to H. Becquerel, C. R. **125**, 679 (1897).

[3] P. Drude, *The Theory of Optics*, Ch. VII.

APPLICABILITY OF VOIGT'S EQUATION

On the basis of the electromagnetic theory Voigt deduces an equation for the rotation, χ, of the plane of polarization of light traversing a distance l of a material in the direction of an applied magnetic field of strength R_0, *viz:*

$$\chi = \lambda l R_0 e/mc^2 \cdot d\mu/d\lambda \tag{1}$$

where e, m, c and λ have their usual meanings and μ is the refractive index of the material. All quantities are in the C.G.S. system. This equation is not applicable to ferromagnetic materials since it is well known[4] that in this case χ is proportional to the intensity of magnetization and not to the magnetic field strength. Voigt himself points this out.[5] It does not suffice to substitute for R_0 the field produced by the magnetization alone, i.e., $4\pi I$ where I is the intensity of magnetization, since Eq. (1) is deduced on the assumption that the equations of motion of an electron in the medium under the action of the incident light and the applied magnetic field are:[6]

$$\begin{aligned} m\ddot{x} + h\dot{x} + kx - (eR_0/c)\cdot\dot{y} &= eX \\ m\ddot{y} + h\dot{y} + ky + (eR_0/c)\cdot\dot{x} &= eY \\ m\ddot{z} + h\dot{z} + kz &= eZ. \end{aligned} \tag{2}$$

In these equations the incident light travels in the direction of the magnetic field, h and k are the friction and elastic constants respectively and the other quantities have their usual meanings. Since eR_0/c cannot be replaced in these equations by $4\pi Ie/c$ it is not permissible to do so in Eq. (1).

As it is not known how to modify the magnetic force factors of Eqs. (2) for paramagnetic materials, the problem is attacked in a different way.

MODIFICATION OF DRUDE'S THEORY FOR X-RAYS

Drude assumes that a material contains rotating ions, electrons, whose magnetic moments may be alined, at least in paramagnetic materials, by the application of a magnetic field. The only effect of the electric field of a light wave is to displace the center of rotation of the electron if the natural frequency of the electron is far removed from that of the incident light. Thus the time rate of change of magnetic flux through a given area is composed of two parts, *viz:* the change which is produced directly by the magnetic vector of the light wave, and the change produced by the motion of the center of rotation of the electron due to the action of the light wave. Drude calculates these quantities and the corresponding current densities and substitutes directly in Maxwell's equations. On this basis he deduces the index of refraction and the rotation of the plane of polarization.

The index of refraction, μ, is given by:

$$\mu^2 = 1 + \sum_h \alpha_h n_h + \sum_k \beta_k n_k \tag{3}$$

[4] P. Drude, reference 3, p. 449.
[5] W. Voigt, reference 2, p. 20.
[6] W. Voigt, reference 2, p. 125.

where the subscripts h and k refer to nonconducting and conducting electrons respectively. n is the number of a particular type of electron per unit volume and α and β are functions of the properties of the medium and the period of the incident light. β is proportional to the square of the period and may be neglected in the case of x-rays since the frequency is very high. This approximation has been shown to be valid by experimental determinations of μ which agree within experimental error with Eq. (3), even for conductors, when β is put equal to zero. In this case h goes from 1 to the total number of different types of electrons present whether conducting or not.

Lorentz[7] also derives an expression for μ, namely:

$$\mu^2 = 1 + e^2/\pi m \cdot \sum_h n_h/(\nu_h^2 - \nu^2) \tag{4}$$

where e and m have their usual values, ν_h is the natural frequency of all of the n_h electrons and ν is the frequency of the incident light. Since the sums in Eqs. (3) and (4) are taken over the same range we can solve for α_h.

$$\alpha_h = e^2/\pi m(\nu_h^2 - \nu^2). \tag{5}$$

Drude also shows that the plane of polarization of plane-polarized light is rotated through an angle, χ, when a beam of light passes through a magnetized substance in a direction parallel to the magnetic field. χ is given by:

$$\chi = f\mu l/2c^2\tau^2 \tag{6}$$

where l is the thickness of the material traversed by the light, c is the velocity of light and $\tau = 2\pi\nu$. By convention χ is positive when the rotation is right-handed to an observer looking in the direction of the field. f is given by the equation

$$f = (1/c) \cdot \sum_h n_h' \alpha_h q_h/T_h + \sum_k n_k' \beta_k q_k/T_k. \tag{7}$$

Drude assumes that an electron of type h moves in an orbit of area q_h with a period T_h. n_h' is the effective number of electrons of this type per unit volume having their magnetic moments parallel to the external magnetic field. It is shown also that the number of lines of magnetic induction per unit area produced by these n_h' electrons is

$$M_h = 4\pi e q_h n_h'/cT_h. \tag{8}$$

The total flux per unit area is

$$4\pi I = \sum_h M_h \tag{9}$$

where I is the intensity of magnetization.

We may neglect the second sum in Eq. (7) as was done in Eq. (3) and combining Eqs. (7) and (8) we get

$$f = \sum_h \alpha_h M_h/4\pi e. \tag{10}$$

[7] H. A. Lorentz, *The Theory of Electrons*, 2nd Ed. p. 149.

It has been shown[8] that for x-rays we can neglect ν_h^2 compared to ν^2. Thus Eq. (4) becomes

$$\left.\begin{aligned} \mu^2 &= 1 - e^2n/\pi m\nu^2 \\ \text{or} \qquad \mu &= 1 - e^2n/2\pi m\nu^2 \end{aligned}\right\}. \tag{11}$$

This is the usual expression for μ.

Here $n = \Sigma_h n_h$ = the total number of electrons per unit volume and since μ differs only slightly from unity, the approximation in the second of Eqs. (11) is justified. Also Eq. (5) becomes

$$\alpha_h = - e^2/\pi m\nu^2. \tag{12}$$

Combining Eqs. (9), (10) and (12) we get

$$f = - \sum_h e^2 M_h/4\pi^2 em\nu^2 = - eI/\pi m\nu^2. \tag{13}$$

Substituting from Eq. (13) in Eq. (6)

$$\begin{aligned} \chi &= - \mu eIl/2c^2\tau^2\pi m\nu^2 \\ &= - 2\pi\mu eIl/mc^2. \end{aligned} \tag{14}$$

It can be seen from Eq. (11) that $\mu < 1$ and α_h is negative. Consequently it appears that the sign of χ has a real significance in this case. For example, iron shows a positive rotation for visible light and in this range $\mu > 1$. Thus the quantity corresponding to α_h is positive for this region and χ is calculated as positive. The minus sign here indicates then that the rotation will be negative. The direction of rotation cannot be predicted for ordinary light from the magnetic properties of the substance but if the direction is known in one case and the calculated χ is of opposite sign in a second case, it is expected that the direction of rotation will differ in the two cases.

The work of several experimenters[9] has shown that the planes of electronic orbits are not rotated in a magnetic field but it may be pointed out that the deduction of Eq. (14) is independent of the mode of magnetization and depends only on the assumption that magnetization is an electronic effect.

It is interesting to note that χ is independent of the wave-length, λ, of the x-rays except in so far as μ is a function of λ. Since μ is very nearly unity for a wide range of x-ray wave-lengths, it is to be expected that χ will be approximately constant for all x-rays.

[8] H. A. Lorentz, reference 7.

[9] M. de Broglie, Le Radium **10,** 186 (1913).
K. T. Compton and E. A. Trousdale, Phys. Rev. **5,** 315 (1915).
A. H. Compton and Oswald Rognley, Phys. Rev. **16,** 464 (1920).
T. D. Yensen, Phys. Rev. **31,** 714 (1928).
J. C. Stearns, Phys. Rev. **35,** 1 (1930).
J. C. Stearns, Phys. Rev. **35,** 292 (1930).

The Experimental Arrangement

The general radiation of effective wave-length about 0.3A from a gas-filled x-ray tube was used in these experiments. This white radiation was about 10 percent polarized thus giving a much greater effective intensity than could be obtained by scattering. The arrangement of apparatus is shown in Fig. 1. The x-rays passed through holes in the pole pieces of an electromagnet, in a direction parallel to the magnetic field, and fell upon a cylindrical carbon scattering block which served as analyzer. The radiation scattered by the carbon block at right angles to the primary beam was detected by a photographic film bent into the shape of a cylinder having the same axis as the carbon block. The diameter of the cylindrical film holder was about 4 cm and of the block about 0.5 cm. The total distance from target to analyzer was about 25 cm. The holes in the pole pieces were just large enough to ensure

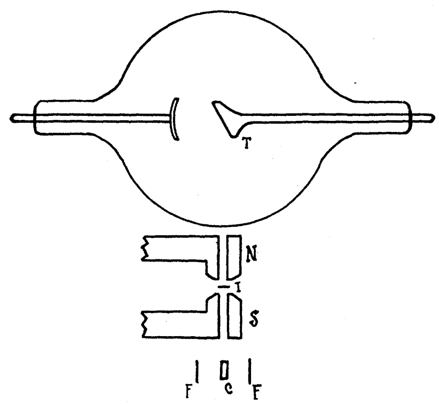

Fig. 1. Arrangement of apparatus. T is the target of the x-ray tube, N and S are the perforated magnetic pole pieces, I is the iron disk, C is the carbon analyzer and F is the photographic film.

uniform illumination over the scattering block. Any desired thickness of iron could be placed between the poles of the magnet.

The x-rays had a predominant polarization with the electric vector in the plane containing the stream of incident cathode rays at the target. The angle which this plane made with the straight line joining cathode and target depended upon the strength of the field produced by the magnet at the center of the x-ray tube. Thus a reversal of the magnetic field produced a rotation of the plane of predominant polarization even if there were no substance between the poles of the magnet. In order to eliminate this effect produced by the change in the direction of the cathode rays photographs were taken with different thicknesses of iron between the poles. The thickness of the iron was never changed enough to affect the magnetic field in the x-ray tube.

Photographs were taken with 0.05 cm and 0.075 cm of iron between the poles of the magnet with the field first in one direction and then in the other. A fixed lead stop between the scattering block and the film and just in front of the latter served to mark a constant angle with the axis of the x-ray tube on the various films used. A microphotometer was used to measure the intensity on the film. As a continuously-recording microphotometer was not available the galvanometer deflections were read directly and the curves plotted. Fig. 2 shows two typical examples. These two curves represent the intensity arriving at the film after passing through 0.075 cm of iron in the magnetic field. Curve (1) was taken for the magnetic field in the direction of propagation of the x-rays and curve (2) with the field reversed. The angular displacement of one of these curves with respect to the other measures the combined effect of the magnetic field on the cathode rays and of any roation of the plane of polarization in transmission through the iron. The process was repeated using 0.05 cm of iron between the poles of the magnet. Since the effect on the cathode rays was the same as in the case for the thicker iron, any difference between the angular displacement of these curves and of those for 0.075 cm of iron measures the rotation of the plane of polarization produced by 0.025 cm of iron upon reversal of the magnetic field.

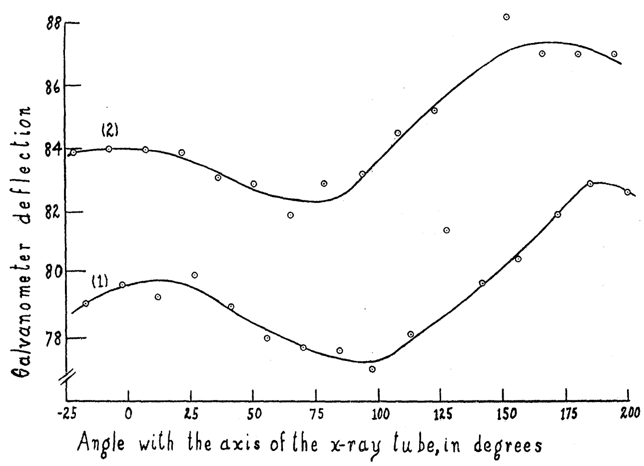

Fig. 2. Microphotometer curves.

Three sets of curves for each thickness of iron were obtained, the average displacements measured and the mean differences found.

The iron used was in the form of disks 0.025 cm thick and of diameter just greater than the holes in the pole pieces. On account of the difficulty of determining the magnetic properties of the iron and of estimating the demagnetizing effect of the free poles, a direct measurement of magnetization was made. As is well known[10] the force between two magnetic poles in contact is $2\pi I^2$ dynes per cm^2. The force required to separate one of the disks from the other two in the field was measured and the intensity of magnetization calculated. This was repeated with but two disks present. Mechanical difficulties rendered these results somewhat inaccurate but they are probably as precise as the measurement of angular displacements. The strength of the magnetic field producing magnetization was measured in order to test Voigt's equation.

[10] S. G. Starling, *Electricity and Magnetism*, p. 283.

If there is appreciable dispersion in the magnetic rotation the apparent percentage of polarization should be less with a magnetic field across the iron than without the field. That is, if the planes of polarization of different wave-lengths were rotated through different angles, the microphotometer curves would be flattened by the application of a field. No appreciable difference in the apparent percentage of polarization was obtained with the iron magnetized and not magnetized. Kartschagin and Tschetwerikowa did obtain a difference in this case but they were using the iron itself as the analyzing scatterer. They assumed that the effective thickness of iron traversed before scattering was equal to the thickness of iron which would absorb one-half of a primary beam of wave-length equal to the wave-length of maximum energy from their x-ray tube. Thus their primary rays travelled various distances in the direction of magnetization before analysis and a decrease in the apparent percentage of polarization is to be expected even if there is no dispersion.

TABLE I.

Trial	$\theta_1^\circ(l=0.05$ cm$)$	$\theta_2^\circ(l=0.075$ cm$)$	$\theta_2^\circ-\theta_1^\circ$ $=\chi_2^\circ-\chi_1^\circ$	$\chi_2^\circ-\chi_1^\circ$	$\chi_2^\circ-\chi_1^\circ$
1	-4.3	-14.4	from experiment	from theory given in this paper	from Voigt's theory for nonferromagnetic materials
2	-4.3	-15.8			
3	-5.8	-15.8			
Average	-4.8	-15.3	-10.5	-14.0	-1.1×10^{-7}

In Table I θ_1 is the apparent rotation produced by a thickness of 0.05 cm of iron, and θ_2 by 0.075 cm, on reversal of the field. Both θ_1 and θ_2 include the effect of the field on the cathode rays. One might expect θ_1 to be much larger compared to θ_2. However the demagnetizing effect of the free poles is considerably reduced on increasing the thickness of the iron so that by Eq. (14) χ is increased by the increase of both l and I. The magnetic field was kept constant at 300 gauss. The intensity of magnetization measured 79 unit poles per cm^2 for 0.05 cm of iron and 97 unit poles per cm^2 for a thickness of 0.075 cm. $\theta_2-\theta_1$ is independent of any magnetic effect on the cathode rays. Values of χ were calculated for these two cases, χ_1 for 0.05 cm of iron and χ_2 for 0.075 cm of iron both from Voigt's theory (Eq. 1) and from Eq. (14). $\chi_2-\chi_1$ is given in Table I for comparison with $\theta_2-\theta_1$ as observed. $d\mu/d\lambda$ was calculated for Eq. (1) from Eq. (11). The minus sign indicates the direction of the rotation in the conventional manner.

CONCLUSIONS

The results given in the last three columns of Table I differentiate between the two theories in a striking and unmistakable way. Voigt's theory is certainly not applicable in this case. The comparatively large estimate of the rotation, 4′, made by Kartschagin and Tschetwerikowa in their case for Voigt's equation came about since their effective wave-length was near the K absorption level of iron. Thus their estimate of $d\mu/d\lambda$, based on the anomalous dispersion theory, was many times that given by Eq. (11). The effective frequency used in the present experiments was certainly so much greater than any of the characteristic frequencies of iron that Eq. (11) contains no appreciable error.

It is very difficult to estimate the probable error in the experimental value of $\chi_2 - \chi_1$. After careful consideration of the various errors which may have entered due to the particular equipment used, and to the photographic method of measuring intensities, the author is forced to conclude that the results given here indicate only the correct order of magnitude of the effect. However it can be concluded with little doubt that the plane of polarization is rotated a few degrees under the conditions described. For these reasons no calculation of the Verdet constant has been made from the experimental results. It is greatly desired that accurate ionization-chamber measurements be made, using a much steadier source of x-rays than was available for this work, and it is hoped that such work, if not done elsewhere in the meantime, may be accomplished in this laboratory in the near future.

The author wishes to take this opportunity to thank Drs. W. and M. L. Rowles, and his wife, Ethel Froman, for their advice and assistance in the experimental part of this work.