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An Interpretation of Cosmic-Ray Phenomena

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Schindler's data on the *transition effects* of the cosmic rays have been interpreted on the assumption that the equilibrium between the primary radiation and its secondary corpuscular rays is different in different media. The ionization behind any thickness of absorbing materials, assumed to be proportional to the flux of secondaries, is calculated in terms of the absorption coefficients, $\nu(m)$, of the primary radiation and the production and absorption coefficients, $\beta(m)$ and $\mu(m, m)$, respectively, of the secondary rays, these coefficients being characteristics of the media. A comparison with the data permits a determination of each of the absorption coefficients as well as the product of each of the production coefficients by the number of primary rays. The results are as follows, expressed in cm^{-1} lead equivalent: $\nu(Pb)=0.0064$; $\nu(Fe)=0.009$; μ(Pb, air), the absorption coefficient in lead for secondaries produced in air $=0.50$; $\mu(Pb, Pb)=0.98$; $\mu(Fe, air)=0.30$; $\mu(Fe, Fe)=0.45$; $\mu(Pb, Fe)=0.72$; $\mu(Fe, Pb)=0.48$; $\beta(Pb)/\beta(Fe)=2.0$. This determination of the absorption coefficients of the primary and secondary radiations allows the following estimates to be made. (a) The lower limit of the average energy of the secondary radiation is about 30 million volts. (b) The average number of secondaries per primary is about 100 in iron and 230 in lead. (c) The energy of a primary cosmic ray, equal to the sum of the energies of its secondaries, is about 2×10^{10} volts.

Until 1929 our only knowledge of the cosmic radiation had been derived from measurements of the ionization in closed vessels at high altitudes and under various thicknesses of absorbing material. Although measurements of this type have yielded values of the absorption coefficient they have revealed little concerning the nature of this radiation or the mechanism by which the ionization is produced. The principal evidence on this point has resulted from experiments with counters and with the Wilson cloud chamber. Skobelzyn[1] first observed the tracks of high energy corpuscular rays which were believed to be associated with the cosmic radiation and Mott-Smith and Locher[2] have identified these tracks with the ionizing rays which produce the coincident discharges of Geiger-Mueller counters. From the coincidence counting rate of these counters several investigators have determined the rate of influx of these rays through our atmosphere and if a reasonable allowance is made for the probability of more than one ray passing through the

[1] Skobelzyn, Zeits. f. Physik **54,** 686 (1929); C. R. **189,** 55 (1929); **194,** 118 (1932).

[2] Mott-Smith and Locher, Phys. Rev. **38,** 1399 (1931).

counters simultaneously[1,3] these results, in conjunction with the number of ion pairs per centimeter of path as determined from the density of droplets in the cloud chamber,[1,3] give a rate of ionization in reasonable agreement with that attributed to the cosmic radiation. These corpuscular rays, however, must be regarded as a secondary effect of the primary radiation, as Skobelzyn,[1] Locher,[3] and Millikan and Anderson[4] have pointed out, for the experiments with the cloud chamber not only show that, in many instances, the rays originate within the material walls of the cloud chamber, but also the energy of these rays is low enough to rule out the possibility of an absorption coefficient comparable with that determined from the electroscope measurements. Moreover, the large number of tracks with positive curvature on Anderson's photographs, show that the secondaries are largely of nuclear origin. As Millikan and Anderson have pointed out, this fact is of the greatest interest because of its invalidation of the use of the Klein-Nishina formula for deriving the energies of the primary rays from absorption measurements even if it can be shown that the primary rays are of the gamma type, for this formula is based upon the assumption of a primary gamma-ray interacting with the extranuclear electrons alone. Although the formula itself must be in error in omitting nuclear absorption there is still the possibility that the underlying ideas are right, i.e., a material substance may be correctly represented as a cloud of free electrons and protons, if the nuclear constituents are included along with the extranuclear electrons. On the other hand the rate of absorption of the primary rays ,as well as the energies of the secondaries, may depend in a characteristic way upon the structure of the nucleus of the absorbing material.

Some recent ionization measurements made by Schindler[5] seem to reveal the first definite information on this subject. In brief, his experiments consisted of a series of measurements of the ionization in a thin-walled vessel placed behind absorbing screens of various materials and of various thicknesses. The curves obtained in the region of an interface between two different media of different nuclear type at once suggest a phenomenon analogous to the readjustment of the equilibrium between a beam of gamma-rays and its secondary beta-rays.[6] In fact, his data may be very simply and accurately interpreted on this basis. To put the matter more exactly the following postulates are required. (1) The rate of production of secondaries in any medium m is proportional to the intensity of the primary radiation, the production coefficient $\beta(m)$ being a characteristic of m. (2) The rate of absorption of the primary rays in m is proportional to their intensity, the absorption coefficient $\nu(m)$ likewise being a characteristic of m. (3) The rate of absorption in a medium m of secondaries produced in a medium n is proportional to their num-

[3] G. L. Locher, Phys. Rev. **39,** 883 (1932).

[4] Millikan and Anderson, Phys. Rev. **40,** 325 (1932).

[5] H. Schindler, Zeits. f. Physik **72,** 625 (1931); see also T. H. Johnson, Phys. Rev. **40,** 468 (1931). B. Rossi has obtained similar results with counters and has made a qualitative interpretation somewhat similar to that presented here, Rend. Lincei, **XV**, 734 (1932).

[6] K. W. F. Kohlrausch, Radioaktivität, page 139 *et seq.*

ber, the absorption coefficient $\mu(m, n)$ being a characteristic of n as well as of m. (4) Ionization is produced by the secondaries alone.[7] In accordance with these assumptions the change in the flux q_n of the secondaries produced in n while passing through the element dx of m is

$$dq_n = -\mu(m, n)q_n dx \tag{1}$$

and the corresponding change in the flux q_m of secondaries produced in m is

$$dq_m = \beta(m)Ne^{-\nu(m)x} - \mu(m, m)q_m dx \tag{2}$$

where N is the number of primary rays per square centimeter per second across the interface between media n and m. Integrals of these equations subject to the conditions at the interface, $x=0$, that $q_m=0$, and $q_n=q_{n_0}$ are

$$q_n = q_{n_0}e^{-\mu(m,n)x} \tag{3}$$

$$q_m = \beta(m)N[e^{-\nu(m)x} - e^{-\mu(m,m)x}]/[\mu(m, m) - \nu(m)]. \tag{4}$$

If I is the average number of ions per centimeter of path produced by a secondary ray in the ionization chamber, the rate of production of ions per cc in the case of the one dimensional problem considered here, is

$$J = (q_n + q_m)I. \tag{5}$$

In Schindler's experiments only the ions produced by the rays included in a cone of half angle 30° about the vertical were measured and hence the one dimensional case may be considered a fairly good approximation. In fact Eq. (5) represents the experimental data quite accurately if a suitable determination of the constants is made.

In the case of the transitions from air to lead and air to iron, respectively the first step in fitting Eq. (5) to the experimental data was the determination of $\nu(m)$. For this determination the last two points, corresponding to the greatest thicknesses of the absorbing material, were used and it was assumed that both of the μ's were large enough to make all but the term containing $e^{-\nu(m)x}$ completely negligible at these thicknesses. The fact that μ's satisfying this condition could then be determined to bring about good agreement between (5) and the data, justified this procedure although it would have been more satisfactory if more extensive data were available at the tail of the curve. These last two points also determine the multiplier $\beta(m)N/[\mu(m, m) - \nu(m)]$, and q_{n_0} is determined from the ionization with no absorbing screen ($x=0$). It remains therefore to determine two exponentials with known multiplying coefficients, the difference between which agrees with the residual of the data after the term in $e^{-\nu(m)x}$ has been subtracted. The choice of $\mu(m, n)$ and $\mu(m, m)$ is therefore very definite and a variation in one of these of the order of ten percent, from the values stated, though the other be adjusted accordingly for the best fit, is sufficient to throw the equation into serious disagreement with the data.

[7] Without seriously altering the results it is possible to assume that the primaries produce ions *per se*, but as this assumption is not required for an explanation of Schindler's results, for simplicity it is not included.

The agreement between the observed and calculated values of the ionization in the case of the air to lead, and air to iron transitions is shown in Tables I and II, the constants having the following values: $q_0(\text{air})I=0.461$ ion pairs per cc per sec.; $\beta(\text{Pb})NI/[\mu(\text{Pb, Pb})-\nu(\text{Pb})]=0.298$ ion pairs per cc per sec.; $\beta(\text{Fe})NI/[\mu(\text{Fe, Fe})-\nu(\text{Fe})]=0.0322$ ion pairs per cc per sec.; $\nu(\text{Pb})=0.0014$ cm²/g electron $=0.0064$ cm^{-1} Pb; $\nu(\text{Fe})=0.002$ cm²/g electron $=0.009$ cm^{-1} Pb; $\mu(\text{Pb, air})=0.11$ cm²/g electron $=0.50$ cm^{-1} Pb; $\mu(\text{Fe, air})=0.066$ cm²/g electron $=0.30$ cm^{-1} Pb; $\mu(\text{Pb, Pb})=0.218$ cm²/g electron $=0.98$ cm^{-1} Pb; $\mu(\text{Fe, Fe})=0.099$ cm²/g electron $=0.45$ cm^{-1} Pb.

The absorption coefficients are expressed both in terms of the units used by Schindler in which unity corresponds to $6.06\cdot10^{23}$ extranuclear electrons per sq. cm (this unit may be conveniently named a gram electron per cm²) and in cm^{-1} lead equivalent. The multiplying constants as determined from the data have been reduced to the number of ions which would be produced under normal atmospheric conditions in one cubic centimeter by the radiation included within the 30° cone of Schindler's experiments.

TABLE I. *Air to lead transition.*

x (g electrons/cm²)	Observed	Calculated	Ratio observed to calculated
0	0.461	0.461	1.000
2.3	.468	.471	0.995
4.1	.465	.468	0.993
9.0	.427	.424	1.005
13.1	.384	.384	1.000
18.0	.349	.349	1.000
27.0	.313	.310	1.010
45.0	.287	.284	1.010
90.0	.262	.262	1.000
135.0	.246	.246	1.000

TABLE II. *Air to iron transition.*

x (g electrons/cm²)	Observed	Calculated	Ratio observed to calculated
0	0.461	0.461	1.000
3.5	.464	.462	1.005
7.0	.454	.453	1.001
14	.421	.422	0.998
21	.392	.392	1.000
35	.354	.345	1.000
70	.295	.302	0.976
105	.269	.269	1.000

In addition to these principal transitions Schindler also gives data for secondary transitions from lead to iron and from iron to lead, in each case starting from various thicknesses of the former substance. In calculating these transitions all but two of the necessary constants have already been determined from the data of the principal transitions, the remaining unknown constants being the absorption coefficients in the second medium of secondaries produced in the first, i.e., $\mu(\text{Fe, Pb})$ and $\mu(\text{Pb, Fe})$.

In calculating the ionization after a variable thickness x of the second substance m and a fixed thickness x_0 of the first substance n it is necessary to take account of the remaining air secondaries which have penetrated n, as well as the corresponding change in the intensity of the primary rays. The equation for the ionization as a function of x in m is therefore

$$\begin{aligned} J = {} & q_0\,(\text{air})\,I\exp\left[-\mu(n,\text{air})x_0+\mu(m,\text{air})x\right] \\ & +\beta(n)NI\left[e^{-\nu(n)x_0}-e^{-\mu(n,n)x_0}\right]e^{\mu(m,n)x}/\left[\mu(n,n)-\nu(n)\right] \\ & +\beta(m)NI\,e^{-\nu(n)x_0}\left[e^{-\nu(m)x}-e^{-\mu(m,m)x}\right]/\left[\mu(m,m)-\nu(m)\right] \end{aligned} \tag{6}$$

in which $\mu(m, n)$ is the only constant remaining to be determined from the secondary transition data. After subtracting all of the terms except that containing $\mu(m, n)$ from the experimental data of the first secondary transition the residuals were very close to pure exponentials with the absorption coefficients $\mu(\text{Pb, Fe}) = 0.159$ cm²/g electron $= 0.751$ cm⁻¹ Pb and $\mu(\text{Fe, Pb}) = 0.107$ cm²/g electron $= 0.48$ cm⁻¹ Pb. The constants thus determined could then be used in calculating the remaining secondary transitions, and the agreement in each case with the experimental data served as an additional check on the values of the constants. The results are shown in Tables III and IV.

TABLE III. *Iron to lead transitions.*

Thickness of iron g electron/cm²	Thickness of lead g electron/cm²	Observed	Calculated	Ratio observed to calculated
38	0	0.347	0.347	1.000
	2.3	0.349	0.351	0.995
	4.1	0.345	0.346	0.997
	18	0.295	0.382	1.048
	45	0.267	0.259	1.030
77	0	0.292	0.292	1.000
	3.2	0.286	0.304	0.940
	13.5	0.261	0.272	0.960

TABLE IV. *Lead to iron transitions.*

Thickness of lead	Thickness of iron	Observed	Calculated	Ratio observed to calculated
18	0	0.349	0.349	1.000
	3.5	.347	.341	1.018
	7.0	.336	.332	1.012
	14	.318	.321	0.991
	28	.306	.310	0.987
	42	.299	.296	1.010
	105	.265	.262	1.012
52	0	.281	.281	1.000
	14	.286	.286	1.000
	35	.279	.284	0.982
	70	.263	.268	0.980
126	0	.250	.250	1.000
	7	.254	.253	1.004
	14	.249	.258	0.965
	35	.245	.257	0.953

Schindler also gives some measurements of the absorption in aluminum and a few other substances but none of these sets of data are sufficiently extensive to make a determination of the constants with a reliability comparable with that of the foregoing data.

Table V contains the absorption coefficients $\mu(m, n)$ of the secondaries produced in medium n and absorbed in medium m and the media are arranged in the order of their atomic weights. This arrangement suggests the following laws: I. Secondaries produced by the cosmic rays in substances of lower atomic weight are the more penetrating. II. The absorption coefficients per gram electron of the secondary cosmic rays are greater in substances of greater atomic weight. To these laws may be added two others which are suggested by the values of the other constants. III. The absorption coefficients per gram electron of the primary radiation are greater in substances of lower atomic weight. IV. The production coefficients per gram electron of the secondary radiation are greater in substances of greater atomic weight.

TABLE V. *The absorption coefficient per gram electron in medium m for secondaries produced in medium n.*

m \ n	air	iron	lead
iron	0.066	0.099	0.107
lead	0.11	0.159	0.218

The multiplying constants of Eq. (5) have the following significance. $q_{n_0}I$ is the number of ion pairs formed per cc per sec. by secondaries in equilibrium in medium n with the primaries of ground level intensity which are included within the 30° cone, and $\beta(m)NI/[\mu(m, m)-\nu(m)]$ is the corresponding number of ion pairs in equilibrium with primaries of the same intensity in medium m. If we define $j^m(\theta)$[8] by the equation

$$\beta(m)NI/[\mu(m, m) - \nu(m)] = 2\pi \int_0^{30^\circ} j^m(\theta) \sin\theta d\theta \qquad (7)$$

and use the distribution function found by Medicus[9] for the secondary rays i.e.,

$$j^m(\theta) = j^m(0)(1 + 4\cos^2\theta)/5$$

then

$$j^m(0) = 1.4\beta(m)NI/[\mu(m, m) - \nu(m)].$$

The values for $j^m(0)$ are as follows

$j^{\mathrm{air}}(0) = 0.64$ ion pairs per sec. per cc
$j^{\mathrm{Fe}}(0) = 0.46$ " " " " " "
$j^{\mathrm{Pb}}(0) = 0.41$ " " " " " ".

THE ENERGIES OF THE SECONDARIES

Although it is impossible to make any strict correlation between the absorption coefficient of the secondary radiation and the energies of the corpuscular rays one may speculate as to the order of magnitude of one of these

[8] $J^m(\theta)$ is the number of ion pairs produced per cc per sec. at atmospheric pressure by secondaries in equilibrium with primaries of ground level intensity in medium m and reduced to unit solid angle in the direction θ with the vertical.

[9] G. Medicus, Zeits. f. Physik **74**, 350 (1932).

quantities from a knowledge of the other. For example, one may postulate that the average range is equal to the reciprocal of the absorption coefficient and that ions requiring about fifty volts per pair are formed at a rate indicated by the density of droplets in the expansion chamber, i.e., about thirty-six pairs per cm in atmospheric air.[1,3] This correlation neglects the possibility of large angle scattering of the secondary rays by nuclear collisions and it assumes that the secondaries are confined to a single direction. Hence the energies calculated in this way must be regarded as a lower limit, although perhaps not far from the right order of magnitude. Making this correlation on the basis of the absorption coefficient in lead, it is found that the air secondaries have an average energy of 30 million electron volts, that is, about one third of the most probable of the energies measured by Anderson. This agreement is perhaps as good as would have been expected from the crudity of the assumptions.

The Density of the Secondaries

The large values of the absorption coefficients of the corpuscular rays derived from Schindler's data at once given an explanation for the fact discovered by Bothe and Kolhorster[10] and by Rossi[11] that the interposition of absorbing material between two counters diminishes the coincidence counting rate in approximate agreement with the absorption coefficient of the primary radiation. An explanation first suggested by Bothe and Kolhorster[10] but rejected by them at that time as being unlikely, is that a single primary ray produces a large number of secondary rays distributed along its path so that coincident discharges of two counters separated by more than a few centimeters of lead, are caused, not by the same secondary, but by different secondaries initiated by the same primary ray. Without the absorbing material a single secondary ray having the proper direction will pass through both counters and, therefore, under this condition the counting rate is a measure of the intensity of the primary radiation N multiplied by the probability P of a primary ray being accompanied by one or more secondaries at any point along its path. With the absorbing material the counting rate is a measure of the number $Ne^{-\nu x}$ of primary rays which penetrate the material multiplied by P^2.

The ratio of counting rates with and without absorbing material is therefore, $Pe^{-\nu x}$. Since this ratio is found, in the experiments cited, to be of the order of $e^{-\nu x}$ it follows that P is of the order of unity. In other words, a primary ray is almost always accompanied by at least one of its secondaries and the coincidence counting rate is approximately a measure of the intensity of the primary radiation.

The average number of secondaries in equilibrium with one primary ray may be estimated from a comparison of the coincidence counting rate of two counters and the rate of production of ions in a closed vessel. The approximate flux of the primary radiation $N_\perp$ taking the results of the counter experi-

[10] Bothe and Kolhorster, Zeits. f. Physik **56,** 751 (1929).
[11] B. Rossi, Zeits. f. Physik **68,** 64 (1931).

ments of Street and Johnson,[12] is 0.0073 per sq. cm per sec. in unit solid angle in the vertical direction. The flux $q_\perp$ of secondaries, on the other hand, may be estimated from the ionization data of Schindler. From the relation $j^m(0) = Iq_\perp$, using the value $I=36$ obtained from the density of droplets in the cloud chamber, we have $q_\perp{}^{\text{air}}=0.018$; $q_\perp{}^{\text{Fe}}=0.013$; $q_\perp{}^{\text{Pb}}=0.011$. Comparing these values with the value 0.0073 for $N_\perp$ we have[13] $q_\perp{}^{\text{air}}/N_\perp=2.5$; $q_\perp{}^{\text{Fe}}/N_\perp=1.8$; $q_\perp{}^{\text{Pb}}/N_\perp=1.5$.

The Average Energy of the Primary Rays

It is now possible to estimate the energy of a primary ray as the sum of the energies of the secondaries produced by a single primary. Since the secondaries are produced largely from the atomic nuclei there is the *a priori* possibility that much of the secondary energy is energy of nuclear disintegration. This possibility must be rejected however because of the extremely high energies found by Anderson and because of the angular distribution of the secondaries. The conclusion is that the energy of the secondary is acquired at the expense of the primary ray. The total number of secondaries produced by a single primary of ground level energy is then approximately equal to $\mu(m, m) q_\perp{}^m/\nu(m)N_\perp$ which has the values: in lead, $0.218\times1.5/0.0014=233$; in iron, $0.099\times1.8/0.002=90$. With the value for iron, this being similar to the material of the cloud chamber and the average energy of a secondary ray as found by Anderson, i.e., 2.5×10^8 electron volts, the average energy of a primary ray, equal to the sum of the energies of its secondaries, is 2.2×10^{10} electron volts. Since these rays have already passed through the atmosphere and have dissipated energy by a continuous production of secondaries along their path, it is necessary to suppose the energies of these primary rays are originally greater than this figure by about a factor of two.

In conclusion it is a pleasure to acknowledge conversations with Dr. J. C. Street during which many of the ideas presented here were developed.

[12] Street and Johnson, Phys. Rev. **40**, 1048A (1932).

[13] These low values for the average number of secondaries per primary seem inconsistent with the conclusion that a primary is almost always accompanied by at least one secondary and it may be necessary to assume that the primary ray is itself an ionizing corpuscular ray. This assumption would not materially alter the interpretation of Schindler's data other than to allow a considerably greater latitude in the choice of absorption coefficients.