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The Acoustic Resonator Interferometer: II. Ultrasonic Velocity and Absorption in Gases

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The derivation of the equivalent electric network of the acoustic resonator interferometer in Part I of this paper has made it possible to develop the theory for the current in a simple resonant circuit in which the electrodes of the piezoelectric plate of the interferometer are connected to the terminals of the variable condenser of the circuit. The special case of this theory in which the circuit is excited at a constant frequency determined by the crevasse frequency of the resonator plate in its given situation with respect to electrodes and associated circuit, when the acoustic path in the interferometer is *detuned* and the resonant circuit is tuned so that its resonant maximum occurs at the same frequency, takes an especially simple form and leads to a direct procedure for determination of ultrasonic velocity and absorption in a gas in terms only of current in the resonant circuit and path-length in the interferometer, all circuit and interferometer constants dropping out. The values of current as a function of path-length obtained experimentally are in complete accord with the theory and data for ultrasonic absorption in air and in CO_2 so far obtained are in agreement with the meager data available by other methods. The role of the coefficient of reflection at the fluid-reflector surface is discussed.

I. EXPERIMENTAL OBJECTIVES AND METHODS

THE vibration of a column of fluid of variable length, one end of which is bounded by a vibrating plane face of an electrical vibrator such as a piezoelectric plate, the other end being bounded by a plane reflector parallel to the vibrating face of the plate, has been considered in Part I[1] of the present paper, and the equivalent electric network of the system was there shown to be the same as that of the quartz plate alone with modified resistance and capacity coefficients. Such a system, associated with an appropriate driving circuit, may be termed an acoustic resonator interferometer, and has already been used for the measurement of compressional velocities in liquids,[2,3,4,5] and in gases,[6] of absorption coefficients in gases,[6] and of the modification of velocity in liquids due to scattering.[5] Specific consideration of some of the methods of measurement of absorption in gases and of the role of the reflection coefficient at the boundary of the fluid will be given in the

[1] J. C. Hubbard, Phys. Rev. **38**, 1011–1019 (1931). Part I will be referred to hereinafter as I.

[2] J. C. Hubbard and A. L. Loomis, Phil. Mag. **5**, 1178–1190 (1928); and A. L. Loomis and J. C. Hubbard, J.O.S.A. and R.S.I. **17**, 295–307 (1928).

[3] E. B. Freyer with J. C. Hubbard and D. H. Andrews, Jour. Am. Chem. Soc. **51**, 759–770 (1929). E. B. Freyer, *ibid.* **53**, 1313–1319 (1931).

[4] E. Klein and W. D. Hershberger, Phys. Rev. **37**, 760–774 (1931).

[5] C. R. Randall, Bur. Stand. Jour. Res. **8**, 79–99 (1932).

[6] J. C. Hubbard, Phys. Rev. **35**, 1442 (1930); **36**, 1668–1669 (1930).

present paper. Mr. W. D. Hershberger[7] has presented independently an analysis of the motion of the fluid column and of its reaction on the piezoelectric plate, his treatment being somewhat more restricted than that of the author, particularly in that he does not consider the effects of coefficients of reflection at the fluid-reflector surface of value less than unity.

Among the methods of measurement which have been used, undoubtedly the simplest is that in which the interferometer electrodes are connected to the terminals of a variable condenser which is part of a simple resonant circuit driven by an oscillation generator of suitable frequency. The measurements consist in this case of a determination of current in the resonant circuit as a function of the length r of the fluid column.[2,3,4,6] The current readings pass through sharp maxima (for gases, minima for liquids) or peaks as r is increased, these peaks having been interpreted as marking the value of r, for resonance in the fluid, successive resonant positions of the reflector being one-half wave-length apart. An analysis of this method for gases is given in the present paper.

In a second method, far more sensitive than the first, the interferometer electrodes are connected to a secondary coil coupled to the primary of an oscillation generator. As r is varied a cyclical variation of frequency of the generator takes place which is compensated by the variation of a vernier condenser in parallel with the capacity of the generator, constancy of frequency being assured by using the double heterodyne beat method.[2,3,6] From the variation of vernier condenser readings as a function of r the acoustic behavior of the medium is deduced. The theory of this method will be given in a later paper.

A third method[5] makes use of the current in the leads to the interferometer itself as a function of r, the circuits being otherwise as in the first method described above. Klein and Hershberger[4] report a variety of experiments in which a number of methods were employed, the interferometer system being treated as an impedance in alternating current measurements.

It is essential that the power dissipated in the acoustic system be so small as to produce negligible temperature effects. In practice it has been found possible to reach the limits of accuracy imposed by the sensitiveness of the thermocouple and galvanometer used in the first method with amounts of power several orders smaller than that which is capable of influencing the results. The power necessary in the second method is still much smaller.

The most important factor causing a departure of the acoustic system from that theoretically expected is to be found in the conditions at the electrodes of the piezoelectric plate. This factor need not be considered if the electrodes are of sufficiently small mass and if their attachment to the plate is of such nature as not materially to modify the crevasse. As will be seen the constants of the crystal including the effect of electrode mounting upon the position and shape of the crevasse may be eliminated from consideration in determinations of velocity and absorption.

[7] W. D. Hershberger, J. Acous. Soc. Am. 3, 263–268 (1931).

II. Method of Forced Vibrations. Ultrasonic Velocity and Absorption in a Gas

As shown in I the acoustic interferometer may be represented by an electric network consisting of a fixed capacity K_1 shunted by a series consisting of the fictitious inductance L, capacity K' and resistance R', where

$$R' = R + AB\rho v P \tag{1}$$

$$1/K' = 1/K + AB\rho v\omega Q \tag{2}$$

R and K being the fictitious series resistance and capacity respectively of the quartz plate itself, A the cross section of the fluid column, B a constant of quartz, and ρ and v, respectively the density of the fluid and the wave velocity in it.

P and Q have the values, in general, with $x=0$ in I, Eqs. (12) and (13), respectively

$$P = \frac{1 - \gamma^3 e^{-4r\alpha} + \gamma(1-\gamma)e^{-2r\alpha}\cos(2r\omega/v)}{1 - 2\gamma^2 e^{-2r\alpha}\cos(2r\omega/v) + \gamma^4 e^{-4r\alpha}} \tag{3}$$

$$Q = \frac{\gamma(1+\gamma)\sin(2r\omega/v)}{1 - 2\gamma^2 e^{-2r\alpha}\cos(2r\omega/v) + \gamma^4 e^{-4r\alpha}} \tag{4}$$

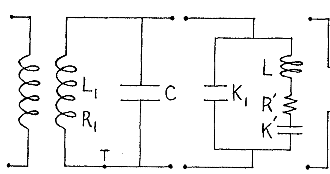

Fig. 1. Resonator circuit with interferometer replaced by its equivalent electric network.

where α is the attenuation factor of particle velocity in the fluid and is equal to one-half the coefficient of absorption, and γ is the coefficient of reflection of particle velocity between the fluid and the reflecting plate. The distance r is that between the vibrating plate and the reflector, i.e., the length of the fluid column.

Let the electrodes of the piezoelectric plate be connected to the terminals of the variable condenser C of the resonant circuit L_1, R_1, C, Fig. 1. The resistance R_1 includes the resistance of the vacuum thermocouple T which is inserted between C and L_1 on the grounded side of the interferometer. Let the inductance L_1 be loosely coupled to the output of an oscillation generator which may be kept at any desired frequency, so that in L_1 we have the induced e.m.f., $E=E_0\, e^{j\omega t}$. We have for the impedances of the various branches of the circuit, putting $\theta=j\omega$: $Z=L_1\theta+R_1$; $Z_1=1/K_1\theta$; $Z_2=L\theta+R'+1/K'\theta$; $Z_3=1/C\theta$.

Solving for i_0, the amplitude of current in the L_1R_1 branch, including the thermocouple heater, we have

$$i_0^2 = \frac{E_0^2\left\{R'^2K'^2\omega^2 + \left[1 - LK'\omega^2 + \frac{K'}{C+K_1}\right]^2\right\}\omega^2}{\left\{\left[\frac{1-(C+K_1)L_1\omega^2}{C+K_1}(1-LK'\omega^2) - \left(R_1R'K' + \frac{L_1K'}{C+K_1}\right)\omega^2\right]^2 + \left[R'K'\frac{1-(C+K_1)L_1\omega^2}{C+K_1} + R_1(1-LK'\omega^2) + \frac{R_1K'}{C+K_1}\right]^2\omega^2\right\}}. \tag{5}$$

Putting $I_0 = E_0/R_1$, the maximum value of i_0 at resonance with the interferometer disconnected, and also putting

$$p = 1 - (C + K_1)L_1\omega^2 \tag{6}$$

$$q = 1 - LK'\omega^2, \quad q_0 = 1 - LK\omega^2 \tag{7}$$

$$\phi_1 = R_1(C + K_1)\omega \tag{8}$$

$$\phi_2 = R'K'\omega \tag{9}$$

and

$$\sigma = i_0/I_0$$

we have

$$\sigma^2 = \frac{\phi_1^2[\phi_2^2 + (q + K'/(C + K_1))^2]}{[pq - \phi_1\phi_2 - (1-p)K'/(C+K_1)]^2 + [\phi_2 p + \phi_1(q + K'/(C + K_1))]^2} \cdot \tag{10}$$

Eq. (10) is an exact description of the current amplitude in the thermocouple heater relative to the current amplitude at resonance in the resonant circuit with the interferometer disconnected, as a function of the decrements of the oscillating circuit and of the interferometer system, of the departure from resonance in the two systems, of the pure capacity $C+K_1$, and of the fictitious capacity K' of the acoustic system. This equation is similar to the one deduced by D. W. Dye[8] for the piezoelectric resonator in a resonant circuit but is complicated by the inclusion of the modified R', K' and ϕ_2 terms introduced because of the interferometer system. Dye's analysis includes a capacity K_2 between the resonator and the resonant electrical circuit to account for the effect of variable electrode distance from the quartz plate. This capacity is here suppressed, the electrode conditions being maintained constant.

In view of the exhaustive study which Dye has made of his equation no extended discussion of its analogue, Eq. (10), is necessary here. It should be pointed out, however, that in general σ^2 as a function of the frequency rises to the usual resonance maximum for $p=0$, and that further, if $q+K'/(C+K_1)=0$ in the same neighborhood there will be a sharp minimum or *crevasse* in the resonance curve. This crevasse is extraordinarily narrow because of the extremely small value of ϕ_2 as compared to ϕ_1, and at its center σ^2 falls to a small fraction of its value on either side. By repeated adjustment of C and ω it is possible to find the value of the frequency for which p and $q+K'/(C+K_1)$

[8] D. W. Dye, Proc. Phys. Soc. Lond. **38**, 399–458 (1926).

are zero simultaneously, provided the reflecting plate is set at a distance $r=(2n+1)\lambda/4$, so that P and Q are both equal to zero and $K'=K$, making $q=q_0$. In practice it is sufficient to find a crevasse for any setting of r, whereupon a rough determination of $\lambda/2$ is made. The setting of r is now made at $\lambda/4$, then by successive adjustment of C and ω the crevasse is made symmetrical. The interferometer is then disconnected and C is varied, ω being kept constant, until i_0 has the maximum value I_0, which is determined. The interferometer is then reconnected and C is adjusted to its former value for a symmetrical crevasse, which completes the adjustment necessary for a determination of σ^2 as a function of r. A series of observations of i taken under these conditions is shown in Fig. 2, curves I and II, in which only maxima

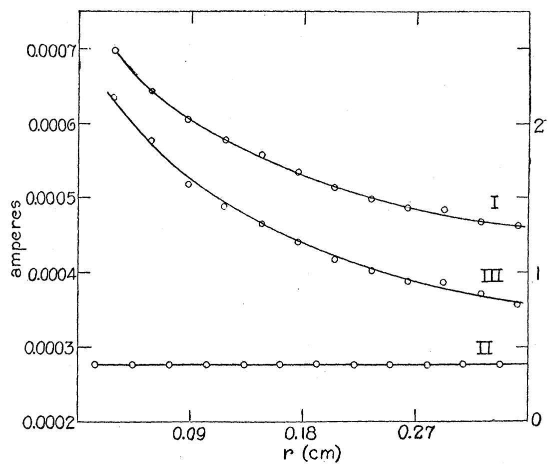

Fig. 2. Curves I and II, maximum and minimum galvanometer readings as a function of r. Curve III, representation of Eq. (17) as a function of the same observations.

and minima of current are shown. The behavior in the immediate neighborhood of a maximum for the same case is shown in Fig. 4, Insert a, the abscissa scale being magnified twentyfold. In the preliminary adjustments just described, we have made $p=0$, $q_0+K/(C+K_1)=0$, and, remembering Eqs. (1), (6) and (7), $q+K'/(C+K_1)=K'AB\rho v\omega Q$, so that

$$\sigma^2 = \frac{(1+SP)^2+S^2Q^2}{[1+SP+1/R\omega\phi_1(C+K_1)]^2+S^2Q^2} \tag{11}$$

where

$$S = AB\rho v/R. \tag{12}$$

When the reflector is at a distance $r=(2n+1)\lambda/4$ from the face of the vibrating plate, if n is small, $P=$zero to a very close approximation, and $Q=0$.[9] Let σ_0 be the value of σ for this reflector position. We have then

[9] See I, Figs. 2 and 3. These figures were drawn for the case in which γ was assumed to be unity. As we shall see later the statement made above is practically always valid for gases.

$$\sigma_0 = 1/[1 + 1/R\omega\phi_1(C + K_1)]$$

or

$$R\omega\phi_1(C + K_1) = \sigma_0/(1 - \sigma_0). \tag{13}$$

The value of σ given by Eq. (11) under the prescribed conditions will have maximum values, σ_m, for values of r such that $Q=0$ and $P=P_m$ given by

$$P_m = (1 + \gamma e^{-2r\alpha})/(1 - \gamma^2 e^{-2r\alpha}). \tag{14}$$

We then have

$$\sigma_m = (1 + SP_m)/[1 + SP_m + (1 - \sigma_0)/\sigma_0], \text{ or}$$

$$S = AB\rho v/R = [\sigma_m(1 - \sigma_0)/(1 - \sigma_m)\sigma_0 - 1]/P_m = \text{constant}. \tag{15}$$

Putting $\gamma = e^{-\beta}$, Eq. (14) becomes $P_m = e^{\beta/2}$ [cosh $(r\alpha+\beta/2)$]/sinh $(r\alpha+\beta)$, or, to a very close approximation, unless r is large

$$P_m = (1 + \beta/2)/(r\alpha + \beta). \tag{16}$$

This value of P_m inserted in Eq. (15) would lead at once to a determination of α and β if we had the means of accurate determination of A, assumed to be the area of the vibrating plate exposed to the fluid. As is well known, only a small portion of the surface of a piezoelectric plate as ordinarily cut is in vibration in a given mode[10,11] though by optical means it can be shown that the entire cross section of the column of a fluid in an interferometer is set into resonant vibration and presumably reacts upon the crystal over the whole of its exposed face. Hershberger[12] has recently pointed out that the assumption that A is to be taken as the area of the plate exposed to the fluid is not valid. As we shall see, however, it is sufficient to assume an effective A, which is constant. In the absence of knowledge of its magnitude we have then in Eq. (15) only a means of computing the relative effects of absorption and reflection. For, substituting P_m from Eq. (16) into (15), we have

$$[(1 - \sigma_0)/\sigma_0][\sigma_m/(1 - \sigma_m) - \sigma_0/(1 - \sigma_0)](r\alpha/\beta + 1) = S' \tag{17}$$

where

$$S' = S(1 + \beta/2)/\beta = \text{constant}. \tag{18}$$

Eq. (17) enables us at once to compute α/β from a series of observations of values of σ_m and the corresponding values of r, and the mean value of σ_0. Fig. 2, curve III, shows the observations of Fig. 2, curves I and II, in terms of Eq. (17), the smooth curve passing through the maxima being the hyperbola given by Eq. (17), where $\alpha/\beta = 6.276$, and $S' = 2.548$. All types of acoustic interferometers which have been used by the method described here yield results for gases in entire accord with the theory showing that within the limits of errors of observation the quantities entering into S', Eq. (18), may be considered as constant.

[10] Nat. Phys. Lab. Report for 1928.
[11] P. Vigoureux, Quartz Resonators and Oscillators, London (1931).
[12] W. D. Hershberger, Physics **2**, 269–273 (1932).

Having determined α/β from a given series of observations, it remains to be seen how α and β may separately be determined. Putting $(1-\sigma_0)/\sigma_0 = a$, combining Eqs. (11) and (13), and solving for $1+SP$, we have

$$1 + SP = (a/b)\{1 + [1 + b - (QSb/a)^2]^{1/2}\} \tag{19}$$

where $b = (1-\sigma^2)/\sigma^2$. Remembering Eq. (15), it is seen that in (19) we have a relation, free of all circuit and interferometer constants, involving only galvanometer readings, values of r, and the values of α and β. Having determined the ratio α/β from a series of observations, using Eq. (17), we may assume a value of α, from which we may compute β and S'. The corresponding value of S is then found by means of Eq. (18). Choosing any value of σ on the r, σ curve, preferably at about one-half the altitude of a peak, we compute the value of b. The corresponding value of r is found from the curve for use in computing P and Q. These functions involve r both in the hyperbolic and circular functions composing them. The argument, $2r\omega/v$ of the circular functions may be found directly if the frequency is known, but may be found independently of frequency from the interferometric determination of $\lambda/2$, since $\theta = 2r\omega/v = 4\Delta r\pi/\lambda$, where Δr is the distance from the point for which σ is chosen to the axis of the peak, or one-half the peak width, since, owing to its extreme narrowness, the symmetry of the peak is not affected by damping to a determinate degree. It is now possible to compute the left and right hand sides of Eq. (19). If the left hand side is larger than the right, the tentative value of α which was chosen is too large, etc. Successive trials lead, usually in three or four steps, to the region of intersection of the curves of the two sides of this equation plotted as functions of α, when by interpolation the value of α satisfying the equation is found. The process may be shortened by noting that $1+SP$ must lie between the values a/b and $(a/b)[1+(1+b)^{1/2}]$ and by using the approximations $P = 4(r\alpha+\beta)/[4(r\alpha+\beta)^2+\theta^2]$, and $Q = 2\theta/[4(r\alpha+\beta)^2+\theta^2]$ in all but the last two steps.

Measurement of velocity

The values of r for successive values of maximum σ^2 given by Eq. (11) are to a degree of precision beyond experimental measurement, independent of the damping,[13] except in excessively absorbing media, and are determined by the maximum values of P, which in turn occur when $\cos 2r\omega/v = 1$, or when $r = n\lambda/2$. It then remains to measure the frequency in order to evaluate the velocity v.

III. Experimental Tests of the Theory

Three interferometers have been built for the purpose of testing the theory. As the results obtained from all of them differ in no important detail, no extensive description of them will be given.

Interferometer I, Fig. 3a, was designed for operation in the open air. It consists of the quartz plate Q resting upon a grounded electrode mounted at the top of a Bakelite support. This support is attached to a plate provided

[13] Cf. also C. D. Reid, Phys. Rev. **35**, 829 (1930).

with adjusting screws so that the face of the plate may be made parallel to the plate glass reflector R. This adjustment was made by viewing fringes in sodium light passed through R and falling upon Q when the two were close together and R was started in retreat from the crystal. The supports for R and Q are mounted upon the ways of a micrometer comparator of great accuracy made by the late Professor H. A. Rowland. The screw is provided with a large head divided into 100 parts, and has a fine adjustment making possible measurements to 10^{-4} mm. This interferometer was particularly useful in testing the effect of various methods of crystal mounting and types of electrodes. Its great accuracy and fine adjustment were used both with the method described in this paper, the crystal being used as a resonator, and with the crystal used as an oscillator, after the methods of Pierce[14] in order to test the effect of variation of velocity with reflector distance reported by

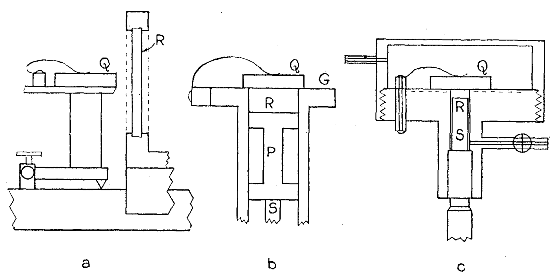

Fig. 3.

Reid.[13] The effect found by Reid was of decreasing amount with increase of frequency, being small with the highest frequencies which he employed. As the frequencies so far used in this laboratory are much higher the effect to be expected might be so small as to escape detection. So far, in the range 400 to 700 k.c. no such effect has been detected by either the resonator or oscillator method, although the interferometer distance has been varied from the face of the crystals used to a distance of 10 cm.

Interferometer II, Fig. 3b, was designed to give the largest possible reaction upon the crystal by the fluid column. It consists of a solid cylinder of brass in which a concentric hole 1″ in diameter was bored. The upper end of the cylinder was ground plane. Upon it rests the crystal Q, the lower face of which is gold sputtered providing the grounded electrode. Contact is made with the upper electrode by means of a light spring. The plunger P is close fitting, with end R ground plane and parallel to the surface G. P is displaced by the micrometer screw S.

Interferometer III, Fig. 3c, was designed to test the effect of the removal of water vapor and CO_2 from the air and to study other gases at room temperature. The end of the micrometer screw serves as the reflector, its parallelism to the surface upon which the crystal rests being tested by sodium fringes, with an optical flat in place of the sputtered crystal. The air or other gas to be tested enters a small channel at the side of the screw, passing up-

[14] G. W. Pierce, Proc. Am. Acad. **60,** 269–302 (1925).

ward and through a small slot under the crystal and into the chamber above, from which it escapes.

The constant frequency oscillator consists of a dynatron oscillator with secondary output connected to the inner grid of a UX222 tube having 45 volts applied to the outer grid. The output of this tube is amplified in two stages of resistance capacity coupling, with nonregenerative tuned output in the last stage. Loosely coupled to the output is the coil L_1 of the resonant circuit which excites the interferometer. Minute variation of the frequency of the dynatron oscillator is provided by a precision condenser of large capacity in series with the condenser of the dynatron circuit. In addition a vernier condenser of very small capacity and with a linear scale of 50 cm length is connected in parallel with the precision condenser so that rapid adjustment for symmetry of crevasse may be made. Currents are measured by a Western Electric vacuum thermocouple and a galvanometer provided with a number of shunts.

Connection to the interferometer is made through a mercury switch, which serves also at any time to connect the interferometer into a Pierce circuit so that studies by both the resonator and oscillator method may be compared, the crystal being under identical conditions. The results of these comparative studies will be presented in another paper.

The most general conclusion which can be drawn from the experiments so

TABLE I. *Galvanometer and screw readings. 597 k.c. First twelve rows give i_m with corresponding peak number m. Column (1), room air, interferometer II: (2), room air, interferometer III: (3), dry air: (4), dry, CO_2—free air: (5), CO_2.*

m	(1)	(2)	(3)	(4)	(5)
1	0.001484	0.000720	0.000698	0.000703	0.000653
2	1432	674	642	647	541
3	1383	634	603	614	484
4	1344	603	577	583	444
5	1295	582	557	557	420
6	1255	563	534	540	394
7	1211	539	513	514	386
8	1163	524	498	498	377
9	1119	508	485	486	
10	1094	504	484	483	
11	1050	492	467	470	
12	1020	483	462	456	
t°C	22.2	23.3	24.0	24.0	23.3
$\lambda/2$	0.02928	0.02944	0.02941	0.02944	0.02389
i_0	0.000235	0.000310	0.000278	0.000279	0.000304
I_0	0.002398	0.002329	0.002275	0.002293	0.002275
r cm	0.2343	0.3538	0.3529	0.3533	0.1433
$2\ \Delta r$	0.00322	0.00159*	0.00157	0.00156	0.00200
i	0.0006	0.000375	0.000350	0.000350	0.000350
r cm	0.3514				
$2\ \Delta r$	0.00325				
i	0.0006				

* Peak structure not satisfactory owing to rapid drift of frequency and necessity of repeated correction.

far carried out is that for every case investigated, in the frequency range 400–700 k.c. and with all three of the interferometers just described, the results for the maxima and minima of the σ, r curve can be expressed within experimental error by the hyperbolic relation given in Eq. (17). The value of α/β and of S' for any set of observations may be determined by the method of least squares. Table I gives the original data for five cases at the frequency 597 k.c. The computed values of α/β and S' for each of the sets are given in Table II. Fig. 2 is drawn for the results of column (3). Fig. 4, Inserts (a) and (b), show the structures respectively of the 12th peak for dry air, and the 6th peak for CO_2. From these and other similar peaks the data for $2\Delta r$ in Table I are obtained. For the purpose of comparison of different sets of results taken

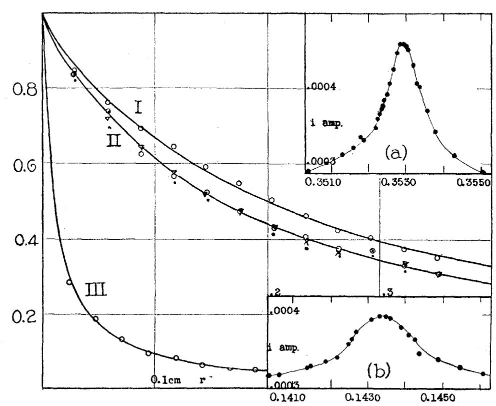

Fig. 4. Reduced observations compared with $1/(r\alpha/\beta+1)$.
Curve I, room air in interferometer II; Curve II, with circles, dry air in interferometer III; Curve III, CO_2 in interferometer III. Insert (a), twelfth current peak for dry air; Insert (b), sixth current peak for CO_2.

with different gases or under different conditions, as with different interferometers, we may plot $[(1-\sigma_0)/\sigma_0][\sigma_m/(1-\sigma_m)-\sigma_0/(1-\sigma_0)]/S'$ as a function of r. This should be compared with its theoretical equivalent, $1/(r\alpha/\beta+1)$. Fig. 4 shows as an example the first of the above functions for the observations of Table I indicated by circles, points, etc., and full curves, I, II, III, for $1/(r\alpha/\beta+1)$ for three of the sets corresponding respectively to columns (1), (3) and (5) of Table I. It is to be noted that all such curves start with ordinate unity at $r=0$.

Absorption measurements

Having determined α/β, it is possible, as has been explained, to find the separate values of α and β from some value of σ and its corresponding r by a method of graphic interpolation. The accuracy with which these quantities may be obtained is affected by the constancy with which the frequency and amplitude of the driving circuit may be maintained. In the absence of a piezoelectrically driven generator of required frequency for driving the resonant circuit containing the interferometer, it has been the practise to test the constancy of frequency at the conclusion of readings for each peak by setting r at the nearest value of $(2n+1)\lambda/4$ and noting whether i is at the minimum value i_0. To test whether the amplitude of output has been changed, the interferometer is disconnected and a reading of I_0 is taken, corresponding to the resonance maximum of the interferometer circuit. If either i_0 or I_0 has changed the intervening observations are rejected. This practise has entailed a loss of time and labor, making it highly desirable to use crystals for driving the generator. However, since the particular crevasse of a given crystal most suitable for interferometric work is often not at one of the principal frequencies of the crystal, the process of accumulation of plates for driving purposes involves first a careful exploration of crevasses for their suitability in the interferometer, and then of preparing oscillator plates, one for each frequency which can be used.

The results given in Table I were taken with a rectangular plate. The crystal was an excellent oscillator and no other crevasses were observed in the neighborhood of its principal crevasse frequency. The peaks were symmetrical and showed no evidence at any time of having any satellites. This crystal was therefore regarded as being particularly suitable for the present purpose of illustration of the theory. The values of α/β, S', α, and γ computed from the results given in Table I are shown in Table II.

TABLE II. *Absorption and reflection coefficients.* $\nu = 597$ *k.c.*

	Interferometer	Gas	t°C	$\lambda/2$ cm	α/β	S'	α	γ
1	II	Room air	22.2	0.02928	5.045	16.63	0.130	0.9745
2	III	Room air	23.3	0.02944	6.384	2.262	0.106*	0.9836
3	"	Dry air	24.0	0.02941	6.276	2.548	0.110	0.9827
4	"	Dry CO_2 free air	24.0	0.02944	6.312	2.561	0.108	0.9831
5	"	CO_2	23.3	0.02389	94.78	5.705	0.702	0.99261

* See remark under Table I.

Data on absorption are very meager. From Neklepajew[15] we have for dry air, $\mu = 0.00073/\lambda^2$, which for $\lambda/2 = 0.02941$ gives $\mu = 0.211$. For α this gives 0.105, in fair agreement with the value 0.110 for dry air in Table II. For CO_2, we have the calculations of Herzfeld and Rice[16] from the velocity measurements of Pierce[14] and by extrapolation from the results of Abello[17] giving respectively for μ, $3\times10^{-12}\nu^2$ and $5.6\times10^{-12}\nu^2$. We have, for $\nu = 5.97 \times 10^5$, from the former, $\alpha = 0.54$, and from the latter $\alpha = 1.0$. These results may be compared with $\alpha = 0.702$ for CO_2 in Table II.

The coefficient of reflection

An interesting feature of these experiments is the large departure from unity of the coefficient of reflection γ. If we take as the coefficient of reflection of air waves at a surface of brass, $\gamma = (R_1 - R_2)/(R_1 + R_2)$, where R_1 for brass

[15] N. Neklepajew, Ann. d. Physik **35,** 175–181 (1911).
[16] K. F. Herzfeld and F. O. Rice, Phys. Rev. **31,** 695 (1928).
[17] T. P. Abello, Proc. Nat. Acad. Sci. **13,** 699 (1927).

is $\rho v = 8.5 \times 3.65 \times 10^5 = 3.1 \times 10^6$, and for air, $R_2 = 40$, we expect γ to be $1 - 1.3 \times 10^{-5}$. The departure from unity in these experiments is actually several orders greater, as may be seen from Table II. In explanation of this we may recall that we are dealing with a resonant system, and that it is to be expected that resonance in the reflector system will result in a motion of the reflector surface depending upon the dimensions and material of the reflector. The coefficient of reflection in terms of motion of reflector surface is considered in Part I of this paper. It is to be noted in Table II that γ changes abruptly from interferometer II to III, and from air to CO_2. Similar changes are noted in passing from one frequency to another. It would be of interest in this connection to use as a reflector a second piezoelectric plate of quartz so that the amplitude of vibration of the reflecting surface could be studied directly. The equations developed for this case, where it was assumed that the reflector was rigid, I, Eqs. (17) and (18) do not apply in so far as γ is not unity. From I, Eqs. (12) and (13) we have putting $x = r$,

$$P_r = [(1+\gamma)(1-\gamma^2 e^{-2r\alpha})e^{-r\alpha} \cos (r\omega/v)]/[1-2\gamma^2 e^{-2r\alpha} \cos (2r\omega/v) + \gamma^4 e^{-4r\alpha}] \quad (20)$$

$$Q_r = [(1+\gamma)(1+\gamma^2 e^{-2r\alpha})e^{-r\alpha} \sin (r\omega/v)]/[1-2\gamma^2 e^{-2r\alpha} \cos (2r\omega/v) + \gamma^4 e^{-4r\alpha}]. \quad (21)$$

These equations, with Eqs. (1) and (2) applied to the reflector, considering its network in combination with a suitable thermogalvanometer connected to its electrodes, would make practicable a detailed study of reflection phenomena. Grossmann and Wien[18] have used a piezoelectric plate as reflector for the study of frequency variation of a radiating crystal oscillator as a function of reflector distance. Grossmann[19] has used an arrangement of two crystals as source and detector for the study of absorption in CO_2, but at this writing details of his method and procedure are not available.

It is of interest to compare the effect on the observations of using different cross sections of fluid column. From Eq. (12), $A = SR/B\rho v$, and from Eq. (13), $R = \sigma_0/(1-\sigma_0)\omega\phi_1(C+K_1)$. In changing under the same conditions from one interferometer to another, $B\rho v$ and $\omega\phi_1(C+K_1)$ remain practically unchanged, so that the areas of cross section of the two columns should be in proportion to the respective values of $S\sigma_0/(1-\sigma_0)$, or $S'\sigma_0\beta/(1-\sigma_0)(1+\beta/2)$. For interferometer II, room air, we have $S' = 16.63$, $\beta = 0.0258$, $\sigma_0/(1-\sigma_0) = 0.1086$, and for interferometer III these quantities have the respective values 2.262, 0.0166, 0.1537, implying a ratio of cross section of interferometer II to III of 8.03. The actual areas were 5.07 and 0.707 cm², having a ratio 7.18. The agreement of these figures is closer than might be expected in view of the remarks following Eq. (16). The particular crystal for which the above numbers are cited vibrates vigorously for the mode considered at two points near opposite corners of its rectangular area, little trace of motion being found over much of its surface. In using such a crystal with interferometer III it is necessary to adjust the crystal so that one of its vibrating areas is exposed to the gas. The nearness to equality of the ratios given above and

[18] E. Grossmann and M. Wien, Phys. Zeits. **32**, 377–378 (1931).
[19] E. Grossmann, Phys. Zeits. **33**, 202 (1932).

similar values in many other cases point to the conclusion that what we have called the effective value of A is nearly equal to the actual area exposed to radiation. It seems probable that this equality would be exact with crystals cut in the manner described by Straubel[20] in which case the absolute value of S in Eq. (15) could be determined and the coefficient of absorption of the gas could be evaluated directly from observations of maxima and minima of σ, using Eqs. (15) and (16), thus avoiding the laborious procedure of analysis of peak form.

A study is in progress, with the methods here outlined, of the ultrasonic absorption constants of several gases, considered as functions of frequency and temperature.

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[20] H. Straubel, Phys. Zeits. **32**, 222 (1931).