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The Hall Effect With Audiofrequency Currents

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Hall e.m.f. in tellurium for currents of frequencies up to 25,000 cycles per sec.—The Hall e.m.f. in tellurium has been investigated with currents of frequencies up to 25,000 cycles per second, with particular reference to any effect of frequency on the magnitude of the Hall coefficient R_H, and on the phase of the Hall e.m.f. relative to the longitudinal current producing it. A Hartley-circuit vacuum-tube oscillator furnishes the current for the specimen, which is placed in a constant magnetic field of about 3000 gauss. The Hall e.m.f., amplified by a resistance-coupled vacuum-tube amplifier, is observed on a cathode-ray oscillograph. A general method of procedure is indicated for use in *comparing alternating e.m.f.'s* as small as a few millivolts or less with respect to magnitude and phase. The current drawn and the power consumed are entirely inappreciable. The results of a comparison method of this sort and a null method agree in the conclusion that in the range studied, the Hall coefficient for tellurium is constant to within about 2 percent for frequencies up to 10,000 c.p.s. and to within 7 percent for the complete range. This agrees with the results of Smith on bismuth. For the specimen used the value found for the Hall coefficient is $R_H = 481$. The two methods also agree in finding the frequency, wave-form, and phase of the Hall e.m.f. the same as those of the longitudinal current producing it. The phase difference, if any, is believed to be less than 2° at frequencies below 20,000 c.p.s. and less than 4° at 25,000 c.p.s.

ALTERNATING-CURRENT HALL EFFECT

THE usual equation for the Hall e.m.f.

$$E_H = R_H HI/d \tag{1}$$

(in which R_H is the Hall coefficient, H the magnetic field, d the thickness of the plate, and I the current) shows that the e.m.f. reverses direction with reversal of the current. The Hall effect may be regarded as a rotation of the equipotential lines in the plate, and it may be readily shown[1] that

$$R_H H = \rho \tan \theta \tag{2}$$

where θ is the angle of rotation and ρ is the resistivity of the material. The angle of rotation is thus seen to be independent of the magnitude and direction of the current.

The number of workers who have investigated the Hall effect with alternating currents is surprisingly small. All investigators[2,3,4,5] agree that the

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[1] Campbell, Galvanomagnetic and Thermomagnetic Effects, Longmans, Green, and Co., 1923.

[2] Des Coudres, Phys. Zeits. **2**, 586 (1901).

[3] Von Traubenberg, Ann. d. Physik **17**, 78 (1905).

[4] Zahn, Ann. d. Physik **36**, 553 (1911).

[5] Smith, Phys. Rev. **35**, 81 (1912).

magnitude of the effect is the same as with direct current, but no further observations of its character seem to have been reported.

In the work described in the present paper, alternating current, furnished by an oscillating vacuum-tube circuit was maintained in a tellurium plate. A unidirectional magnetic field produced an alternating Hall e.m.f. which was then amplified by the use of a vacuum-tube amplifier and observed on a cathode-ray oscillograph.

An arrangement of this sort has two important advantages over those previously used for studying the a.c. Hall effect. In the first place, practically no current is drawn from the circuit for the purpose of measurement (observations indicating that even the d.c. grid current is less than 10^{-9} amp.), and thus we are able to deal with an actual Hall e.m.f. rather than with a Hall current, which may disturb conditions in the plate, and which in any case necessitates corrections for the inductive effects of leads and measuring instruments. Secondly, observations can be made of the frequency, wave form, and phase of the Hall e.m.f. as well as of its magnitude.

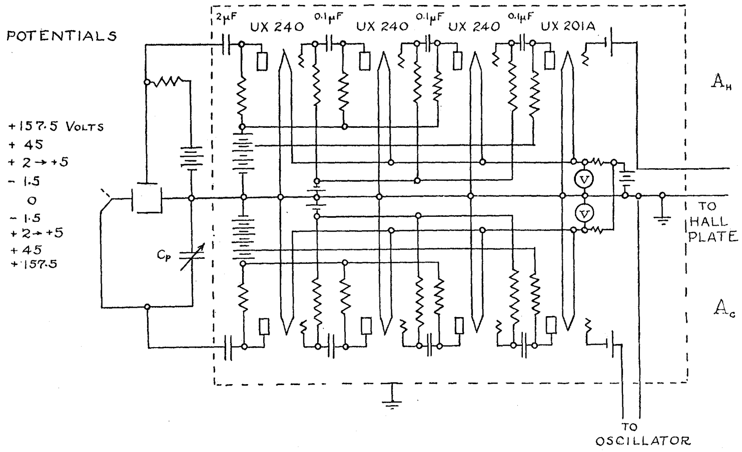

Fig. 1. Amplifier—oscillograph circuit diagram.

Tellurium was chosen because of its extremely large positive Hall coefficient. Bismuth, the material used in the high frequency work of Smith[5] (the only other observer to work at higher than commercial frequencies), has a negative coefficient of smaller magnitude.

EXPERIMENTAL ARRANGEMENT

Oscillator

The vacuum-tube oscillator was of the conventional Hartley circuit type, utilizing a UX 250 tube and giving a maximum power output of almost five watts. It was unshielded and was placed about twelve feet from the other apparatus, it being found by experiment that this distance was sufficient to prevent direct transfer of energy to the shielded amplifier and was more satisfactory than shielding. The inductance was a flat multilayer coil with air core. For the purpose of electrostatic shielding a grounded strip of solid copper was placed around the coil, the lapped ends being insulated from each other in order to avoid electromagnetic shielding. Outside this, ten turns of wire formed a pick-up coil sufficient to maintain a half-ampere current in the tellurium plate.

The specimen was shielded from the oscillator and other electrical disturbances by being placed in a grounded copper box.

Hall plate and magnetic circuit

The tellurium plate itself, of dimensions $4\times2\times0.25$ cm, was cast in a rectangular hole cut in an asbestos plate and supported on a second asbestos plate rotated 45° with respect to the first, to give support to the lead wires. Platinum wires running through a groove between the asbestos plates were melted into the tellurium when the plate was cast. These made three fixed contacts with the plate—one at each end and one on one side.

A fourth contact P on the opposite side, was a movable pin which made contact with the upper surface of the plate. This pin was mounted on a carriage, similar to the tool-rest on a lathe, which gave by adjustment of micrometer screws two-dimensional motion in the plane of the specimen. The pressure of the contact at P could be adjusted by a thumbscrew.

The plate was mounted in the air gap of a magnetic circuit in which fields up to slightly more than 4000 gauss could be obtained.

Amplifiers

The two amplifiers, placed in a second large copper box, each consisted of a UX 201A tube followed by three stages of resistance-coupled amplification employing the "high-mu" UX 240 tubes. This gave a voltage amplification of 40,000 when needed. The amplification was controlled by varying the filament voltage.

One amplifier served to amplify the Hall e.m.f. or an ohmic p.d., while the other, merely in order to impress on the oscillograph a voltage of the same frequency as the first but of variable phase, amplified the voltage picked up by a single-turn coil about a foot away from the oscillator. This coil could be rotated to vary the amplitude. The variation of phase was accomplished by the use of a variable condenser C_P placed across the amplifier output. When this variation was not sufficient, a resistance was placed in series with the condenser and the potential across one or the other placed on the oscillograph.

It was not desirable to connect the oscillograph directly across the plate resistance of the last amplifier tube, because of the potential difference caused by the d.c. plate current. An output transformer did not prove satisfactory because of variations in magnitude and phase of the output with change of frequency and conditions of operation of the amplifier. Placing a two microfarad blocking condenser in the line to the oscillograph, however, accomplished the desired result without appreciable reduction of the a.c. voltage to be measured.

Oscillograph

The cathode-ray oscillograph, a Bedell-Reich Oscilloscope,[6] was used to examine the output of the first amplifier, either with the horizontal linear time axis or with the horizontal sinusoidal time axis provided by the second amplifier, or with no horizontal displacement at all. Amplified stabilization, as described by Bedell and Kuhn,[7] was employed with the linear time axis.

A method of frequency measurement[8] by means of a calibration of the current charging the condenser in the "sweep voltage" circuit of the linear time axis of the oscilloscope was employed.

In order to avoid the distortion[9] often produced in the oscillograph tube when the electric field across the vertically-deflecting plates is zero, a 45-volt B-battery in series with a resistance was placed across these plates. The spot was then brought back to the center of the screen by a permanent magnet above the tube. Experience showed that this eliminated the distortion and introduced no appreciable additional change in the beam.

PROCEDURE

I. Comparison method

The first method consisted of the comparison of the Hall e.m.f. with an ohmic potential difference as nearly as possible in phase with the current in the tellurium plate and proportional to it. Comparison of the magnitudes of the two voltages was followed by a comparison of their phases. The magnitude comparison involved merely the measurement of the oscillograph deflections, first when the Hall e.m.f. was placed across the amplifier A_H and then when, by a switch, connections were shifted to place the ohmic potential difference across the same amplifier.

A similar plan was followed in the phase comparison, but in this case the second amplifier A_C was utilized. With the ohmic potential difference on the vertically-deflecting plates of the oscillograph, the phase of the comparison voltage E_C was varied by variation of condenser C_P until there was no phase difference. By the shift of connections already mentioned the Hall effect itself could now be amplified by the same amplifier and placed across the vertically-deflecting plates. By this means the phase of the Hall e.m.f. could be compared with that of E_C and thus with that of the ohmic potential difference. Any phase difference showed itself in an opening of the straight line into an ellipse. Measurements of the amplitude of the vertical and horizontal deflections separately and of the major and minor axes of the ellipse gave data for phase computations.

After several other methods of obtaining the ohmic potential difference had proved unreliable, it was realized that a potential difference E_B along the edge of the plate itself, obtained by displacing the movable pin contact P a fixed distance from its initial position P_1, was certain to be in phase with the

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[6] Bedell and Reich, Journal Am. Inst. E. E. **46,** 563 (1927).

[7] Bedell and Kuhn, Rev. Sci. Inst. **1,** 227 (1930).

[8] Wood, Rev. Sci. Inst. **3,** 378 (1932).

[9] Bedell and Kuhn, Phys. Rev. **36,** 993 (1930).

current and proportional to it, providing only that the inductive reactance of the plate was negligible in comparison with its resistance. Calculation indicated that this was true.

II. Null method

The second or null method of observation, obviously suggested by the other method, employed the potential difference E_B along the side of the plate to balance out the Hall e.m.f. Accordingly with the amplifier A_H connected to terminals P and Q on an equipotential line as before, the magnetic field was impressed across the plate and the resulting Hall e.m.f. E_H was balanced out by a displacement of point P parallel with the edge of the tellurium plate from its former position P_1 to a new position P_2, so that P_2 and Q lay on the same new equipotential, as shown by zero vertical displacement of the oscillograph. No horizontal deflection whatever was employed. In the comparison method, E_B was made only approximately the same size as E_H; here they were made exactly equal by this adjustment for zero deflection. The amplifier and oscillograph were thus used here only as a potential-detecting instrument.

The location of P_2 should be dependent only on the angle of rotation of the equipotentials, and thus should be independent of the current and of the actual size of the Hall e.m.f. and consequently independent of the amplification. With P_2 left fixed, the frequency was varied in order to observe whether P_2 and Q still remained at the same potential.

RESULTS

A detailed summary of some of the experimental results obtained by each method will now be given.

In the first place, the production of ellipses and straight lines on the oscillograph screen showed that the frequency of the Hall effect was the same as that of the current in the tellurium plate. There was hardly any doubt that this result was to be expected, yet apparently no direct observations of this sort had been made previously. Furthermore the fact that the ellipses and lines were undistorted showed that the wave-form of the Hall e.m.f. was essentially the same as that of the current producing it.

Values obtained in the $E_H - E_B$ comparison are given in Table I. The numerical values of the oscillograph deflections recorded are of no particular

TABLE I. *Amplitude and phase comparisons at different frequencies.*
$I=0.48$ amp. $H=3100$ gauss.

f (c.p.s.)	Magnitude			Phase			
	$2E_H$	$2E_B$	E_H/E_B	$2E_H$	$2E_c$*	$2b$	ϕ
750	7.63 cm	7.48 cm	1.02	7.5 cm	7.4 cm	<0.3 cm	$<3^\circ 20'$
2750	7.34	7.19	1.02	7.8	7.6	<0.3	$<3^\circ 10'$
5500	7.53	7.34	1.03	7.4	7.2	<0.3	$<3^\circ 20'$
10000	7.27	7.25	1.00	7.2	7.2	<0.3	$<3^\circ 30'$
19200	6.30	6.10	1.03	6.1	6.3	<0.3	$<3^\circ 50'$
25000	2.75	2.70	1.02	2.9	3.0	<0.3	$<8^\circ 20'$

* Adjusted to phase equality with E_B.

significance; it is the variation of their ratio with frequency that is of interest. The column headed ϕ gives the phase angle between E_B and E_H, as computed by the method given below from the experimental observations in the preceding three columns.

In the null method, when P was placed at the equipotential point P_2 at a given frequency by making the oscillograph deflection zero, it was observed that at each other frequency within the range studied the oscillograph deflection, if any, was too small to be measured. It could not have been greater than 0.2 cm. This held true even when the amplification (at all but the frequencies above 10,000 c.p.s.) was increased to a value which would have produced a Hall e.m.f. deflection three or four times those previously used (in the comparison method). At frequencies above 10,000 c.p.s. the full amplification obtainable was used at all times.

A similar test was also made without the magnetic field, and, to the same precision as before, the location of P_1 was also found to be unchanged with frequency.

The distance between P_1 and P_2 when a field of 2900 gauss was used was observed to be approximately 0.022 cm. This corresponds to an angle θ of rotation of the equipotentials of about 38′.

INTERPRETATION OF RESULTS

Combining the Hall equation

$$E_H = R_H \cdot (H/d) \cdot I \tag{1}$$

with

$$E_B = IZ_B \tag{3}$$

where Z_B is the impedance between the two points P_1 and P_2 along the edge of the tellurium plate, we obtain

$$E_H/E_B = (H/d)(1/Z_B)R_H. \tag{4}$$

Thus the question of the constancy of Z_B with frequency is seen to be of importance here.

The inductance of the tellurium plate was calculated[10] to be 0.024 microhenry, corresponding to an inductive reactance of 0.0038 ohm at 25,000 c.p.s. The value of the resistance of the plate, measured directly, (2.5 ohms) is about 660 times this value, and hence Z_B should be independent of frequency within the precision of the measurements. The constancy of the E_H/E_B ratio as shown in Table I then indicates that there is no change in R_H with frequency.

The question of the determination of phase from measurements of a given ellipse has already been discussed elsewhere.[11] The formula developed there is

$$\sin \phi = ab/A_x A_y \tag{5}$$

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[10] Bureau of Standards Circular No. 74 (*Radio Instruments and Measurements*), page 246.

[11] Wood, Rev. Sci. Inst. **2**, 644 (1931).

where a and b are the semi-axes of the ellipse, and A_x and A_y half the horizontal and vertical deflections respectively. Measurements of all four quantities were made, and the phase angle calculated by this formula. The four quantities are also connected by the relation:

$$A_x^2 + A_y^2 = a^2 + b^2. \tag{6}$$

This relation was used as a check on the measurements.

The phase angle ϕ_{BI} between E_B and the current I should be such that

$$\tan \phi_{BI} = X_B/R_B. \tag{7}$$

The argument to show that this ratio of inductive reactance X_B to resistance R_B along the edge of the plate is negligible has already been given in the discussion of the magnitude ratio E_H/E_B.

Thus the phase of E_B is negligibly different from that of I. From Table I it may be seen that ϕ the phase angle between E_B and E_H, if different from zero, is smaller than can be detected by this method.

In the null method the simple fact that the locations of P_1 and P_2 are unchanged with frequency variation, leads to the conclusion that both magnitude and phase of the Hall effect are independent of frequency.

Since the oscillograph deflection remained zero to within 2 mm in, say, 150 mm at frequencies of 10,000 c.p.s. or below, we may say that the Hall coefficient is constant (or proportional to the impedance, if our calculations in regard to the negligibility of inductive reactance should be in error) to within 1.5 percent. At a frequency of 25,000 c.p.s. we may say that it is constant to within 2 mm in 30 mm or 7 percent. Thus this method checks with greater precision the result of the comparison method in finding no change of Hall coefficient with frequency.

The very fact that it is possible to find a point P_2 at the same potential as Q, when the magnetic field is turned on, shows that there is no phase difference between the Hall e.m.f. and the current producing it. Calculations indicate that a phase difference of 2° could have been detected at frequencies below 20,000 c.p.s. and a phase difference of 4° at 25,000 c.p.s.

Magnitude of the Hall Coefficient

Although the primary object of this work was merely to observe changes in the magnitude and phase of the Hall effect with frequency, no particular attention being paid to making a careful measurement of the actual value of the Hall coefficient R_H, it is possible to make an approximate calculation of its value, from observations made in a comparison method similar to that already described, but which involved the use of the potential across a small series resistance.

The comparison with the small series resistance proved unsatisfactory only at the higher frequencies. Therefore, since the average low-frequency value of the E_H/E_R ratio was 1.115 and $Z_R = R_R = 0.005$ ohm, we are able to utilize an equation similar to that already derived for the E_H/E_B ratio, namely,

$$E_H/E_R = (H/dZ_R)R_H \tag{8}$$

in order to calculate R_H. Other numerical values are $H=2900$ gauss, and $d=0.25$ cm, the latter dimension being somewhat uncertain. Thus we find for the Hall coefficient

$$R_H = 481$$

in reasonable agreement with values reported by other observers working exclusively with direct currents, as given in Table II (taken from Campbell's monograph[1]).

TABLE II. *Tellurium Hall coefficient.*

Temp.	H	R_H	Observer
20°C	2800	530	Ettingshausen and Nernst
27.1°	Immaterial	784	Wold
65°	4000	430	Lloyd
—	—	500	Zahn
21.8°	8800	I 536	Smith
20.3°	8800	II 621	Smith

CONCLUSIONS

It would seem manifestly desirable to continue the work to higher frequencies. In view of the radical changes in technique rendered necessary at radiofrequencies, this was not attempted in the present investigation. The null method, in particular, would seem to lend itself readily to this extension, and work upon it is now in progress.

The combination of two amplifiers and oscillograph offers, it would seem, a powerful tool for the investigation at different frequencies, of the phase and magnitude of small voltages other than the Hall e.m.f. The comparison procedure here outlined could be followed with little change.

Finally, the author desires to express heartfelt gratitude to Professor Merritt, who suggested the problem and without whose continued interest it could not have been carried to completion, as well as to many other friends whose advice and suggestions have been extremely helpful.